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On Pro-p Cappitt Groups with finite exponent

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To Pavel Zalesskii on his 60th birthday

Abstract. A pro-p Cappitt group is a pro-p group G such that $\widetilde{S}(G) = \overline{\langle L \leqslant_c G \,|\, L \ntrianglelefteq G\rangle}$ is a proper subgroup (i.e. $\widetilde{S}(G) \neq G$). In this paper we prove that non-abelian pro-p Cappitt groups whose torsion subgroup is closed and it has finite exponent. This result is a natural continuation of main result of the first author [7]. We also prove that in a pro-p Cappitt group its subgroup commutator is a procyclic central subgroup. Finally we show that pro-2 Cappitt groups of exponent 4 are pro-2 Dedekind groups. These results are pro-p versions of the generalized Dedekind groups studied by Cappitt (see Theorem 1 and Lemma 7 in [1]).

Keywords. Generalized Dedekind groups, pro-p Cappitt groups, torsion groups.

2020 Mathematics Subject Classification. 20E34, 20E18.

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1. Introduction

Profinite groups are totally disconnected compact Hausdorff topological groups. Such groups can be seen as projective limits of finite groups (see Ribes–Zalesskii [9] or Wilson [12]).

We recall that a totally disconnected compact Hausdorff group G is called a profinite torsion group (or periodic) if all of its elements are of finite order.

One motivation for studying these groups is the following open question due Hewitt–Ross (see Open Question 4.8.5b in [9]): Is a profinite torsion group necessarily of finite exponent?

In celebrated paper [13], Zel′manov proves that a finitely generated pro-p torsion group is finite. Therefore the question above is open for infinitely generated torsion groups.

In [3] and [4], Herfort proves that a profinite group G whose order is divisible by infinitely many different primes has a procyclic subgroup with the same property. Otherwise if G is a profinite torsion group then the order of G is divisible by only finitely many distinct primes.

As main result in this paper we prove that a infinitely generated non-abelian pro-p Cappitt group has finite exponent. This result is the profinite version of second part of Theorem 1 in [1]. In [7] the first author of this paper proves the first part of the our main result (see Theorem A). Also we obtained other related results of independent interest (see Theorem B).

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2. Preliminaries

The standard notation is in accordance with [7], [9] and [12].

We recall that an abstract group (finite or infinite) is called a *Dedekind group* if every subgroup is normal. As in [1], for any group G, we define $S(G)$ to be the subgroup of G generated by all the subgroups which are not normal in G, i.e. $S(G) = \langle N \mid N \leq G \text{ and } N \ntrianglelefteq G\rangle$. Note that G is a Dedekind group if and only if $S(G) = \{1\}$. Also $S(G)$ is a characteristic subgroup of G and the quotient group of G by $S(G)$ is a Dedekind group. A group is to be a *Cappitt group* if it satisfies $S(G) \neq G$, these groups are generalizations of Dedekind groups (see for example Kappe and Reboli [5] or Cappitt [1]).

In the profinite version, as in [7] and [8], a *profinite Dedekind group* will be a profinite group in which every closed subgroup is normal. A profinite non-abelian Dedekind group will be called a *profinite Hamiltonian group*.

Remark 1. If each procyclic subgroup of a profinite group G is normal, then G is a profinite Dedekind group (see Remark 1, p. 90-91 in [8]).

A. Porto and V. Bessa [8] classified these groups as follow

Theorem 2. *A profinite group G is Dedekind if, and only if, G is abelian or there exists a finite set of odd primes J and a natural number e such that*

$$G \cong Q_8 \times \widetilde{E} \times \prod_{p\in J}\left(\prod_{i=1}^{e}\left(\prod_{m(i,p)} C_{p^i}\right)\right),$$

where Q_8 is the quaternion group of order 8, $\widetilde{E}$ is an elementary abelian pro-2 group and each $m(i,p)$ is a cardinal number. In particular, if G is a profinite Hamiltonian group then G has finite exponent.

Define G' as the topological closure in G of the abstract commutator subgroup $[G,G] = \langle [g,h] \mid g,h \in G\rangle$, where $[g,h] = g^{-1}h^{-1}gh$ is the commutator of the elements $g,h \in G$. The abelianization of group G will be denoted by $G^{ab} = G/G' = G/\overline{[G,G]}$. If G is a profinite group, G^{ab} is an abelian profinite group.

Definition. *A* profinite Cappitt group *is a profinite group G such that $\widetilde{S}(G) = \overline{\langle L \leqslant_c G \mid L \ntrianglelefteq G\rangle}$ is a proper subgroup of G.*

This profinite version was defined firstly in [7]. For finite groups we have that $\widetilde{S}(G) = S(G)$.

Using Theorem 2, Porto gave the following characterization of profinite Cappitt groups.

Theorem 3. *Let G be a profinite group satisfying $\{1\} \neq \widetilde{S}(G) \neq G$. Then G can be expressed as direct product $\widetilde{H} \times \widetilde{K}$, where $\widetilde{H}$ is a pro-p Cappitt group for some prime p and $\widetilde{K}$ is a profinite Dedekind group that does not contain elements of order p. Such G is nilpotent profinite of class at most 2.*

Denote by $\mathbb{Z}_p$ the p-adic integers. Consider $|z|$ the order of an element z of a group G. For a group G we denote $\mathrm{tor}(G) = \{x \in G \mid |x| < \infty\}$ the subset of the elements of torsion of G. As in the abstract case (see 16.2.7 in [6]), when G is a nilpotent pro-p group, then $\mathrm{tor}(G)$ is a subgroup of G (not necessarily closed). A main result of this paper is the following

Theorem A. *Let G be a non-abelian pro-p Cappitt group. Then G' is a procyclic central subgroup. Moreover, if $\mathrm{tor}(G) \leqslant_c G$ then G has finite exponent.*

Remark 4. The Theorem A has immediate proof when G is a pro-2 Hamiltonian group since $G' = C_2$ is central because G has nilpotent class 2 (see Corollary 1 in [7]), and also G has finite exponent by Theorem 2. The condition to be non-abelian is necessary in the Theorem A since $\mathbb{Z}_p$ is an abelian non-periodic group satisfying all hypothesis.

Now we present some preliminaries results that will are used throughout in the proof of the main theorem.

Lemma 5. *Let $G = \varprojlim_{i\in\mathscr{I}}\{G_i,\phi_{ij}\}$ where $\{G_i,\phi_{ij},\mathscr{I}\}$ is a surjective inverse system of finite groups (with discrete topology). Then*

$$\widetilde{S}(G) = \varprojlim_{i\in\mathscr{I}}\{S(G_i),\phi_{ij}|_{S(G_i)}\},$$

where $\{S(G_i),\phi_{ij}|_{S(G_i)},\mathscr{I}\}$ is the associated surjective inverse system.

Proof. See Proposition 1 in [7]. $\square$

The following facts are direct consequences of the definition of subgroups $S(G), S(L)$, $S(G/J)$ and the Correspondence Theorem. In the case of abstract groups, these can be found in remark on page 312 in [1] or Lemma 2.2 in [5].

Remark 6. If G is an abstract group with $S(G) \neq G$ and J is a normal subgroup of G contained in $S(G)$, then $S(G/J) \neq G/J$, furthermore if L is a subgroup of G containing elements of $G \setminus S(G)$ then $S(L) \neq L$.

Analogously to the abstract case, similar results can be shown in the category of profinite groups, as follows.

Remark 7. If G is a profinite Cappitt group (i.e. $\widetilde{S}(G) \neq G$) and J is a closed normal subgroup of G contained in $\widetilde{S}(G)$, then $\widetilde{S}(G/J) \neq G/J$, furthermore if L is a closed subgroup of G containing elements of $G \setminus \widetilde{S}(G)$ then $\widetilde{S}(L) \neq L$.

The following result is similar to Lemma 1 in [7], we will repeat the same argument for the convenience of the reader.

Lemma 8. *Let G be as in the Lemma 5. If $G \neq \widetilde{S}(G)$ then $G_i \neq S(G_i)$, $\forall\ i \in \mathscr{I}$.*

Proof. Suppose that there is a G_i such that $\widetilde{S}(G_i) = S(G_i) = G_i$. Denote by $\phi_i : G \twoheadrightarrow G_i$ the continuous canonical projection of the inverse limit described in Lemma 5. By assumption G_i is generated by all its non-normal subgroups. Let $\overline{L}$ be one of these subgroups and consider L_i its inverse image by ϕ_i, clearly L_i is a non-normal closed subgroup of G containing $\ker(\phi_i)$. Therefore $\ker(\phi_i) \leqslant \widetilde{S}(G)$ and by Remark 7 have $G/\ker(\phi_i) \neq \widetilde{S}(G/\ker(\phi_i)) = S(G/\ker(\phi_i))$, so $|S(G_i)| < |G_i| < \infty$, a contradiction. $\square$

In accordance with the notations of the previous lemmas we obtain the following.

Lemma 9. *Let G be a pro-p Cappitt group. Then G' is a procyclic central subgroup.*

Proof. Since Cappitt groups are nilpotent of class at most 2 (see Corollary 1 in [7]) we have $G' = \overline{[G,G]} \leqslant Z(G)$. Consider $G = \varprojlim_{i\in\mathscr{I}}\{G_i,\phi_{ij}\}$ where $\{G_i,\phi_{ij},\mathscr{I}\}$ is a surjective inverse system of finite p-groups (with discrete topology). From Lemma 8 it follows that $G_i \neq S(G_i)$, $\forall\ i \in \mathscr{I}$. Now, if any G_i is abelian we have $(G_i)' = \{1\}$. On the other side, if G_i is a non-abelian p-group then from Theorem 1 in [1], we have $(G_i)'$ is a finite cyclic p-group. Note that $\phi_i(G') = (G_i)'$, $\forall\ i \in \mathscr{I}$, hence from Corollary 1.1.8 in [9] we have $G' = \varprojlim_{i\in\mathscr{I}}\{(G_i)',\phi_{ij}\}$ whence it follows that G' is procyclic. $\square$

The preceding result is a generalization of the Corollary on page 314 in Cappitt [1]. Kappe and Reboli show that the proof given by Cappitt was incorrect and give a new proof for this fact, this is the content of Theorem 4.2 in Kappe and Reboli [5].

Remark 10. If G is a non-abelian pro-p Cappitt group, then G is nilpotent of class 2 (see Corollary 1 in [7]). In this case $G' \leqslant Z(G)$ and therefore $[[x,y],z] = 1$, $\forall\ x,y,z \in G$. By Lemma 5.42 (i) in Rotman (see [11]), we have that $[x^n,y] = [x,y^n] = [x,y]^n$, $\forall\ n \in \mathbb{Z}$.

Lemma 11. *Let G be a non-abelian pro-p Cappitt group. If $a \in G \setminus \widetilde{S}(G)$ has infinite order, then a is central.*

Proof. Let a be an element in $G \setminus \widetilde{S}(G)$ of infinite order, hence $|a| = p^{\infty}$ and $\overline{\langle a \rangle} \cong \mathbb{Z}_p$. If a is not central in G then there is $g \in G$ such that $[a, g] \neq 1$, moreover $g^{-1}ag \in \overline{\langle a \rangle}$ because $\overline{\langle a \rangle} \lhd G$. By Remark 10 we have $g^{-1}aga^{-1} = [g, a^{-1}] = [g, a]^{-1} = [a, g] \in \overline{\langle a \rangle}$. Since $\mathbb{Z}_p$ is a free pro-p group on $\{1\}$, there is a unique continuous epimorphism $\zeta : \mathbb{Z}_p \to \overline{\langle a \rangle}$ such that $\zeta(1) = a$, therefore each element of $\overline{\langle a \rangle}$ can be written as $a^{\lambda} = \zeta(\lambda)$ for some $\lambda \in \mathbb{Z}_p$ (see similar notation in Section 4.1 on [9]). Since $\overline{\langle a \rangle}$ is infinite it follows that ζ is an isomorphism (see Proposition 2.7.1 in [9]). In particular $1 \neq [a, g] = a^{\delta}$ for some $\delta \in \mathbb{Z}_p$. Let $(n_i)_{i \in \mathbb{N}}$ be a sequence of integers converging to δ on $\mathbb{Z}_p$, in other words choose the n_i such that $\lim_{i \to \infty} n_i = \delta$ (note that $\overline{\langle a \rangle} = \mathbb{Z}_p$ is a complete metric space and $\overline{\mathbb{Z}} = \mathbb{Z}_p$). Adapting the argument from Lemma 4.1.1 in [9] to $\mathbb{Z}_p$ we have $\lim_{i \to \infty} [a, g]^{n_i} = [a, g]^{\delta}$. By properties of limits of sequences in metric spaces together with Remark 10 we obtain

$$[a, g]^{\delta} = \lim_{i \to \infty} [a^{n_i}, g] = \lim_{i \to \infty} (a^{n_i})^{-1} \cdot \lim_{i \to \infty} g^{-1} \cdot \lim_{i \to \infty} a^{n_i} \cdot \lim_{i \to \infty} g = [a^{\delta}, g].$$

As G' is central (see Remark 10) we have $[a, g] = a^{\delta} \in Z(G)$. So $[[a, g], g] = [a^{\delta}, g] = [a, g]^{\delta} = 1$, which implies that $a^{2 \cdot \delta} = \zeta(2 \cdot \delta) = 1$, a contradiction because $\mathrm{Ker}(\zeta) = \{0\}$. Therefore a is central in G. □

Proposition 12. *Let G be a non-abelian pro-p Cappitt group and* $\mathrm{tor}(G)$ *is a closed subgroup of G. Then G is a periodic group.*

Proof. Suppose that G contains elements of infinite order. Since G is nilpotent whose class is two (see Corollary 1 in [7]), it is generated by these elements, not all of which lie in $\widetilde{S}(G)$ otherwise we would have $G = \overline{\langle G \setminus \widetilde{S}(G) \rangle} = \overline{\langle \mathrm{tor}(G) \rangle} = \mathrm{tor}(G)$, a contradiction. Let a be an element in $G \setminus \widetilde{S}(G)$ of infinite order. From Lemma 11 a is central. Now if w is a non-central element of $G \setminus \widetilde{S}(G)$ then $aw, a^2 w$ are non-central elements of infinite order, so both lie in $\widetilde{S}(G)$. Thus $a^2 w w^{-1} a^{-1} = a \in \widetilde{S}(G)$, contradicting the choice of a. Therefore every element of $G \setminus \widetilde{S}(G)$ is central and thus G is abelian since $G \setminus \widetilde{S}(G)$ generates G, a contradiction. □

Example. Note that the condition $\widetilde{S}(G) \neq G$ in the Proposition 12 is necessary, for instance $G = \left(\prod_{r \in \mathbb{N}} Z_r\right) \times Q_8$ is a non-abelian pro-2 non-periodic group with $\mathrm{tor}(G) = Q_8$ and $Z_r \cong \mathbb{Z}_2$, $\forall\ r \in \mathbb{N}$. We show that $\widetilde{S}(G) = G$. Fix $n \in \mathbb{N}$, indeed, it is sufficient we consider the following families of procyclic subgroups of G isomorphic to $\mathbb{Z}_2$, that we will denote by

$$I_n = \overline{\langle (0, \ldots, 0, 1_n, 0, \ldots, 0, i) \rangle},$$
$$J_n = \overline{\langle (0, \ldots, 0, 1_n, 0, \ldots, 0, j) \rangle},$$
$$K_n = \overline{\langle (0, \ldots, 0, 1_n, 0, \ldots, 0, k) \rangle},$$

where $1_n := (0, \ldots, 0, 1_n, 0, \ldots, 1)$ is the generator of $Z_n \cong \mathbb{Z}_2$. Observe that

$$(0, \ldots, 0, 1_n, 0, \ldots, 0, i)^j = (0, \ldots, 0, 1_n, 0, \ldots, 0, -i) \notin I_n,$$

so inductively I_n is not a normal subgroup of G. Similarly, it can be shown also that J_n and K_n are not normal subgroups of G for each $n \in \mathbb{N}$. Therefore by the definition of $\widetilde{S}(G)$ we have that I_n, J_n and K_n are contained in $\widetilde{S}(G)$. Now consider the following elements of $\widetilde{S}(G)$, $x_n = (0, \ldots, 0, 1_n, 0, \ldots, i)$ and $y_n = (0, \ldots, 0, 1_n, 0, \ldots, 0, j)$. Note that $x_n \cdot y_n^{-1} = (0, \ldots, 0, -k) := -k$, so $k \in \widetilde{S}(G)$. Similarly $i, j \in \widetilde{S}(G)$ and this implies that Q_8 is contained in $\widetilde{S}(G)$. Since $(0, \ldots, 0, 1_n, 0, \ldots, 0, k)$ and $(0, \ldots, 0, -k)$ are in $\widetilde{S}(G)$ we have $1_n := (0, \ldots, 1_n, 0, \ldots, 0, 1)$ belongs to $\widetilde{S}(G)$, and therefore $\widetilde{S}(G) = G$.

3. Proof of the Theorem A

We are now ready to prove a main result of this paper.

Proof. The fact that G' is a procyclic central subgroup follows from Lemma 9. If G is Dedekind (i.e. pro-2 Hamiltonian group) the result follows immediately from Theorem 2 and Remark 4. From now on we will consider that $\widetilde{S}(G) \neq \{1\}$. Since $\mathrm{tor}(G)$ is closed in G, from Proposition 12 follows that G is a pro-p torsion group. If G is finitely generated as pro-p group then from celebrated Zel′manov Theorem (see Theorem 4.8.5c in [9]) we have that G is a finite p-group and the result follows. Therefore we can suppose that G is an infinitely generated pro-p and $\{1\} \neq \widetilde{S}(G) \neq G$. Also since G is a torsion group and G' is procyclic, it follows from Proposition 2.7.1 [9] that $G' = C_{p^n}$ for some $n \in \mathbb{N}$ fixed.

Suppose by contradiction that G has no finite exponent. So there are p-elements of G, say, $x_1, x_2, \ldots, x_k, \ldots, (k \in \mathbb{N})$, such that $|x_k| \to \infty$ for $k \to \infty$ (elements of unlimited order).

Let $G^{ab} = G/\overline{[G,G]}$ be the abelianization of G. It is straightforward prove that G^{ab} is an abelian pro-p torsion group because G is a torsion group. From Theorem 4.3.8 in [9] we obtain that

$$G^{ab} = \prod_{m(1)} C_p \times \prod_{m(2)} C_{p^2} \times \ldots \times \prod_{m(e)} C_{p^e},$$

where each $m(i)$ is a cardinal number, e is some natural number, with at least one $m(i)$ infinite because G^{ab} is infinitely generated.

For each $g \in G$ we have $(gG')^{p^e} = G'$ since p^e is the finite exponent of G^{ab}. Therefore for all $k \in \mathbb{N}$ we have

$$G' = (x_k G')^{p^e} = x_k^{p^e} G' \Longrightarrow x_k^{p^e} \in G' = C_{p^n}.$$

Thus we obtain

$$({x_k}^{p^e})^{p^n} = x_k^{p^{e+n}} = 1.$$

Therefore the order of each x_k is limited by p^{e+n}, $\forall\ k \in \mathbb{N}$, a contradiction. We conclude that G has finite exponent. $\square$

4. Pro-2 Cappitt groups of exponent 4

The following result shows that the only pro-2 Cappitt groups of exponent 4 are pro-2 Dedekind groups.

Theorem B. *If G is a pro-2 group of exponent 4 then either $\widetilde{S}(G) = \{1\}$ or $\widetilde{S}(G) = G$.*

Proof. Suppose that $\{1\} \neq \widetilde{S}(G) \neq G$. It is clear that not every procyclic subgroup of G is normal, otherwise $\widetilde{S}(G) = \{1\}$ (see Remark 1). Then consider $J = \overline{\langle x \rangle}$ a non-normal procyclic subgroup of G. Let $H = \overline{\langle x, y, z \rangle} \leqslant_c G$ such that $y \notin N_G(J)$ and $z \notin \widetilde{S}(G)$. By construction and Remark 6 we have that H is a 3-generated group of exponent 4 with $\{1\} \neq \widetilde{S}(H) \neq H$ and $\overline{\langle z \rangle} \trianglelefteq_c H$. Since H has exponent 4, we have that H is a finitely generated pro-2 torsion group, it follows from celebrated Zel′manov Theorem that H is a finite non-abelian 2-group (see Theorem 4.8.5c in [9]). From Corollary 1 in Porto [7] and Lemma 9 we have that G and H are nilpotent groups of class 2 with H' being cyclic, so $H' \cong C_2$ or $H' \cong C_4$. Take a minimal set of generators of H chosen to lie outside $\widetilde{S}(H) = S(H)$, say, $\{h_1, h_2, \ldots, h_l\}$, where $l \geq 2$. Note that $H' = \big\langle [h_i, h_j] \mid i, j \in \{1, 2, \ldots, l\}, i \neq j \big\rangle$ (see p. 129 in [10]). Let $H_i = \langle h_i \rangle$ for each $i \in \{1, 2, \ldots, l\}$. Since $h_i \notin S(H)$ we have $H_i \trianglelefteq H$ and so $[h_i, h_j] \in H_i \cap H_j$ for each $i, j \in \{1, 2, \ldots, l\}$. Note that $[h_i, h_j]$ belongs to H_m for every $i, j, m \in \{1, 2, \ldots, l\}$, so $H' \leqslant H_m$ for each $m \in \{1, 2, \ldots, l\}$. If $H' \cong C_4$ we have that $H' = H_1 = H_2 = \cdots = H_m$, a contradiction since H is not abelian. Therefore H' has order 2. Note that H is a finite 2-group and it is 3-generated with exponent 4, so it is well-know that its order is at most 64. A GAP computation using [2] shows that H is either Dedekind or $\widetilde{S}(H) = S(H) = H$, a contradiction. $\square$

For example, a GAP [2] check yields, the SmallGroup(16, 6), SmallGroup(27, 4) and SmallGroup(64, 28) are finite Cappitt groups with commutator subgroup C_2, C_3 and C_4, respectively.

In Corollary 4.4 in [5], Kappe and Reboli prove that if G is a p-group Cappitt, then G' is a finite cyclic p-group. Finally, in view of this result, example above, Lemma 9 and Theorem A, we can pose the following questions.

5. Open questions about profinite Cappitt groups

(1) Are there non-abelian non-periodic pro-p Cappitt groups? If G is not torsion, is G virtually p-adic?

(2) If G is a pro-p group satisfying $\{1\} \neq \widetilde{S}(G) \neq G$, do we always have that $G' \cong C_{p^n}$ for some $n \in \mathbb{N}$?

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