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Três ensaios sobre regulação, telecomunicações e digitalização: perspectivas teóricas, empíricas e institucionais

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Orientador: Professor Doutor Rogério Mazali

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Resumo

Esta tese é composta por três ensaios que abordam, respectivamente, sanção ótima, regulação da concorrência e a digitalização. O primeiro ensaio modela as estruturas de incentivo ótimas nos processos de sanção monetária conduzidos pela autoridade regulatória de telecomunicações no Brasil. A imposição de multas tem sido historicamente uma ferramenta utilizada pelo regulador para fazer cumprir as regulamentações que estabeleceu. Desde 2012, as entidades reguladas têm a prerrogativa de um desconto de 25% na penalidade imposta, desde que não contestem e paguem na primeira instância. Este estudo investiga os comportamentos implícitos do regulador e das entidades reguladas em decisões relacionadas à conformidade com as normas e aos pagamentos de multas, bem como examina, em termos teóricos, as condições para estabelecer esquemas de incentivo ótimos para a rápida e eficiente resolução de infrações regulatórias. Os resultados indicam as políticas *first-best* e *second-best* a serem seguidas pelo regulador. A política *first-best* envolve desenvolvimento institucional ao lado de sanções máximas fixas, enquanto a política *second-best* se relaciona a políticas de incentivo alinhadas em primeira e segunda instância. O estudo oferece orientações para implementar a política *second-best* com base nos parâmetros do modelo. O segundo ensaio analisa o impacto de remédios assimétricos impostos a provedores de serviços com Poder de Mercado Significativo em municípios brasileiros pouco competitivos nos mercados atacadistas de linhas dedicadas, transporte de dados de alta capacidade e infraestrutura de rede de acesso fixo para transmissão de dados. Utilizando uma combinação de análise de Diferenças em Diferenças e técnicas de Propensity Score Matching, este trabalho investiga como o controle de preços no atacado influencia a concentração de mercado, densidade de serviço, participação de mercado de pequenos provedores, investimentos em fibra, queixas de usuários e qualidade percebida, especialmente em relação a preços e excelência do serviço. As descobertas revelam uma variedade de efeitos em relação a remédios diversos e indicadores, culminando em uma proposição que defende a completa desregulamentação do mercado de infraestrutura de rede de acesso fixo para transmissão de dados, a parcial desregulamentação de linhas dedicadas e um foco intensificado no mercado atacadista de transporte de dados de alta capacidade. O terceiro ensaio analisa como a mudança tecnológica trazida pela digitalização pode impactar a vida econômica em diversos aspectos. A rápida evolução de tecnologias digitais, acelerada pelo impacto da pandemia de COVID-19, está remodelando fundamentalmente as economias e alterando padrões de crescimento. Apesar de seus benefícios, a adoção da transformação digital apresenta desafios, desencadeando disputas sociais com vencedores e perdedores. Este estudo se concentra em cinco características econômicas principais desta transformação tecnológica digital: os impactos macroeconômicos da digitalização, incluindo o i) paradoxo da produtividade; e ii) remodelação de estratégias de desenvolvimento devido à crescente relevância econômica do setor de serviços; iii) ampliação das desigualdades sociais devido à divisão digital; iv) aumento da concentração de mercado liderada pelo avanço das tecnologias da informação; e v) como novas tecnologias, como a inteligência artificial, podem impactar o futuro dos mercados de trabalho. A análise é conduzida por meio de uma revisão da literatura para cada uma dessas características econômicas, concluindo com implicações de política.

Abstract

This thesis consists of three essays that address, respectively optimal sanctioning, competition regulation and digitalization. The first essay models optimum incentive structures in monetary sanctioning processes led by the regulatory telecommunications authority in Brazil. The imposition of fines has historically been a tool used by the regulator to enforce the regulations it has established. Since 2012, regulated entities have the prerogative of a 25% discount on the imposed penalty, provided they do not contest it and pay in the first instance. This study investigates the implicit behaviours of the regulator and regulated entities in decisions regarding compliance with norms and fine payments, as well as to examine, in theoretical terms, conditions for establishing optimal incentive schemes for the quick and efficient resolution of regulatory infractions. The results indicate first and second-best policies to be followed by the regulator. The first-best policy involves institutional development alongside fixed maximal sanctioning, while the second-best policy relates to aligned first and second-instance incentive policies. The study offers guidelines for implementing the second-best policy based on the model's parameters. The second essay examines the impact of asymmetric remedies imposed on service providers with Significant Market Power in Brazilian uncompetitive municipalities within the wholesale markets of dedicated lines, high-capacity data transport, and fixed access network infrastructure for data transmission. Employing a combination of Difference-in-Difference analysis and Propensity Score Matching techniques, this inquiry explores how wholesale price regulation influences market concentration, service density, small providers' market share, fiber investments, user grievances, and perceived quality, particularly with respect to pricing and service excellence. The findings unveil a spectrum of effects across diverse remedies and indicators, leading to a proposition advocating the complete deregulation of fixed access network infrastructure for data transmission, the partial deregulation of dedicated lines, and a heightened focus on the pivotal high-capacity data transport wholesale market. The third essay analyses how technological change brought by digitalization may impact economic life in several separate but correlated features. The rapid evolution of digital technology, accelerated by the impact of the COVID-19 pandemic, is fundamentally reshaping economies and altering growth patterns. Despite its benefits, embracing Digital Transformation presents challenges, triggering societal upheavals with winners and losers. This study focuses on five main economic features of this digital technological transformation: the macroeconomic impacts of digitalization, including the i) productivity paradox; and the ii) reshaping of development strategies due to the augmenting economic relevance of the service sector; iii) widening social inequalities due to the digital divide; iv) increased market concentration led by the ascent of information technologies; and v) how new technologies such as artificial intelligence may impact the future of labour markets. The analysis is conducted through a literature review for each of those economic features, concluding with policy implications.

Sumário

1 APRESENTAÇÃO DA TESE

1.1 INTRODUÇÃO

A regulação econômica tem passado por inovações ao longo do tempo, tanto para acomodar mudanças tecnológicas e de mercado, quanto para internalizar mudanças de políticas baseadas na experiência passada. Essas inovações, contudo, merecem uma maior fundamentação do ponto de vista técnico, cabendo à Academia um papel crucial, fornecendo suporte com base no estado da arte em diversas frentes.

O objetivo desta tese é então explorar como a regulação econômica, especialmente no que concerne à regulamentação de telecomunicações no Brasil, pode ser aperfeiçoada. Isso é feito considerando questões do passado não resolvidas, questões do presente que ainda estão sob revisão regulamentar, e questões do futuro que podem vir a ser objeto de regulação.

Esses três tipos de questões encontram problemas práticos específicos: uma questão do passado se refere ao modo pelo qual o regulador sanciona regulados por não cumprimento de normas; uma questão do presente se refere à eficácia de remédios regulatórios e a estratégia de prioridade do regulador no que tange à sua política pró-competição; e uma questão do futuro se refere à nova forma de regulação, tendo em vista o surgimento e consolidação de novas tecnologias digitais.

Este estudo se debruça sobre cada um desses tópicos em três artigos independentes. O primeiro tema é tratado sob um arcabouço de cunho teórico; o segundo tema é explorado do ponto de vista empírico; e o terceiro tema é investigado sob uma perspectiva institucional.

Assim, busca-se tratar dos respectivos assuntos microeconômicos/regulatórios por meio de três distintas metodologias e enfoques: modelagem em Teoria dos Jogos; modelagem econométrica; e análise qualitativa via revisão literária.

As subseções subsequentes detalham a contextualização e relação de cada artigo com a tese. Ademais, resumem os principais resultados de cada ensaio.

1.2 ARTIGO TEÓRICO "SANÇÕES EM TELECOMUNICAÇÕES: UMA INVESTIGAÇÃO SOBRE A INTERAÇÃO ESTRATÉGICA ENTRE REGULADOR E REGULADO"

Desde sua fundação, a Agência Nacional de Telecomunicações (Anatel) tem autoridade para estabelecer normas específicas para empresas reguladas e capacidade para fiscalizar e impor sanções em casos de não conformidade. Consequentemente, o mecanismo regulatório de sanções monetárias tem servido consistentemente como uma ferramenta convencional em resposta a violações de normas regulatórias.

Em resumo, o regulador estabelece as regras e o regulado decide segui-las ou não. Quanto maior a complexidade da norma e sua aplicação, maior é a divergência de entendimentos sobre se elas foram ou não cumpridas. Neste contexto de incertezas e assimetria de informações, o regulador deve agir *ex-post* às advertências que recebe sobre o não cumprimento das normas, decidindo se sanciona ou não o regulado e, se sim, em quanto. Com a sanção monetária imposta, o regulado decide se paga ou contesta a multa.

Ao longo do tempo, o não cumprimento das multas levou o regulador brasileiro a criar, em 2012, um desconto de 25% nas multas para que as empresas paguem a sanção sem contestação, com o objetivo de acelerar os processos procedimentais e eventualmente levar à realização dos objetivos regulatórios.

O tamanho deste desconto é objeto de um importante debate, uma vez que há evidências de sua eficácia na indução de pagamento na primeira instância, mas uma falta de eficiência, pois pode não ser o fator de redução ideal, considerando os objetivos sociais de forçar as empresas a cumprirem a regulação, em um cenário de política *first-best*, mas também de compensar a sociedade por sua não conformidade, em um cenário de *second-best*. Portanto, a definição de um fator de redução ideal das multas constitui uma ferramenta política relevante, a qual este artigo investiga.

O objetivo deste estudo é aprofundar nas decisões estratégicas tomadas tanto pelo regulado quanto pelo regulador no que diz respeito ao cumprimento das normas setoriais de telecomunicações - pelo primeiro - e à aplicação - pelo último. Dentro desta atribuição, um modelo em Teoria dos Jogos é desenvolvido, sob o qual os mecanismos de incentivo podem ser explorados de forma ótima.

Em um cenário em que a aplicação da lei é certa, o indivíduo que comete um ato prejudicial terá que pagar uma multa porque é estritamente responsável. Este indivíduo pode fazê-lo porque espera obter um ganho com isso, mas ele não internaliza o dano imposto à sociedade ao fazê-lo.

No presente estudo, a ação da empresa regulada de telecomunicações, que é neutra ao risco, é analisada, na qual sua decisão é baseada principalmente em uma avaliação utilitarista de seus ganhos ilícitos versus as possíveis penalidades decorrentes disso. Do ponto de vista do regulador, uma função de utilidade é modelada considerando sua natureza como um fabricante de regras benevolente.

O estudo apresenta um modelo sequencial que é resolvido estrategicamente por técnicas de indução inversa, com foco em três possíveis políticas de incentivo de primeira instância pelo regulador: baixos, médios e altos incentivos de primeira instância. Posteriormente, o sistema de crenças é integrado, levando à análise dos equilíbrios Bayesianos perfeitos.

Este primeiro artigo está estruturado em cinco seções principais, além de uma introdutória. A segunda seção apresenta uma revisão da literatura que apoia a estratégia de modelagem do estudo, a terceira seção delineia o modelo teórico e sua solução, a quarta apresenta as implicações e diretrizes políticas do estudo, a quinta conclui e a sexta serve como uma seção complementar onde são fornecidos cálculos detalhados.

1.3 ARTIGO EMPÍRICO "REGULAÇÃO PRÓ-COMPETIÇÃO EM TELECOMUNICAÇÕES: UMA AVALIAÇÃO DE IMPACTO"

Os serviços de telecomunicações eram predominantemente fornecidos por empresas estatais até a década de 1980, quando uma onda de liberalização do mercado começou, levando à privatização do setor em países desenvolvidos e em desenvolvimento. Essa mudança trouxe mudanças estruturais e institucionais nas relações Estado-Mercado, que deram origem ao conceito atual de regulação e à criação de agências reguladoras em todo o mundo.

A agência reguladora brasileira de telecomunicações, a Agência Nacional de Telecomunicações (Anatel), foi fundada durante esse contexto de privatização, já no final do século, quando forças políticas se uniram permitindo que essas mudanças ocorressem. A transição para um Estado Regulador exigiu a implementação de novos instrumentos governamentais para evitar o poder monopolístico entre os novos incumbentes privatizados. Como resultado, esse marco regulatório no Brasil capacitou a Anatel a reavaliar periodicamente a regulamentação, visando promover a competição e garantir a conformidade com os desenvolvimentos tecnológicos e de mercado.

Desde então, a Anatel adotou uma abordagem regulatória pró-competição, com foco especial em sua regulamentação de metas de competição, conhecida como “Plano Geral de Metas de Competição” (PGMC), aprovada em 2012. O PGMC passou por sua primeira revisão em 2018. Nesta versão revisada, a Anatel fez um esforço para adaptar vários remédios nos mercados atacadistas de acordo com o nível de competição observado no setor varejista.

O princípio subjacente desta regulamentação é que a competição é avaliada no nível varejista e, dependendo da extensão dos problemas identificados lá, remédios regulatórios são impostos aos provedores com poder de mercado significativo (PMS) no nível atacadista. Assim, a política regulatória pró-competição no setor de telecomunicações brasileiro desde 2018 pode ser descrita como um processo de ajuste fino, abordando diferentes questões de competição e oferecendo soluções sob medida por meio de suas intervenções regulatórias.

Dado o rápido avanço tecnológico em toda a cadeia de valor das Tecnologias da Informação e Comunicação (TICs) e telecomunicações, bem como a profunda dinâmica de mercado, a calibração contínua da política é crucial. Além disso, é importante avaliar o impacto de instrumentos regulatórios anteriores, seguindo preferencialmente procedimentos empíricos estabelecidos, aderindo às diretrizes de melhores práticas para avaliação de desempenho.

O quadro regulatório foi projetado de tal forma que os municípios que enfrentam problemas de competição mais pronunciados recebam medidas de controle de preços no atacado, um remédio regulatório mais intensivo, enquanto outros com características econômicas e sociais semelhantes não o fazem.

O design da regulamentação permite então a avaliação das intervenções identificando grupos tratados e de controle, possibilitando a avaliação dos impactos causados por regulamentos específicos. Ao examinar esses tratamentos diferenciais, este segundo artigo busca obter *insights* sobre a eficácia e os resultados das medidas regulatórias implementadas, a partir de um modelo econométrico.

A estratégia empírica empregada neste estudo utiliza a técnica estatística de diferenças em diferenças para avaliar os impactos regulatórios na banda larga fixa em municípios brasileiros. Ademais, é feito um pareamento via *propensity score* entre os grupos de tratamento e controle. Especificamente, a análise se concentra em municípios que foram submetidos a regulação assimétrica no controle de preços no atacado de três mercados relevantes diferentes, comparando-os com municípios que não passaram por tal regulação. Ao empregar este método, o estudo visa medir e entender os efeitos das intervenções regulatórias nos serviços de banda larga fixa em diferentes áreas geográficas.

O artigo empírico inclui uma seção sobre detalhes regulatórios da política pró-competição em telecomunicações no Brasil, uma revisão da literatura, a metodologia, apresentação dos resultados dos modelos econométricos, conclusões e informações complementares.

1.4 Artigo de revisão de literatura "Mudança Tecnológica e suas Características Econômicas na Era Digital: Revisão da Literatura e Diretrizes"

A rápida evolução da tecnologia, esperada para se intensificar devido ao impacto da pandemia de COVID-19, está alterando fundamentalmente as economias e seus padrões de crescimento. As tecnologias digitais são transformadoras não apenas em termos econômicos, mas também socialmente, alterando modelos de negócios, relações humanas e estruturas institucionais.

A mudança tecnológica hoje em dia não pode ser dissociada do grau de Transformação Digital que alcançou todos os setores das economias e com o aspecto revolucionário que está assumindo. Isso é cada vez mais significativo desde a revolução da Tecnologia da Informação e Comunicação (TIC) nos anos 70. Enquanto a revolução industrial levou um século para influenciar a globalização, a revolução das TICs levou uma década, e com a Transformação Digital, esses fenômenos agora ocorrem simultaneamente. Enquanto o progresso tecnológico nas Revoluções Industriais da Manufatura era sobre a duplicação da matéria, na Economia Digital, agora é sobre a duplicação de elétrons, cuja capacidade técnica tem experimentado um crescimento consistente de dois dígitos por quase quatro décadas.

A revolução das TICs marcou uma mudança fundamental onde as telecomunicações, antes consideradas periféricas, agora se destacam como elemento central e indispensável, pois reduziu radicalmente o custo de mover ideias. A probabilidade de que as tecnologias digitais possam de fato assumir características de Tecnologia de Propósito Geral (GPT), por permitirem aplicações em diferentes indústrias e setores, destaca o impacto inovador que podem exercer, comparando este momento com as revoluções industrial e de TICs.

No entanto, esse ritmo de evolução tecnológica inevitavelmente desencadeia convulsões sociais, levando tanto a beneficiários quanto a pessoas adversamente afetadas. O artigo de revisão de literatura analisa os obstáculos apresentados pela Transformação Digital sob cinco principais aspectos econômicos e fornece diretrizes para os ramos regulatórios e legislativos enfrentarem alguns de seus problemas.

Uma questão macroeconômica que pode reganhar destaque no contexto da crescente digitalização é a versão atualizada do paradoxo da produtividade moderna, que afirma que o rápido desenvolvimento das tecnologias de informação e comunicação nas últimas duas décadas coincidiu com uma desaceleração generalizada no crescimento da produtividade agregada.

Outro aspecto relevante é como a robótica e a telemigração em um quadro de crescimento liderado por serviços podem reformular as estratégias de desenvolvimento e o balanço de pagamentos de países em desenvolvimento de maneira positiva ou negativa. Em suma, a DigiTech pode permitir que muitos mercados emergentes exportem diretamente a fonte de sua vantagem comparativa, dependendo de como suas instituições assim o permitam.

Além disso, as desigualdades sociais podem ser ampliadas com o aumento da divisão digital, quando essas tecnologias não são adotadas por todos, mantendo uma lacuna entre indivíduos, famílias, empresas e áreas geográficas em diferentes níveis socioeconômicos, tanto em relação à oportunidade quanto à capacidade de acessar TICs e ao uso da Internet para uma ampla variedade de atividades.

No âmbito da ordem econômica, o surgimento das tecnologias da informação pode potencialmente alimentar a concentração de poder de mercado nas mãos de poucas empresas digitais dominantes. A dinâmica de mercado influenciada por essas mudanças pode levar a economia a um equilíbrio menos competitivo, no qual essas entidades poderosas poderiam explorar sua vantagem, criando distorções que poderiam contrariar os impactos positivos da inovação. Esses trade-offs de criar incentivos para a inovação versus o risco potencial de diminuição do bem-estar social devem ser considerados nas respostas às evoluções tecnológicas e de mercado.

Outro aspecto relevante da mudança tecnológica digital diz respeito ao seu impacto no mercado de trabalho. Por um lado, há argumentos alarmistas de que os avanços iminentes em IA e robótica significarão o fim do trabalho humano, enquanto muitos economistas, por outro lado, afirmam que, porque os avanços tecnológicos no passado eventualmente aumentaram a demanda por trabalho e salários, não há motivo para se preocupar que desta vez será diferente.

O objetivo deste terceiro artigo é então analisar a literatura sobre esses cinco aspectos econômicos principais relacionados à Transformação Digital: produtividade, globotics, divisão digital, poder de mercado e futuro do trabalho, concluindo por implicações de política para o caso brasileiro.

2 CONCLUSÃO

Cada artigo possui um objetivo distinto para questões independentes entre si, mas complementares quando se pensa em termos macro no arcabouço institucional em que o regulador se insere.

Os resultados do artigo teórico indicam políticas *first* e *second-best* a serem seguidas pelo regulador. A política *first-best* envolve o desenvolvimento institucional ao lado de uma sanção máxima fixa, enquanto a política *second-best* diz respeito a políticas de incentivo de primeira e segunda instância alinhadas. O estudo oferece diretrizes para implementar as políticas *first* e *second-best* com base nos parâmetros do modelo.

As descobertas do artigo empírico, por sua vez, são derivadas de quatro modelos distintos. Elas oferecem insights sobre os efeitos de vários remédios regulatórios nos indicadores de desempenho do mercado e competição.

O Modelo 1 explora remédios regulatórios para o mercado de exploração industrial de linhas dedicadas. O Modelo 2 avalia os efeitos das medidas de controle de preços de transporte de dados de alta capacidade, contendo evidências de um impacto significativo no desempenho do mercado e alguns indicadores de experiência do consumidor, a serem confirmados no Modelo 4. O Modelo 3, por outro lado, analisa os instrumentos regulatórios da infraestrutura de rede de acesso fixo para transmissão de dados via par de cobre em taxas de transmissão iguais ou inferiores a 12 Mbps, revelando a ausência de resultados impactantes. Por último, no Modelo 4, a integração dos dois últimos remédios de mercado atacadista mencionados relata resultados que implicam uma competição aprimorada no mercado varejista.

As descobertas revelam uma série de efeitos em vários remédios e indicadores, culminando em uma recomendação que apoia a completa desregulamentação da infraestrutura de rede de acesso fixo para transmissão de dados, desregulamentação parcial de linhas dedicadas, juntamente com um foco enfatizado no mercado atacadista crítico de transporte de dados de alta capacidade. Este estudo traz à tona a importância da avaliação da padronização pró-competitiva, o que exige um nível elevado de escrutínio e reflexão quando se trata de elaborar políticas e fazer os ajustes regulatórios necessários.

O artigo de revisão de literatura, por fim, analisa diversos canais pelos quais a Transformação Digital pode agir e demandar ação por parte do poder público no que tange a endereçar falhas de mercado, prevenir perturbações sociais e promover cooperação internacional.

Algumas políticas são ressaltadas, tais como aquelas que enderecem as causas de clivagem de produtividade entre empresas, incluindo a inclusão digital e capacitação para melhor aproveitamento dos benefícios de tecnologias digitais; a própria oferta de serviços de telecomunicações a preços acessíveis, diminuindo a divisão digital de primeiro nível; a promoção da capacidade de aproveitamento dos ganhos de TICs e tecnologias digitais, por meio de formação de capital humano; a possibilidade de contar com novas formas de regulação para endereçar possíveis novas falhas de mercado; estratégias de desenvolvimento econômico, considerando novas dinâmicas de mercado com a crescente adoção de tecnologias digitais; e balanceamento entre incentivo à inovação e proteção social, considerando efeitos dessas novas tecnologias.

Em suma, os resultados apontam alternativas para alcançar maior eficiência a partir dos instrumentos regulatórios existentes e passíveis de alteração regulamentar, assim como de que forma novos objetivos sociais podem ser implementados.

3 ARTIGOS INTEGRANTES DA TESE

Sanctions in Telecommunications: an Inquiry into the Strategic Interaction between Regulator and Regulated

Sanções em Telecomunicações: Uma Investigação sobre a Interação Estratégica entre Regulador e Regulado

ABSTRACT

This study models optimum incentive structures in monetary sanctioning processes led by the regulatory telecommunications authority in Brazil. The imposition of fines has historically been a tool used by the regulator to enforce the regulations it has established. Since 2012, regulated entities have the prerogative of a 25% discount on the imposed penalty, provided they do not contest it and pay in the first instance. This study investigates the implicit behaviours of the regulator and regulated entities in decisions regarding compliance with norms and fine payments, as well as to examine, in theoretical terms, conditions for establishing optimal incentive schemes for the quick and efficient resolution of regulatory infractions. The results indicate first and second-best policies to be followed by the regulator. The first-best policy involves institutional development alongside fixed maximal sanctioning, while the second-best policy relates to aligned first and second-instance incentive policies. The study offers guidelines for implementing the second-best policy based on the model's parameters.

Keywords: Monetary Sanctions; Deterrence Theory; Game Theory; Telecommunications Compliance and Enforcement; Incentives Mechanisms.

RESUMO

Este estudo modela as estruturas de incentivo ótimas nos processos de sanção monetária conduzidos pela autoridade regulatória de telecomunicações no Brasil. A imposição de multas tem sido historicamente uma ferramenta utilizada pelo regulador para fazer cumprir as regulamentações que estabeleceu. Desde 2012, as entidades reguladas têm a prerrogativa de um desconto de 25% na penalidade imposta, desde que não contestem e paguem na primeira instância. Este estudo investiga os comportamentos implícitos do regulador e das entidades reguladas em decisões relacionadas à conformidade com as normas e aos pagamentos de multas, bem como examina, em termos teóricos, as condições para estabelecer esquemas de incentivo ótimos para a rápida e eficiente resolução de infrações regulatórias. Os resultados indicam as políticas *first-best* e *second-best* a serem seguidas pelo regulador. A política *first-best* envolve desenvolvimento institucional ao lado de sanções máximas fixas, enquanto a política *second-best* se relaciona a políticas de incentivo alinhadas em primeira e segunda instância. O estudo oferece orientações para implementar a política *second-best* com base nos parâmetros do modelo.

Palavras-chave: Sanções Monetárias; Teoria da Dissuasão; Teoria dos Jogos; Conformidade e Fiscalização em Telecomunicações; Mecanismos de Incentivo.

1 INTRODUCTION

The proposition that lawbreaking rates respond to risks and benefits is called the deterrence hypothesis. It is an application of demand theory to one of the most critical issues in law enforcement. The hypothesis asserts that people significantly respond to the incentives institutionally created. Therefore, increasing the resources that society allocates to the identification, conviction, and enforcement of sanctions for offenders should reduce the incidence and social costs associated with behaviours not in compliance with regulatory standards.

Since its foundation, the Brazilian telecommunications regulatory agency (Anatel) has been endowed with the authority to set forth specific norms for regulated firms and the capability to enforce and impose sanctions in cases of non-compliance. Consequently, the regulatory mechanism of monetary sanctions has consistently served as a conventional tool in response to violations of normative standards.

In sum, the regulator sets the rules and the regulated decides on following them or not. The greater the complexity of the norm and its enforcement, the greater is the divergence of understandings on whether they were or not complied. In this context of uncertainties and information asymmetry, the regulator must act ex-post to the alerts it receives on non-compliance of norms, deciding whether to sanction the regulated or not and, if so, on how much. With the monetary sanction imposed, the regulated decides whether to pay or contest the fine.

Throughout time, the non-compliance of fines has led the Brazilian regulator to create, in 2012, a 25% discount on fines for firms to pay the sanction without contesting them, with the purpose of accelerating procedural processes and eventually leading to the achievement of regulatory goals[1].

The size of this discount is of important debate, given that there is some evidence on its efficacy on first instance payment induction, but a lack on its efficiency, since it might not be the optimal reduction factor, considering the social goals on forcing firms to comply with regulation, in a first-best scenario, but also to compensate society by its non-compliance, in a second-best scenario. Therefore, the definition of an optimal factor reduction[2] of fines constitutes a relevant policy tool.

[1] The Regulation on the Application of Administrative Sanctions was approved by the Resolution nº 589 of Board of Directors of Anatel: Resolução nº 589, de 7 de maio de 2012.
[2] This variable shall be denoted in the subsequent section as f.

The purpose of this study is to delve into the strategic decisions made by both the regulated and the regulator on telecommunications sectoral norm compliance – by the former – and enforcement – by the latter. Within this assignment, we develop a theoretical model similar to Polinsky and Shavell (2007) under which the incentive mechanisms could be optimally explored[3].

In a scenario where enforcement is certain, the individual who commits a harmful act will have to pay a fine because he is strictly liable. This individual may do it so because it is expected to obtain a gain from it, but he does not internalize the harm imposed to society by doing so.

In the present study, the action of the risk-neutral telecommunications regulated firm is analysed, in which its decision is based primarily by an utilitarian assessment on its wrongdoing gains versus the possible penalties from it derived. From the regulator's point of view, an utility function is modelled considering its nature as a benevolent rule maker.

The study presents a sequential model that is strategy-wise solved by backward induction techniques focusing on three possible first instance incentive policies by the regulator: low, medium and high first instance incentives. Subsequently, the system of beliefs is integrated, leading to the analysis of perfect Bayesian equilibriums.

The results indicate first and second-best policies to be followed by the regulator. The first-best policy involves institutional development alongside fixed maximal sanctioning, while the second-best policy relates to aligned first and second-instance incentive policies. The study offers guidelines for implementing the second-best policy based on the model's parameters.

To achieve this goal, the study is structured into five main sections, in addition to this introductory one. The second section displays a literature review that supports the modelling strategy of the study, the third section outlines the theoretical model and its solution, the fourth presents the study's policy implications and guidelines, the fifth concludes and the sixth serves as an ancillary section where detailed calculations are provided.

2 LITERATURE REVIEW

According to Becker (1968), the probability and severity of punishment deter crime. Therefore, the fine should be maximal, as it is a costless transfer, while the probability of detection and conviction is costly.

There is a substantial body of literature addressing the optimal probability and magnitude of fines, exploring Becker's (1968) provocative finding that uniformly maximal penalties consistently result in maximal deterrence. Stigler (1970) contends that more severe crimes should incur harsher penalties to establish "marginal deterrence". However, Posner (1985) highlights a trade-off between marginal deterrence and total deterrence.

[3] The model refers to the monetary sanctions of the basic theory when enforcement is certain. The notation was altered to the one it is utilized in the modelling of this study.

Additionally, he emphasizes that society can maintain marginal deterrence with uniform penalties by adjusting the probability of capture.

In this context, a substantial body of more contemporary literature seeks to challenge the conventional conclusion drawn by Becker (1968), suggesting that maximal sanctioning may be nonoptimal.

Polinsky and Shavell (1979), for instance, demonstrate that less than maximal penalties are efficient when there are crimes where the private benefit to the offender exceeds the social cost of the criminal activity. In such cases, it would not be efficient to deter all crimes. If the private benefit of the crime never surpasses the social cost, maximal penalties would still be considered optimal.

Kaplow (1990) states that the optimal fine may not necessarily be the maximum fine when the enforcement cost is sufficiently positive. Both punishment and apprehension incur costs, and, therefore, the allocation of resources between them is determined by weighing the marginal benefit against the marginal cost, akin to a standard optimization problem in economics[4].

To Bebchuk and Kaplow (1993), when the government assesses the difficulty of apprehending individuals only after investing enforcement resources, the optimal sanction is maximal only for those individuals who prove to be the most challenging to apprehend. However, when the government lacks information about the difficulty of apprehension, either before or after allocating enforcement resources, the optimal sanction may be less than the maximum.

These previous models are challenged by two types of criticism (GAROUPA, 1997). The first relates to the fact that their frameworks are predominantly static, lacking the capacity to account for long-term versus short-term considerations. Scholars have then introduced an intertemporal approach to the specification of social welfare, offering a valuable perspective for dynamically designing optimal policies. Two primary findings are commonly explored: past criminal activity tends to foster future crime, and while more severe punishment deters crime over the long term, its impact can be relatively modest in the short term (DAVIS, 1988; LEUNG, 1991, 1995; SAH, 1991). It is worth noting that this study does not delve into intertemporal modelling, as the game is played once, not in repetitive interactions. Despite its intrinsic role in the sanctioning process, forming a continuum of administrative processes that define the extent of monetary sanctions based on various situations, it is perceived that a static model is sufficient to address the issues it aims to tackle.

The second line of criticism pertains to the absence of strategic behaviour, suggesting a need for a more game-theoretic oriented approach (GAROUPA, 1997). Some authors delve into a game-theoretic formulation of criminal decisions and their repercussions, particularly in terms of the deterrence effect stemming from alterations in punishment (TSEBELIS, 1990, 1993; COX, 1994). This study does model strategic behaviour, as it is an objective on defining optimum solutions for first and second instance incentives,

[4] This rationale is utilized in the theoretical model, in the first optimization problem of the Regulator.

considering the policy tools by the regulator, given possible decisions taken by firms on payment, contestability and judicialization.

Friehe and Mungan (2020) demonstrate that regulatory sanctions falling within an intermediate range can lead to subtle issues that are not evident in simple enforcement models. By considering the possibility that firms may face varying probabilities of non-compliance detection, the authors underscore that intermediate sanctions may conflict with aspects of both static and dynamic efficiency, illustrating circumstances under which welfare is better served by a low sanction than an intermediate sanction.

Following Friehe and Mungan (2020), this study pursues the rationale of not only optimal sanctioning but also on optimal incentive schemes for efficient resolution of sanctioning processes in the administrative level.

3 THE MODEL

3.1 BASIC SETUP

The public enforcement of laws, involving the deployment of government agents to identify and penalize violators of legal rules, is a matter of evident significance. For instance, regulators strive to avoid infractions of environmental, safety, consumer protection, and public utilities laws (POLINSKY and SHAVELL, 2007).

Before presenting the model of this study, a basic theoretical setup is introduced based on the model by Polinsky and Shavell (2007). This section discusses the premises that are assumed or not in our model.

The gains of non-compliance Π_I are viewed as excess profits derived from non-compliance to sectoral regulation, $z(\Pi_I)$ is the density of gains among firms, CP is the harm caused to society if the harmful act is committed, M is the fine, Π_R is the standard profit based on regulatory compliance.

The basic model presents that the fine cannot exceed Π_R. Here, it will be considered that the fine will not surpass $\bar{M} < \Pi_R$. In the basic model, the firm will not cooperate with the regulation if its gains outperform the minimum between the fine and the threshold level of gain under the fault-based sanctioning rule $\Pi_I > \min(M, \widehat{\Pi}_I)$. In this study, enforcement is not certain, and thus the firm might not comply with the regulation and still not be sanctioned.

In the basic model, social welfare is determined by subtracting the harm caused from the gains individuals derive from committing the harmful act. Therefore, the problem for the enforcement authority is to maximize social welfare by selecting the optimal fine M. Social welfare is not directly influenced by the imposition of fines, as the payment of a fine is presumed to be a socially costless transfer of money. Given that individuals who engage in harmful acts are those whose gains surpass the fine, social welfare is

$$\int_{\min(M,\widehat{\Pi}_I)}^{\infty} (\Pi_I - CP) z(\Pi_I) d\Pi_I$$

There are costs involved in the sanctioning process. For instance, the two classic types of statistical errors can occur in the public enforcement of law, incurring respective costs for society, which are internalized by the regulator in this study. The Type I error, or false positive, is when an individual who should not be found liable might mistakenly be held responsible — referred to as a “mistaken conviction”. The Type II error, or false negative, is when a firm that should be found liable might mistakenly not be held responsible, referred to as a “mistaken acquittal”.

Png (1986) recalls that, if the probability of detection is fixed, which is the case for this study, increasing the fine becomes necessary to counterbalance the deterrent-diluting effects caused by the error costs.

The overarching challenge in public law enforcement can be conceptualized as the maximization of social welfare. In this context, social welfare denotes the benefits individuals derive from their conduct, subtracted by the costs they bear to prevent harm, the harm they inflict, the expenses incurred in apprehending violators, and the costs associated with imposing sanctions, which may encompass any additional costs related to risk aversion (POLINSKY and SHAVELL, 2007). The authors detail the main issues of these concepts, which are below summarized.

The state faces then four pivotal policy decisions when it comes to law enforcement. The first pertains to the sanctioning rule, where it can be either strict — imposing sanctions whenever harm (or expected harm) is determined — or fault-based, applying sanctions only if the party failed to adhere to a prescribed standard of behaviour or regulatory requirement.

The analysis of this study applies to the fault-based sanctions imposed by the telecommunications regulator of Brazil, after the administrative process indicated a norm violation. Therefore, the sanctioning rule is considered as given.

The second choice involves the nature of the sanction: whether it is monetary, non-monetary, or a combination of both. This study focus solely on the monetary sanctions imposed by the regulator.

A third decision revolves around determining the magnitude of the sanction. This decision is depicted in the first optimization problem of the theoretical model here presented, in which there is the definition on both the magnitude of the sanction (M) and the factor reduction it may take for the process to be finalized in the administrative branch in first instance (f).

Lastly, the fourth choice centers on the probability of detecting offenders and administering sanctions. This probability is contingent on the allocation of public resources towards identifying violators and substantiating their liability, among other factors, which shall be further explored.

This probability was modelled as an exogenous signal of Nature in which information asymmetries are presented and different explanations, as above suggested, can be given to the probability of imposing sanctions.

3.2 GAME DESCRIPTION

The regulator creates norms establishing guidelines and procedures to be followed by the regulated firms. These norms range from consumer protection to pro-competition regulation. The regulated can either comply with the regulation or not, that is, the firm can cooperate or not cooperate with the regulatory authority.

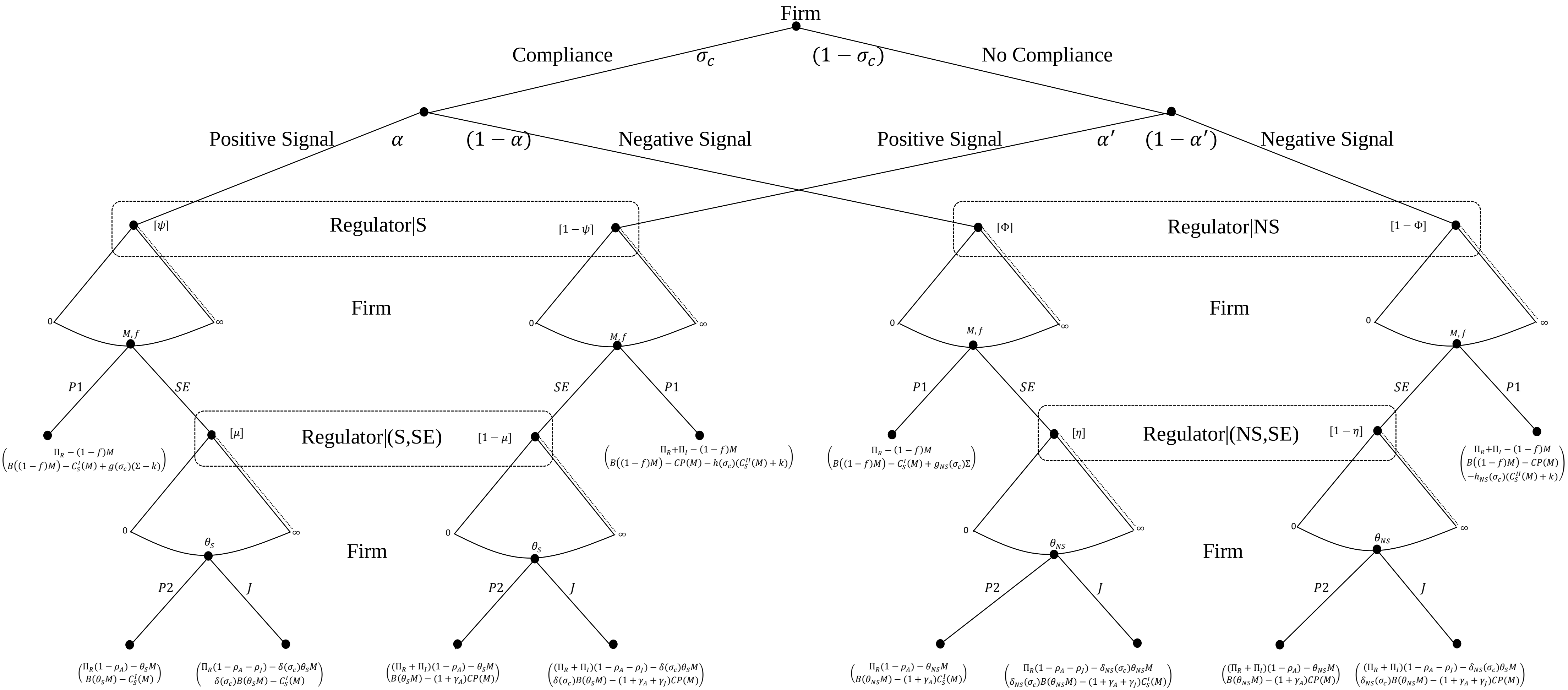
The game starts with this decision of the firm, to comply (or not) with certain regulatory rule. The probability of cooperation chosen by the firm is denoted as σ_c. Afterwards, Nature issues a signal, which can be positive or negative. A positive signal indicates to the Agency that the norm has been broken and a negative one signals that it has not. Note that if the firm cooperates, the positive signal occurs with probability α (conversely the negative signal has probability $1-\alpha$); analogously, if the firm did not cooperate, Nature issues a positive signal with probability α'. In thesis, the odds of issuing a positive sign is higher when the firm hasn't complied with the sectoral regulation, that is, $\alpha' \geq \alpha$.

The Regulator observes the type of the signal, positive or negative, but not the action of the firm, thus, the Regulator must make decisions with imperfect information. The Regulator then decides whether to impose a monetary sanction or not. If the decision is to fine the regulated, the Regulator choses the amount M of the fine. Furthermore, the Regulator can give incentives to a rapid resolution in first instance: if the regulated is penalized by a fine, it is granted a fine reduction factor of order f (a discount of f% on M) if the sanction is paid by the fined firm without being contested. In sum, the decision of the Regulator is then to define the level M of the sanction ($0 \leq M \leq \bar{M}$) and the incentive structure f in an optimization problem, given the unobserved action of the firm (compliance or not compliance).

After the definition of the sanction by the Regulator, the Regulated decides whether to pay the fine in first instance or to contest the decision of the regulatory authority. If the firm pays the fine giving up the right to contest, the game ends and it gets a payoff of its profits minus the fines. In the scenario that the firm complies with the norms, its payoff is $\Pi_R - (1-f)M$, where Π_R represents a "regulatory compliance profit" (henceforth regulatory profit), the firm has excess profits Π_I, having, in the end, a payoff of $\Pi_R + \Pi_I - (1-f)M$. Of course, if the decision of the Regulator is not to sanction, $M = 0$, the game ends as if the Regulated has decided to pay the null fine, not contesting the decision of no penalty.

As for the Regulator, the prospects of the payment occurring in the first instance represents a second-best scenario, compared to the first-best one in which there is no infringement in the first place. The payoff of the Regulator is modelled by an endogenous utilitarian Laffer curve function $B(.)$, in which the utility of the Regulator increases with the sanction received M, but until a certain point.

Figure 1 The Game



Source: Author's elaboration.

In the scenario that the firm has cooperated and paid a fine M in first instance, its payoff is $B\big((1-f)M\big) - C_i^I(M) + g(\sigma_c)(\Sigma - k_i)$, in which the Regulator incurs a false-positive cost of type I error, this cost reasonably being proportionate to the sanction imposed. Function $g(\sigma_c)$, dependent on the cooperation probability, internalizes that even with zero sanction, the Regulator can gain maximum utility Σ, if there was indeed cooperation, deducted by a cost k_i, which is positive $k_S = k$ when the Nature's signal is positive and $k_{NS} = 0$, in case of negative signal.

If the firm has not cooperated in the first place, there is a higher cost associated with society's welfare loss when the social goals internalized by regulation are not pursued. In this case, the Regulator internalizes CP, obtaining a payoff of $B\big((1-f)M\big) - CP(M) - h(\sigma_c)(C_i^{II} + k_i)$.[5] The Regulator internalizes a possible false-negative Type II error, multiplied by $h(\sigma_c)$ and associated with a cost $k_i = k_S = k_{NS} = k$.

It is assumed that the Regulator's utility function takes a quadratic form with an endogenous β parameter, taking the form of $B((1-f)M) = \left(\beta - \frac{1}{2}(1-f)\right)(1-f)M$. This functional form is proposed for simplicity sake, so that $B((1-f)M)$ attains a maximum at $f = 1 - \beta$, in case of an internal solution.

The decision of the firm is to pay in first instance, gaining the f reduction factor, or to contest the sanction, asking for an administrative suspension of the fine. If the decision of the Firm is to contest the sanction, the Regulator finds itself again with an imperfect information decision, having to decide on the plea made by the fined provider. In this decision node, the Regulator determines the new sanction ($M' = \theta M$) after being contested: in a continuum spectrum, the Regulator can either cancel the sanction ($\theta = 0$); reduce the fine ($\theta < 1$); maintain the fine ($\theta = 1$); or increase the sanction ($\theta > 1$). This optimization process is done for each case: in the one in which the Regulator receives a positive signal by Nature, it chooses the optimal θ_S; when there is no signal, the choice is regarding the optimal revision of the fine θ_{NS}.

After the reassessment of the sanction by the regulatory authority, the game ends with the Firm deciding whether to pay the updated fine M' in the administrative level or to challenge the sanction through a judiciary process. If the choice is to pay the updated fine, despite having cooperated in the first place, the Regulated's payoff is $\Pi_R(1-\rho_A) - \theta_i M$. That is, it will have the regulatory compliance profit deducted with two costs: the administrative process cost $\rho_A \Pi_R$, which is modelled as a proportion of Π_R and the payment of the updated fine $\theta_i M (i = S, NS)$. If there is a positive signal, the Firm's payoff is $\Pi_R(1-\rho_A) - \theta_S M$; otherwise, it is $\Pi_R(1-\rho_A) - \theta_{NS} M$. The analogous interpretation holds henceforth for the other payoffs dependent on θ_i. On the other side, if the Regulated has not cooperated, it will count also with the excess profits and the administrative costs associated with it, hence the provider's payoff shall be $(\Pi_R + \Pi_I)(1-\rho_A) - \theta_i M$, $i = S, NS$.

In this case of payment in the second administrative instance, if there has been cooperation of the norm, the Regulator obtains a utility derived from the payment by an

[5] Note that the costs are dependent on the amount of the sanction, based on the notion of socially costly sanctions, presented by Kaplow (1990).

exogenous Laffer curve type, subtracting the associated Error I costs: $B(\theta_S M) - C_S^{ErroI}(M)$ and $B(\theta_{NS} M) - (1+\gamma_A)C_{NS}^{I}(M)$. Conversely, if there hasn't been cooperation and the firm decides to pay the fine right after having contested, the Regulator ends up with $B(\theta_i M) - CP(M)(1+\gamma_A), i = S, NS$, in which there is an additional cost γ_A in this process.

In sum, if the Firm has cooperated, the payoffs are $(U^F, U^A) = (\Pi_R(1-\rho_A) - \theta_i M;\ B(\theta_i M) - C_i^I(M)$ for each one of the signals S and NS. If the Firm has not cooperated with the norms, the payoffs are $(U^F, U^A) = \big((\Pi_R + \Pi_I)(1-\rho_A) - \theta_i M;\ B(\theta_i M) - CP(M)(1+\gamma_A)\big), i = S, NS$.

It is assumed that the Regulator's utility function takes a quadratic form with an exogenous $\hat{a}$ parameter, taking the form of $B(\theta_i M) = \left(\hat{a} - \frac{1}{2}\theta_i\right)\theta_i M$.[6] This functional form is proposed for simplicity sake, so that $B(\theta_i M)$ is maximum when $\theta_i = \hat{a}$.[7]

The Firm may then decide not to pay the fine, taking the case to the court of justice. If the Regulated wins the case, then the fine is abolished. Otherwise, the fine is maintained in its integrity, the probability of conviction in court is $\delta(\sigma_c)$. In this situation, the Firm has an additional cost of the judicial process ρ_J, proportional to its profits. Hence, its payoff after judicialization is $\Pi_R\big(1-\rho_A-\rho_J\big) - \delta^i(\sigma_c)\theta_i M, i = S, NS$, if there is cooperation in the first node and $(\Pi_R + \Pi_I)\big(1-\rho_A-\rho_J\big) - \delta^i(\sigma_c)\theta_i M, i = S, NS$, if there is not.

From the perspective of the Regulator, the infringement of the sectoral legislation alongside judicialization is the worst-case scenario, once it fails on compliance and enforcement. Here, the factor $\delta(\sigma_c)$ also plays a role given that the Regulator's payoff now depends on the court's decision. Therefore, its payoff considering judicialization is $\delta(\sigma_c)B(\theta_S M) - C_S^I(M)$, for a positive sign, and $\delta^{NS}(\sigma_c)B(\theta_{NS} M) - \big(1 + \gamma_A + \gamma_J\big)C_{NS}^I(M)$, for a negative one, if there is cooperation in the first node; and $\delta^i(\sigma_c)B(\theta_i M) - \big(1+\gamma_A+\gamma_J\big)CP(M), i = S, NS$, if there is not.[8] The probability of conviction given non-compliance is higher than the probability of conviction given compliance ($\delta(\sigma_c = 0) > \delta(\sigma_c = 1)$).The idea is that, when the welfare loss is not tackled "within the industry", negative externalities persist, and society bears the costs γ_J of direct disutility to consumers, imbalances in competition in the industry and increasing bureaucracy.

3.3 *GAME SOLUTION*

The game is solved through backward induction. Therefore, the game actions are analysed in the following order: i) the decision of the firm to pay the fine in second instance or to judicialize; ii) the optimal decision of the Agency relating the reassessment of the sanction; iii) the decision of the Firm of whether paying the fine in first instance or contesting it; iv) the decision of the Agency regarding optimal sanctioning structure; v) the compliance decision of the Firm.

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[6] For a more detailed explanation on the modelling of $B(\theta_i M)$ and $B((1-f)M)$, see Appendix B.1.

[7] $\frac{dB}{d\theta} = \hat{a} - \theta = 0 \Leftrightarrow \theta^* = \hat{a}$.

[8] For simplicity, the notation may take the form as $\delta^S(\sigma_c) = \delta(\sigma_c)$.

After considering all possible strategy solutions, attention is drawn to the belief systems of the resulting equilibriums, in which Requirements 1 to 4 are analysed.

Requirement 1: at each information set, the player with the move must have a belief about which node in the information set has been reached by the play of the game. For a non-singleton information set, a belief is a probability distribution over the nodes in the information set; for a singleton information set, the player's belief put probability one on the single decision node.

Requirement 2: given their beliefs, the players strategies must be sequentially rational. That is, at each information set, the action taken by the player with the move (and the player's subsequent strategy) must be optimal given the player's belief at that information set at the other player's subsequent strategies.

Requirement 3: at information sets on-the-equilibrium-path beliefs must be determined by the Bayes' rule and the players' equilibrium strategies.

Requirement 4: At information sets that are off-the-equilibrium-path, beliefs are determined by Bayes' rule and the players' equilibrium strategies where possible.

A Perfect Bayesian Equilibrium consists of strategy profiles and belief systems satisfying Requirements 1 to 4. The objective of the study is to analyse these possible equilibria.

3.3.1 Nature issues a positive signal

3.3.1.1 Judicialization as the final action

3.3.1.1.1 Compliant Firm

The final action of the game pertains to the firm's decision either to pay the updated sanction imposed by the Regulator $\theta_S M$ or to pursue judicial action, seeking more favourable outcomes in the judiciary branch compared to the administrative one.

The firm that has complied with the norms will judicialize if:

$$\Pi_R(1-\rho_A-\rho_J)-\delta(\sigma_c=1)\theta_S M > \Pi_R(1-\rho_A)-\theta_S M$$

$$\Leftrightarrow \ \theta_S > \frac{\rho_J \Pi_R}{M(1-\delta(\sigma_c=1))} = \overline{\theta_S}$$

(Eq. 1)

Putting it into words, the higher the fine imposed compared to the regulatory profit, the greater the perception of absolution, and the higher the fine discounted from the judiciary in relation to procedural costs, the greater will be the tendency to litigate the process.

Additionally, the threshold $\overline{\theta_S}$ over which judicialization is certain is augmented with the judiciary costs $\rho_J \Pi_R$ proportionate to the fine M itself, that is, if taking the case to court is extremely costly to the Firm, then it will pay the fine, despite having cooperated with the norms.

An innocent firm may choose to pay the fine anyway if, for example, extremely low fines are imposed or for firms that do not count with a legal staff (or do not have scale for it).

In this scenario the type I errors are not corrected by the judiciary system, being the burden paid by both the Firm and the Regulator as a false-positive error cost.

3.3.1.1.2 Non-compliant Firm

If the firm has not complied with the norms, its final action is, analogous to the one previously outlined, to either pay in second instance or to pursue judicial action, leaving the decision to the court.

The firm that has not cooperated will judicialize if:

$$(\Pi_R + \Pi_I)(1 - \rho_A - \rho_J) - \delta(\sigma_c = 0)\theta_S M > (\Pi_R + \Pi_I)(1 - \rho_A) - \theta_S M$$

$$\Leftrightarrow \ \theta_S > \frac{\rho_J(\Pi_R + \Pi_I)}{M(1 - \delta(\sigma_c = 0))} = \overline{\overline{\theta_S}}$$

(Eq. 2)

Note that the probability of conviction given non-cooperation is reasonably higher than the probability of conviction given cooperation ($\delta(\sigma_c = 0) > \delta(\sigma_c = 1)$), which implies that the denominator of $\Leftrightarrow \theta S > \frac{\rho_J \Pi_R}{M(1 - \delta(\sigma_c = 1))} = \overline{\theta_S}$

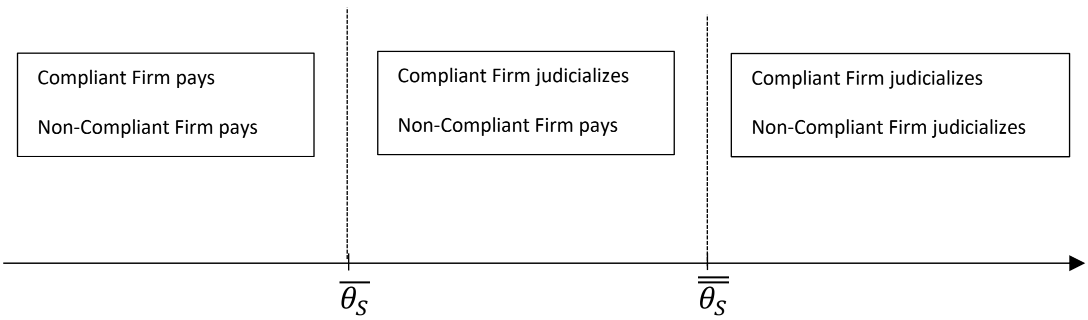
(Eq. 1) is greater than the one from$\Leftrightarrow \theta S > \frac{\rho_J(\Pi_R + \Pi_I)}{M(1 - \delta(\sigma_c = 0))} = \overline{\overline{\theta_S}}$

(Eq. 2) ($1 - \delta(\sigma_c = 1) > 1 - \delta(\sigma_c = 0)$). Considering that $\rho_J \Pi_I \gg 0$, there is a spectrum of θs to be analysed, from the perspective of reassessment optimization on the behalf of the Agency, given that $\overline{\overline{\theta_S}} \gg \overline{\theta_S}$ and firms behave differently along this line.

3.3.1.2 The reassessment of sanctions

In this phase of the game, the Regulator's optimal fine reassessment is analysed, considering whichever decision might have been taken by the Firm on the decision of paying in the second instance or judicializing the process. This is because if the updated fine is low enough ($\theta_S < \overline{\theta_S}$), the Firm will pay the fine, regardless of guilt; if the updated fine is high enough ($\theta_S > \overline{\overline{\theta_S}}$), the Firm will judicialize, regardless of cost; but if the updated fine is in an intermediary range ($\overline{\theta_S} < \theta_S < \overline{\overline{\theta_S}}$), the Compliant Firm will judicialize and the Non-Compliant one will pay in the second instance.

Figure 2 Spectrum of $\boldsymbol{\theta_S}$



Source: Author's elaboration.

There are, therefore, three cases to be analysed under this optimization problem: i) if $\theta_S \leq \overline{\theta_S}$, the Regulator will choose θ_S to maximize its expected utility when the Compliant and Non-Compliant Firms pay; ii) if $\overline{\theta_S} \leq \theta_S \leq \overline{\overline{\theta_S}}$, the Regulator will choose θ_S to maximize its expected utility when the Compliant Firm judicializes and the Non-Compliant Firm pays; and if $\theta_S \geq \overline{\overline{\theta_S}}$, the Regulator will choose θ_S to maximize its expected utility when the Compliant and Non-Compliant Firms judicialize.

The optimization problem of the Regulator for the first range of θ_S *(*if $\theta_S \leq \overline{\theta_S}$*)* is:[9]

$$\underset{\theta_S}{Max}\, \mu[B(\theta_S M) - C_S^I(M)] + (1-\mu)[B(\theta_S M) - (1+\gamma_A)CP(M)] \text{ } s.t. \text{ } \theta_S \leq \overline{\theta_S}$$

(Eq. 3)

The optimization problem of the Regulator for the second range of θ_S *(*if $\overline{\theta_S} \leq \theta_S \leq \overline{\overline{\theta_S}}$*)* is:[10]

$$\underset{\theta_S}{Max}\, \mu[\delta(\sigma_c)B(\theta_S M) - C_S^I(M)] + (1-\mu)[B(\theta_S M) - (1+\gamma_A)CP(M)] \quad s.t. \quad \theta_S \geq \overline{\theta_S} \text{ } and \text{ } \theta_S \leq \overline{\overline{\theta_S}}$$

(Eq. 4)

The optimization problem of the Regulator for the third range of θ_S *(*if $\theta_S \geq \overline{\overline{\theta_S}}$*)* is:[11]

$$\underset{\theta_S}{Max}\, \mu[\delta(\sigma_c)B(\theta_S M) - C_S^I(M)] + (1-\mu)\big[\delta(\sigma_c)B(\theta_S M) - (1+\gamma_A+\gamma_J)CP(M)\big] \quad s.t. \text{ } \theta_S \geq \overline{\overline{\theta_S}}$$

(Eq. 5)

The respective solutions are in Appendices A.1, A.2 and A.3. These separate solutions are broadly analysed through the final objective function of the regulator. In sum, the utility of the regulator in the decision node in which Nature emits a positive signal and the regulated contests is

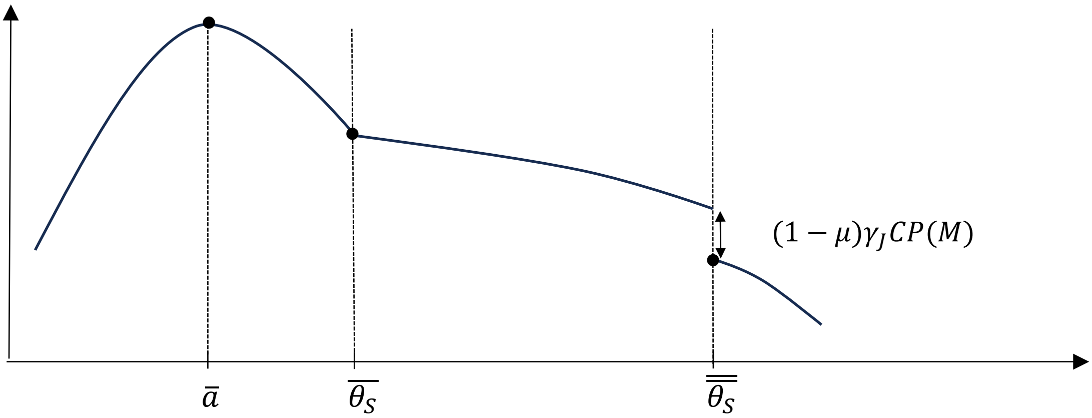
$$U^A(\theta_S | N = S, F^2 = F^3 = SE) = \begin{cases} B(\theta_S M) - \mu C_S^I(M) + (1-\mu)(1+\gamma_A)CP(M), & if \text{ } \theta_S \leq \bar{\theta}_S \\ B(\theta_S M)[1-\mu(1-\delta(\sigma_c))] - \mu C_S^I(M) - (1-\mu)(1+\gamma_A)CP(M), & if \text{ } \bar{\theta}_S \leq \theta_S \leq \overline{\overline{\theta_S}} \\ \delta(\sigma_c)B(\theta_S M) - \mu C_S^I(M) - (1-\mu)\big(1+\gamma_A+\gamma_J\big)CP(M), & if \text{ } \theta_S \geq \overline{\overline{\theta_S}} \end{cases}$$

(Eq. 6)

Considering first that the Agency's optimal second instance incentive is below the lower threshold $\bar{\theta}_S$, that is, $arg\,\underset{\theta_S}{max}\, B(\theta_S M) = \bar{a} < \bar{\theta}_S$.

[9] For the calculations, see Appendix A.1.
[10] For the calculations, see Appendix A.2.
[11] For the calculations, see Appendix A.3.

Figure 3 Sanction reassessment if $\hat{a} = \bar{a}$

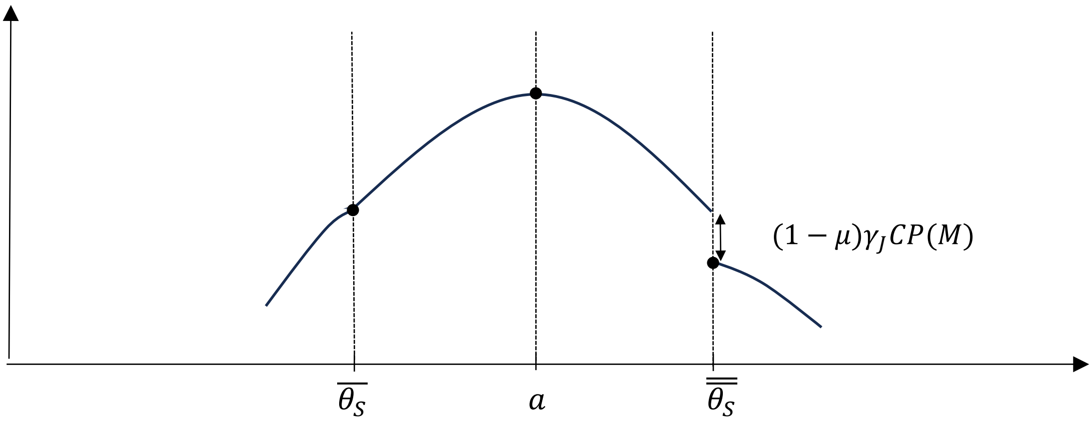


Source: Author's own elaboration.

Considering the discontinuities and the solutions for each range (calculated in the "A" Appendices and represented as black dots in Figure 3), it can be stated that internal optimal solution $\bar{a}$ surpasses the corner solutions, given that $U^A(\bar{a}) > U^A(\overline{\theta_S}) > U^A(\overline{\overline{\theta_S}})$.

Considering now that the Agency's optimal second instance incentive is an intermediate one, that is, $\overline{\theta_S} < arg \max_{\theta_S} B(\theta_S M) = a < \overline{\overline{\theta_S}}$.

Figure 4 Sanction reassessment if $\hat{a} = a$

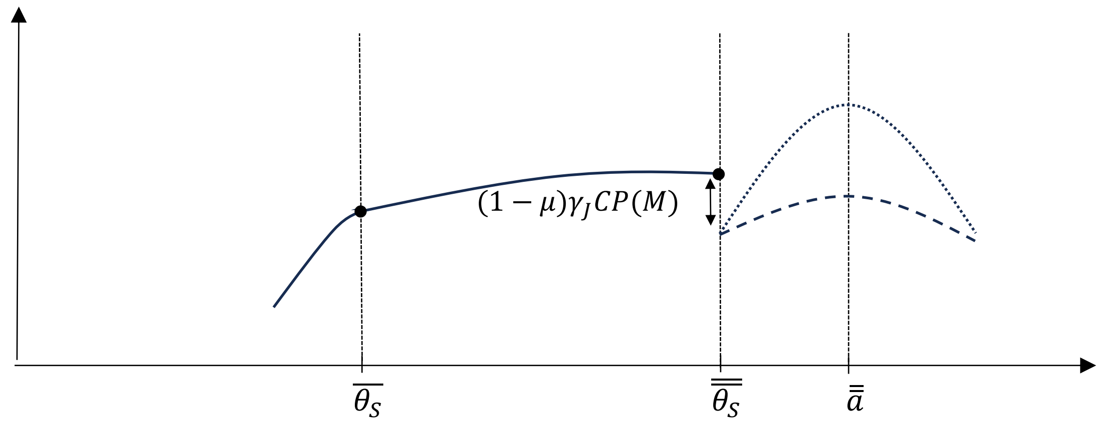


Source: Author's own elaboration.

Considering the discontinuities and the solutions for each range (calculated in the "A" Appendices and represented as black dots in Figure 4), it can also be stated that internal optimal solution a surpasses the corner solutions, given that $U^A(a) > U^A(\overline{\theta_S}) > U^A(\overline{\overline{\theta_S}})$.

Considering lastly that the Agency's optimal second instance incentive is a bold one, that is, $arg \max_{\theta_S} B(\theta_S M) = \bar{\bar{a}} > \overline{\overline{\theta_S}}$.

Figure 5 Sanction reassessment if $\hat{a} = \bar{\bar{a}}$



Source: Author's own elaboration.

Considering the discontinuities and the solutions for each range (calculated in the "A" Appendices and represented as black dots in Figure 5**Figure 4**), it can be asserted that the upper corner solution is preferable to the lower one, as $U^A(\overline{\overline{\theta_S}}) > U^A(\overline{\theta_S})$, but any considerations regarding the optimal internal solution depend on the parameters.[12] Consequently, this resolution can be either the global maximal internal solution, with $\theta_S^* = \bar{\bar{a}}$, represented by the dotted line, or the corner solution, with $\theta_S^* = \overline{\overline{\theta_S}}$, represented by the dashed line.

In the first scenario, if $\theta_S^* = \bar{\bar{a}}$, it is certain that both compliant and non-compliant firms will judicialize, leading the regulator to the worst-case scenario.

In the setting that $\theta_S^* = \overline{\overline{\theta_S}}$, as a tie-breaker, we assume that, if indifferent, the Firm chooses to judicialize. Hence, the Regulator would find itself in a position similar to the one previously outlined.

Therefore, regardless of the Laffer curve type of $B(\theta_S M)$, the optimal solution of the sanction reassessment is the internal solution $\theta_S^* = \hat{a}$, $\hat{a}$ being either $\bar{a}$ or a.

The highest second instance incentive can then already be ruled out, since the payoffs associated with these outcomes are the ones with judicialization.

3.3.1.3 To pay or not to pay

3.3.1.3.1 Low second instance incentive ($\theta_S^* = a$)

In the low second instance incentive structure, the response of the Agency is higher than in the high second instance incentive structure $(a > \bar{a})$, that is, in this scenario, the Compliant Firm judicializes.

Therefore, in the decision node in which the Firm decides whether to pay the fine in first instance, obtaining the f discount, or to contest the sanction, the analysis can be summarized as the conditions regarding the prevailing payoffs.

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[12] For more details on the calculations, see Appendix A.4.

The Non-Compliant Firm chooses to pay in first instance if:

$$\Pi_R + \Pi_I - (1 - f)M > (\Pi_R + \Pi_I)(1 - \rho_A) - aM$$

$$\Leftrightarrow f > 1 - \frac{\rho_A(\Pi_R + \Pi_I)}{M} - a = \bar{f}$$

(Eq. 7)

Alternatively, the Compliant Firm chooses to pay in first instance if:

$$\Pi_R - (1 - f)M > \Pi_R\big(1 - \rho_A - \rho_J\big) - \delta(\sigma_c = 1)aM$$

$$\Leftrightarrow f > 1 - \frac{(\rho_A + \rho_J)\Pi_R}{M} - \delta(\sigma_c = 1)a = \bar{\bar{f}}$$

(Eq. 8)

Thus, there is a spectrum of factor reduction values f upon which the Regulator can operate its first instance incentive mechanism, considering $0 \leq \bar{f} < \bar{\bar{f}} < 1$.[13] If the offered rebate is too low *($f < \bar{f}$)*, firms will ask for a suspension of the fine, as prospects are better under the second instance decision; if the rebate is too high *($f > \bar{\bar{f}}$)*, firms will pay in the first instance, no matter the value of the sanction; but if the first instance incentive is between $\bar{f}$ and $\bar{\bar{f}}$, the Regulator can encourage Compliant Firms to contest and Non-Compliant Firms to pay the fine.

Figure 6 Spectrum of f given low second instance incentive

Source: Author’s elaboration.

3.3.1.3.1 High second instance incentive ($\theta_S^* = \bar{a}$)

In the high second instance incentive structure, the response of the Agency is higher than in the low second instance incentive structure $(a > \bar{a})$, that is, in this scenario, both Compliant and Non-Compliant Firm pays in the second instance.

Therefore, in the node where the Firm decides whether to pay the fine at a discount f or appeal, the Firm’s decision rule can be summarized as below outlined.

The Non-Compliant Firm will choose to pay in first instance if:

$$\Pi_R + \Pi_I - (1 - f)M > (\Pi_R + \Pi_I)(1 - \rho_A) - \bar{a}M$$

[13] For a detailed analysis, see Appendix B.2.

$\Leftrightarrow f > 1 - \frac{\rho_A(\Pi_R + \Pi_I)}{M} - \bar{a} = \breve{f}$

(Eq. 9)

Alternatively, the Compliant Firm will choose to pay in first instance if:

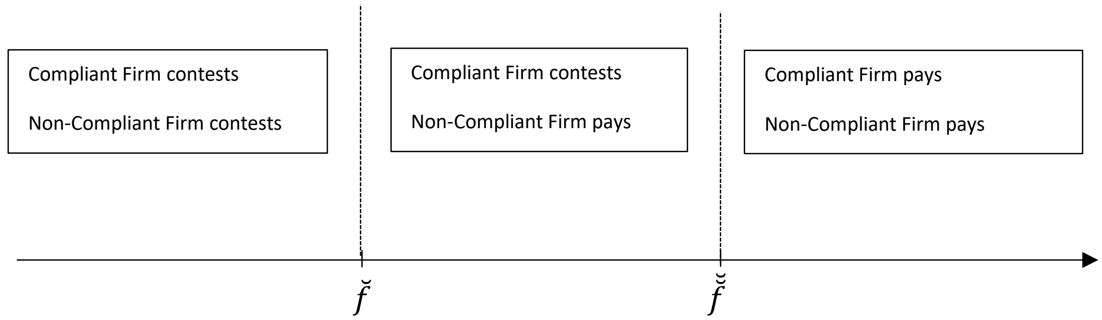
$\Pi_R - (1 - f)M > \Pi_R(1 - \rho_A) - \bar{a}M$

$\Leftrightarrow f > 1 - \frac{\rho_A \Pi_R}{M} - \bar{a} = \breve{\breve{f}}$

(Eq. 10)

That is, there is a spectrum of factor reduction values f upon which the Regulator can operate its first instance incentive mechanism, considering $0 \leq \breve{f} \ll \breve{\breve{f}} < 1$. If the offered discount is too low *($f < \breve{f}$)*, firms will ask for suspensive effect of the fine, expecting for a better perspective under the second instance decision; if the discount is too high *($f > \breve{\breve{f}}$)*, firms will pay in the first instance, no matter the value of the sanction; but if the first instance incentive is between $\breve{f}$ and $\breve{\breve{f}}$, the Regulator can encourage Compliant Firms to contest and Non-Compliant Firms to pay the fine.

Figure 7 Spectrum of f given high second instance incentive



Source: Author's elaboration.

Note that the condition $0 \leq \bar{f} < \bar{\bar{f}} < 1$ demanded additional proof under the auspices of Appendix B.2. Here, the condition $0 \leq \breve{f} < \breve{\breve{f}} < 1$ is trivial, since $\breve{\breve{f}} = 1 - \frac{\rho_A \Pi_R}{M} - \bar{a}$, by subtracting $\frac{\rho_A \Pi_I}{M}$ from both sides, it leads to $\breve{\breve{f}} - \frac{\rho_A \Pi_I}{M} = 1 - \frac{\rho_A \Pi_R}{M} - \bar{a} - \frac{\rho_A \Pi_I}{M} = \breve{f} \Leftrightarrow \breve{\breve{f}} = \breve{f} + \frac{\rho_A \Pi_I}{M}$.

3.3.1.4 Sanctioning Optimization

3.3.1.4.1 Low second instance incentive ($\theta_S^* = a$)

Receiving the positive signal from Nature, the Regulator has, therefore, three cases to be analysed under this optimization problem: i) if $f \leq \bar{f}$, the Regulator will choose M and f to maximize its expected utility when the Compliant and Non-Compliant Firms contest; ii) if $\bar{f} \leq f \leq \bar{\bar{f}}$, the Regulator will choose M and f to maximize its expected utility when the Compliant Firm contests and the Non-Compliant Firm pays; and iii) if $f \geq \bar{\bar{f}}$, the Regulator will choose M and f to maximize its expected utility when the Compliant and Non-Compliant Firms pay.

3.3.1.4.1.1 *Low first instance incentive ($f < \bar{f}$)*

The optimization problem of the Regulator for the first range of f ($f < \bar{f}$) is:[14]

$$\underset{M,f}{Max}\, \psi[\delta(\sigma_c)B(\theta_S^* M) - C_S^I(M)] + (1-\psi)[B(\theta_S^* M) - (1+\gamma_A)CP(M)] \quad s.t. \quad f \leq \bar{f}\ ,$$
$$M \geq 0 \text{ and } M \leq \bar{M} < \Pi_R$$

(Eq. 11)

For $f < \bar{f}$, there is no room to encourage first instance payment, and thus the Firm will always contest. Therefore, under the optimization conditions, the solution for the incentive mechanism may take any value $0 < f^* < \bar{f}$. For more details, see Appendix C.1.

If the utility function of the Regulator hits its maximum for $\theta_S = a > 1$, that is, the reassessment of sanctions is always steep ($M' > M$), then $\bar{f} = 0$, given that f only takes non-negative values. In this case-scenario, there would always be a $f = \varepsilon > 0$ in which at least the Non-Compliant Firm would pay, ruling out this range for f.

Conversely, if the Regulator's utility is such that $\theta_S = a < 1$, indeed, the solution for the incentive mechanism may take any value $0 < f^* < \bar{f}$. These solutions will be analysed in detail thereafter. For more details, see Appendix C.1.

3.3.1.4.1.2 *Medium first instance incentive ($\bar{f} \leq f \leq \bar{\bar{f}}$)*

The optimization problem of the Regulator for the second range of f (if $\bar{f} \leq f \leq \bar{\bar{f}}$) is:[15]

$$\underset{M,f}{Max}\, \psi[\delta(\sigma_c)B(\theta_S^* M) - C_S^I(M)] + (1-\psi)[B((1-f)M) - CP(M) -$$
$$h(\sigma_c)(C_S^{II}(M) + k)] \ s.t.\ f \geq \bar{f},\ f \leq \bar{\bar{f}},\ M \geq 0 \text{ and } M \leq \bar{M} < \Pi_R$$

(Eq. 12)

For a medium first instance incentive, two potential solutions emerge: $f^* = \bar{\bar{f}}$ and $f^* = 1 - \beta_2^*$. These solutions will be analysed in detail thereafter. For more details, see Appendix C.2.

3.3.1.4.1.3 *High first instance incentive ($f \geq \bar{\bar{f}}$)*

The optimization problem of the Regulator for the third range of f (if $f \geq \bar{\bar{f}}$) is:[16]

$$\underset{M,f}{Max}\, \psi\big[B\big((1-f)M\big) - C_S^I(M) + g(\sigma_c)(\Sigma - k)\big] + (1-\psi)[B((1-f)M) - CP(M) -$$
$$h(\sigma_c)(C_S^{II}(M) + k)] \ s.t.\ f \geq \bar{\bar{f}}\ M \geq 0 \text{ and } M \leq \bar{M} < \Pi_R$$

(Eq. 13)

For a high first instance incentive, two potential solutions emerge: $f^* = \bar{\bar{f}}$ and $f^* = 1 - \beta_4^*$. These solutions will be analysed in detail thereafter. For more details, see Appendix C.3.

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[14] For the calculations, see Appendix C.1.

[15] For the calculations, see Appendix C.2.

[16] For the calculations, see Appendix C.3.

3.3.1.4.1 High second instance incentive ($\theta_S^* = \bar{a}$)

Receiving the positive signal from Nature, the Regulator has, therefore, three cases to be analysed under this optimization problem: i) if $f \leq \breve{f}$, the Regulator will choose M and f to maximize its expected utility when the Compliant and Non-Compliant Firms contest; ii) if $\breve{f} \leq f \leq \breve{\breve{f}}$, the Regulator will choose M and f to maximize its expected utility when the Compliant Firm contests and the Non-Compliant Firm; and iii) if $f \geq \breve{\breve{f}}$, the Regulator will choose M and f to maximize its expected utility when the Compliant and Non-Compliant Firms pay.

3.3.1.4.1.1 *Low first instance incentive ($f < \breve{f}$)*

The optimization problem of the Regulator for the first range of f (if $f < \breve{f}$) is:[17]

$$\underset{M,f}{Max}\, \psi[B(\theta_S^* M) - C_S^I(M)] + (1-\psi)[B(\theta_S^* M) - (1+\gamma_A)CP(M)] \;\; s.t. \;\; f \leq \breve{f}\,, \;\; M \geq 0 \;\; and \; M \leq \bar{M} < \Pi_R$$

(Eq. 14)

For $f < \breve{f}$, there is no room to encourage first instance payment, the firms will always contest. Therefore, under the optimization conditions, the solution for the incentive mechanism may take any value $0 < f^* < \breve{f}$. For more details, see Appendix C.4.

If the utility function of the Regulator hits its maximum for $\theta_S = \bar{a} > 1$, that is, the reassessment of sanctions is always steep *($M' > M$)*, then $\breve{f} = 0$, given that f only takes non-negative values. In this case-scenario, there would always be a $f = \varepsilon > 0$ in which at least the Non-Compliant Firm would pay, ruling out this range for f.

Conversely, if the Regulator's utility is such that $\theta_S = \bar{a} < 1$, indeed, the solution for the incentive mechanism may take any value $0 < f^* < \breve{f}$. This is a more reasonable scenario, given that, if judicial costs were higher than the penalty, that is, $\rho_J \Pi_R > M(1 - \delta(\sigma_c = 1)$ which implies that $\bar{\theta}_S > 1$, there would not be a great incentive to judicialize, ultimately. These solutions will be analysed in detail thereafter. For more details, see Appendix C.4.

3.3.1.4.1.2 *Medium first instance incentive ($\breve{f} \leq f \leq \breve{\breve{f}}$)*

The optimization problem of the Regulator for the second range of f (if $\bar{f} \leq f \leq \breve{\breve{f}}$) is:[18]

$$\underset{M,f}{Max}\, \psi[B(\theta_S^* M) - C_S^I(M)] + (1-\psi)[B((1-f)M) - CP(M) - h(\sigma_c)(C_S^{II}(M) + k)]$$
$$s.t. \; f \geq \breve{f}, f \leq \breve{\breve{f}}, M \geq 0 \text{ and } M \leq \bar{M} < \Pi_R$$

(Eq. 15)

For a medium first instance incentive, two potential solutions emerge: $f^* = \breve{\breve{f}}$ and $f^* = 1 - \beta_6^*$. These solutions will be analysed in detail thereafter. For more details, see Appendix C.5.

[17] For the calculations, see Appendix C.4.
[18] For the calculations, see Appendix C.5.

3.3.1.4.1.3 *High first instance incentive ($f \geq \breve{\breve{f}}$)*

The optimization problem of the Regulator for the third range of f (if $f \geq \bar{\bar{f}}$) is:[19]

$$\underset{M,f}{Max}\, \psi\left[B\left((1-f)M\right) - C_S^I(M) + g(\sigma_c)(\Sigma - k)\right] + (1-\psi)[B((1-f)M) - CP(M) - h(\sigma_c)(C_S^{II}(M) + k)] \; s.t. \; f \geq \breve{\breve{f}} \; M \geq 0 \; and \; M \leq \bar{M} < \Pi_R$$

(Eq. 16)

For a high first instance incentive, two potential solutions emerge: $f^* = \breve{\breve{f}}$ and $f^* = 1 - \beta_8^*$. These solutions will be analysed in detail thereafter. For more details, see Appendix C.6.

3.3.2 Nature issues a negative signal

3.3.2.1 Judicialization as the final action

3.3.2.1.1 Compliant Firm

This scenario relates to the one in which no signal from Nature has been received and the Firm contests the fine in the first instance. After the reassessment of the sanction from the part of the Regulator, the Firm can either pay the fine in second instance or take the case to court.

The firm that has cooperated will judicialize if:

$$\Pi_R(1 - \rho_A - \rho_J) - \delta_{NS}(\sigma_c = 1)\theta_{NS}M > \Pi_R(1 - \rho_A) - \theta_{NS}M$$

$$\Leftrightarrow \; \theta_{NS} > \frac{\rho_J \Pi_R}{M(1 - \delta_{NS}(\sigma_c = 1))} = \bar{\theta}_{NS}$$

(Eq. 17)

In essence, the higher the fine imposed compared to the regulatory profit, the stronger the perception of absolution, and the higher the fine discounted from the judiciary in relation to procedural costs, the greater is the inclination to litigate the process. Furthermore, the threshold $\bar{\theta}_{NS}$ after which judicialization becomes inevitable increases with judiciary costs $\rho_J \Pi_R$ proportionate to the fine M itself. In other words, if taking the case is excessively expensive for the firm, the fine will be paid even if it has cooperated with the norms.

3.3.2.1.2 Non-compliant Firm

The firm that has not cooperated will judicialize if:

$$(\Pi_R + \Pi_I)(1 - \rho_A - \rho_J) - \delta_{NS}(\sigma_c = 0)\theta_{NS}M > (\Pi_R + \Pi_I)(1 - \rho_A) - \theta_{NS}M$$

$$\Leftrightarrow \; \theta_{NS} > \frac{\rho_J(\Pi_R + \Pi_I)}{M(1 - \delta_{NS}(\sigma_c = 0))} = \bar{\bar{\theta}}_{NS}$$

(Eq. 18)

[19] For the calculations, see Appendix C.6.

It's important to note that the probability of conviction given non-cooperation is significantly higher than the probability given cooperation *(*$\delta_{NS}(\sigma_c = 0) > \delta_{NS}(\sigma_c = 1)$*)*, implying that the denominator of $\Leftrightarrow \theta NS > \frac{\rho_J \Pi_R}{M(1-\delta_{NS}(\sigma_c=1))} = \bar{\theta}_{NS}$

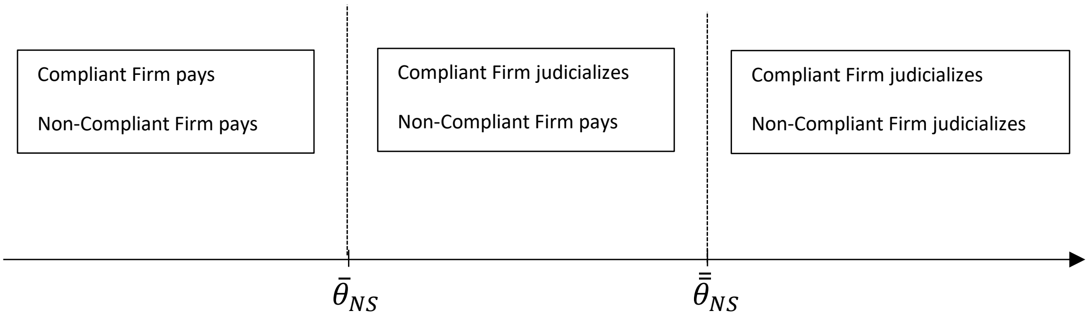
(Eq. 17) is greater than the one from (Eq. 18)*(*$1 - \delta_{NS}(\sigma_c = 1) > 1 - \delta_{NS}(\sigma_c = 0)$*)*. Considering that $\rho_J \Pi_I > 0$, a spectrum of θ_{NS} needs to be analysed. This analysis is conducted from the perspective of reassessment optimization on the behalf of the Agency, given that $\bar{\bar{\theta}}_{NS} > \bar{\theta}_{NS}$ and firms behave differently along this line.

In addition, it is relevant to highlight that $\delta_{NS}(\sigma_c) < \delta_S(\sigma_c)$, for $0 < \sigma_c < 1$. That is, the probability of administrative sanction alleviation is higher if there is no signal from Nature. In other words, the prospect for a better deal via the judiciary system is clearer in this part subgame. Hence, we can affirm that $\bar{\theta}_{NS} < \overline{\theta_S}$ and $\bar{\bar{\theta}}_{NS} < \overline{\overline{\theta_S}}$.

3.3.2.2 The reassessment of sanctions

In this phase of the game, the optimal fine reassessment of the Regulator is analysed, considering whichever decision the Firm might be taking on the decision of whether to pay the fine on the second instance or take the case to court. If the updated fine is low enough *(*$\theta_{NS} < \bar{\theta}_{NS}$), the Firm will pay the fine, regardless of guilt; if the updated fine is high enough *(*$\theta_{NS} > \bar{\bar{\theta}}_{NS}$*)*, the Firm will judicialize, regardless of cost; but if the updated fine is in an intermediary range *(*$\bar{\theta}_{NS} < \theta_{NS} < \bar{\bar{\theta}}_{NS}$ *)*, the Compliant Firm will judicialize and the Non-Compliant one will pay in the second instance.

Figure 8 Spectrum of $\boldsymbol{\theta_{NS}}$



Source: Author's elaboration.

There are, therefore, three cases to be analysed under this optimization problem: i) if $\theta_{NS} \leq \bar{\theta}_{NS}$, the Regulator will choose θ_{NS} to maximize its expected utility when the Compliant and Non-Compliant Firms pay; ii) if $\bar{\theta}_{NS} \leq \theta_{NS} \leq \bar{\bar{\theta}}_{NS}$, the Regulator will choose θ_{NS} to maximize its expected utility when the Compliant Firm judicializes and the Non-Compliant Firm pays; and iii) if $\theta_{NS} \geq \bar{\bar{\theta}}_{NS}$, the Regulator will choose θ_{NS} to maximize its expected utility when the Compliant and Non-Compliant Firms judicialize.

The optimization problem of the Regulator for the first range of θ_{NS} *(*$\theta_{NS} \leq \bar{\theta}_{NS}$*)* is:[20]

[20] For the calculations, see Appendix D.1.

$\underset{\theta_{NS}}{Max}\, \eta[B(\theta_{NS}M) - (1+\gamma_A)C_{NS}^I(M)] + (1-\eta)[B(\theta_{NS}M) - (1+\gamma_A)CP(M)]$
s.t. $\theta_{NS} \leq \bar{\theta}_{NS}$

(Eq. 19)

The optimization problem of the Regulator for the second range of θ_{NS} (if $\bar{\theta}_{NS} \leq \theta_{NS} \leq \bar{\bar{\theta}}_{NS}$) is:[21]

$\underset{\theta_{NS}}{Max}\, \eta\left[\delta_{NS}(\sigma_c)B(\theta_{NS}M) - (1+\gamma_A+\gamma_J)C_{NS}^I(M)\right] + (1-\eta)[B(\theta_{NS}M) - (1+\gamma_A)CP(M)]$ s.t. $\theta_{NS} \geq \bar{\theta}_{NS}$ and $\theta_{NS} \leq \bar{\bar{\theta}}_{NS}$

(Eq. 20)

The optimization problem of the Regulator for the third range of θ_{NS} (if $\theta_{NS} \geq \bar{\bar{\theta}}_{NS}$) is:[22]

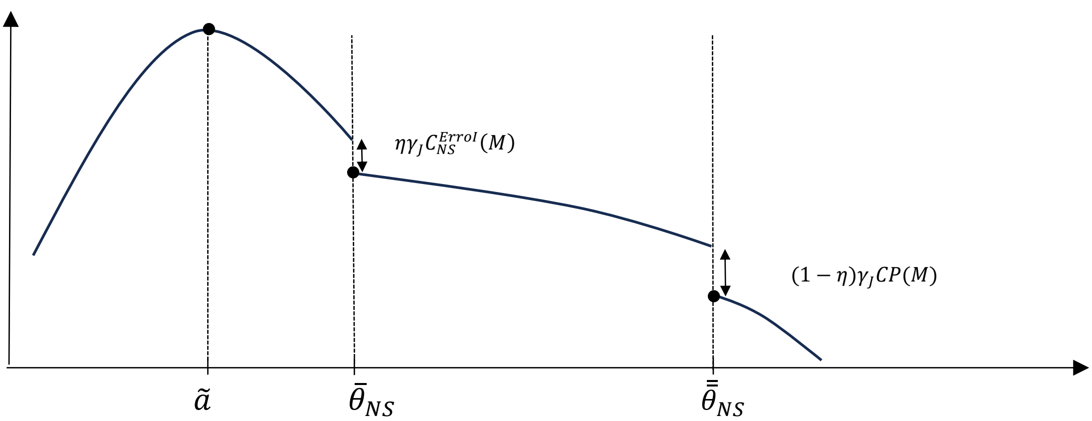
$\underset{\theta_{NS}}{Max}\, \eta\left[\delta_{NS}(\sigma_c)B(\theta_{NS}M) - (1+\gamma_A+\gamma_J)C_{NS}^I(M)\right] + (1-\eta)\left[\delta_{NS}(\sigma_c)B(\theta_{NS}M) - (1+\gamma_A+\gamma_J)CP(M)\right]$ s.t. $\theta_{NS} \geq \bar{\bar{\theta}}_{NS}$

The respective solutions are in Appendices D.1, D.2 and D.3. These separate solutions are broadly analysed through the final objective function of the regulator. In sum, the utility of the regulator in the decision node in which Nature emits a positive signal and the Firm contests is:

$$U^A(\theta_{NS}|N = \mathrm{NS}, F^4 = F^5 = SE) = \begin{cases} B(\theta_{NS}M) - \eta(1+\gamma_A)C_{NS}^I(M) - (1-\eta)(1+\gamma_A)CP(M), & if\ \theta_{NS} \leq \bar{\theta}_{NS} \\ B(\theta_{NS}M)[1-\eta(1-\delta_{NS}(\sigma_c))] - \eta\left(1+\gamma_A+\gamma_J\right)C_{NS}^I(M) - (1-\eta)(1+\gamma_A)CP(M), & if\ \bar{\theta}_{NS} \leq \theta_{NS} \leq \bar{\bar{\theta}}_{NS} \\ \delta_{NS}(\sigma_c)B(\theta_{NS}M) - \mu\left(1+\gamma_A+\gamma_J\right)C_{NS}^I(M) - (1-\mu)\left(1+\gamma_A+\gamma_J\right)CP(M), & if\ \theta_{NS} \geq \bar{\bar{\theta}}_{NS} \end{cases}$$

Considering first that the Agency's optimal second instance incentive is below the lower threshold $\bar{\theta}_{NS}$, that is, $arg\,\underset{\theta_{NS}}{max}\, B(\theta_{NS}M) = \tilde{a} < \bar{\theta}_{NS}$.

Figure 9 Sanction reassessment if $\hat{a} = \tilde{a}$



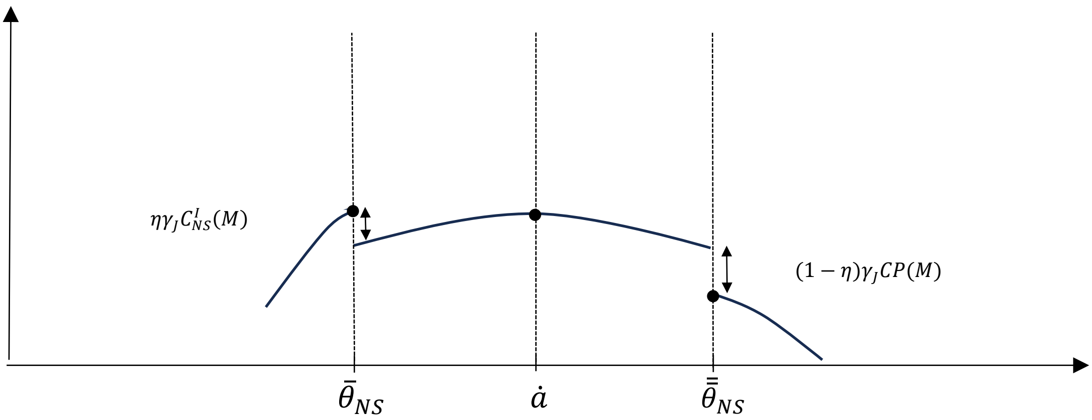
Source: Author's own elaboration.

[21] For the calculations, see Appendix D.2.
[22] For the calculations, see Appendix D.3.

Considering the discontinuities and the solutions for each range (calculated in the "D" Appendices and represented as black dots in Figure 9), it can be stated that internal optimal solution $\tilde{a}$ surpasses the corner solutions, given that $U^A(\tilde{a}) > U^A(\bar{\theta}_{NS}) > U^A(\bar{\bar{\theta}}_{NS})$.

Considering now that the Agency's optimal second instance incentive is an intermediate one, that is, $\bar{\theta}_{NS} < arg \max_{\theta_{NS}} B(\theta_{NS}M) = \dot{a} < \bar{\bar{\theta}}_{NS}$.

Figure 10 Sanction reassessment if $\hat{a} = \dot{a}$

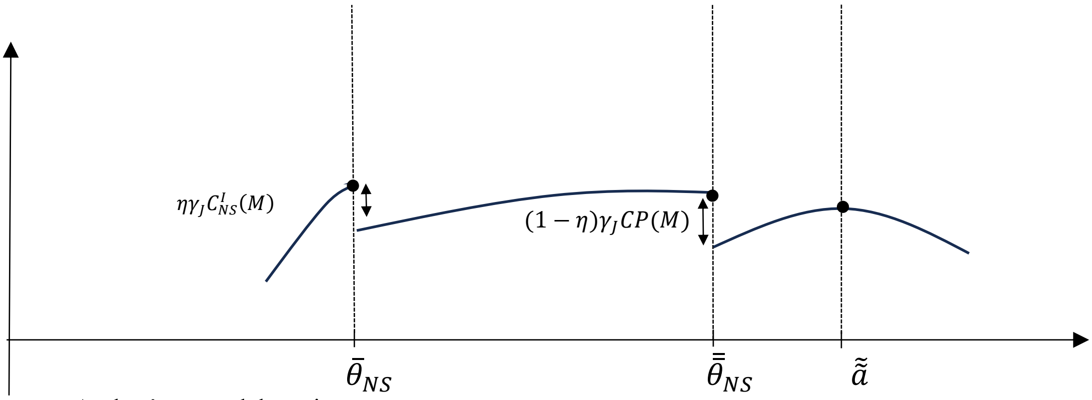


Source: Author's own elaboration.

Considering the discontinuities and the solutions for each range (calculated in the "D" Appendices and represented as black dots in Figure 10), it cannot be stated that internal optimal solution $\dot{a}$ surpasses the lower corner solution, given that $U^A(\bar{\bar{\theta}}_{NS}) < U^A(\dot{a}) < U^A(\bar{\theta}_{NS})$. This is due to the false positive costs in this scenario[23].

Considering lastly that the Agency's optimal second instance incentive is a fierce one, that is, $arg \max_{\theta_{NS}} B(\theta_{NS}M) = \tilde{\tilde{a}} > \bar{\bar{\theta}}_{NS}$.

Figure 11 Sanction reassessment if $\hat{a} = \tilde{\tilde{a}}$



Source: Author's own elaboration.

[23] For the calculations, see Appendix D.4.

Considering the discontinuities and the solutions for each range (calculated in the "D" Appendices and represented as black dots in Figure 11), it can be asserted that the lower corner solution is preferable than the upper one, as $U^A(\bar{\theta}_{NS}) > U^A(\bar{\bar{\theta}}_{NS}) > U^A(\tilde{\tilde{a}})$.[24]

Therefore, regardless of the Laffer curve type of $B(\theta_{NS}M)$, the optimal solution of the sanction reassessment is the internal solution $\theta^*_{NS} = min\{\hat{a}, \bar{\theta}_{NS}\}$, $\hat{a}$ being either $\tilde{a}$,$\dot{a}$ or $\tilde{\tilde{a}}$.

Given the corner solutions when $\theta_{NS} > \bar{\theta}_{NS}$, it can be stated that optimality only will be attained when $\hat{a} = \tilde{a}$. In other words, a generic formula for this solution would be $\theta^*_{NS} = \min\{\hat{a}, \bar{\theta}_{NS}\}$. However, regardless of the utility function, the decision of the Regulator will always be of high second stance incentive, which means that it does not make sense to exogenously assume that the utility function can take the form of $\left(\dot{a} - \frac{1}{2}\theta_{NS}\right)\theta_{NS}M$ and $\left(\tilde{\tilde{a}} - \frac{1}{2}\theta_{NS}\right)\theta_{NS}M$, because the analysis will converge to $\theta^*_{NS} = \bar{\theta}_{NS}$. Therefore, when not receiving the signal, the regulator will choose a reassessment factor that creates incentives for both compliant and non-compliant firms to pay in second instance, that is, $\theta^*_{NS} = \tilde{a}$.

3.3.2.3 To pay or not to pay

3.3.2.3.1 High second instance incentive ($\theta^*_{NS} = \tilde{a}$)

We now move to the decision node where the firm determines whether to pay the initial fine, thereby receiving the discount f or to contest the penalty. The analysis can be succinctly summarized by examining the payoffs players obtain in each case.

The Non-Compliant Firm will choose to pay in first instance if:

$$\Pi_R + \Pi_I - (1 - f)M > (\Pi_R + \Pi_I)(1 - \rho_A) - \tilde{a}M$$

$$\Leftrightarrow f > 1 - \frac{\rho_A(\Pi_R + \Pi_I)}{M} - \tilde{a} = \tilde{f}$$

(Eq. 21)

Alternatively, the Compliant Firm will choose to pay in first instance if:

$$\Pi_R - (1 - f)M > \Pi_R\left(1 - \rho_A - \rho_J\right) - \delta_{NS}(\sigma_c = 1)\tilde{a}M$$

$$\Leftrightarrow f > 1 - \frac{(\rho_A + \rho_J)\Pi_R}{M} - \delta_{NS}(\sigma_c = 1)\tilde{a} = \tilde{\tilde{f}}$$

(Eq. 22)

That is, there is a spectrum of rebate values f upon which the Regulator can operate its first instance incentive, considering $0 \leq \tilde{f} \ll \tilde{\tilde{f}} < 1$.[25] If the offered discount is too low ($f < \tilde{f}$), firms will ask for suspensive effect of the fine, expecting for a better perspective under the second instance decision; if the discount is too high ($f > \tilde{\tilde{f}}$), firms will pay in the first instance, no matter the value of the sanction; but if the first instance incentive is

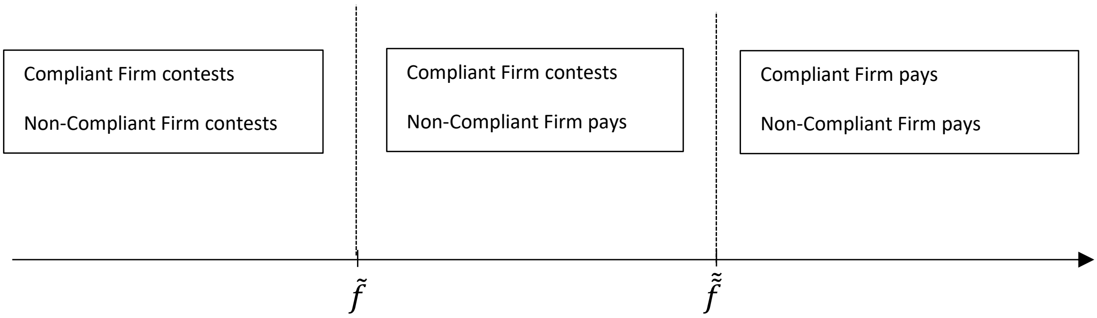
[24] For the calculations, se Appendix D.4.

[25] The calculations conducted in Appendix B.2 is analogous to this case.

between $\tilde{f}$ and $\tilde{\tilde{f}}$, the Regulator can encourage Compliant Firms to contest and Non-Compliant Firms to pay the fine.

Note that $\tilde{f} > \bar{f}$ and $\tilde{\tilde{f}} > \bar{\bar{f}}$, that is, when there is no positive signal from the Nature, the minimum thresholds to pay in first instance are superior, that is, the offered discounts must be higher.

Figure 12 Spectrum of f without signal



Source: Author's elaboration.

3.3.2.4 Sanctioning Optimization

3.3.2.4.1 High second instance incentive ($\theta_{NS}^{*} = \tilde{\alpha}$)

Receiving the positive signal from Nature, the Regulator has, therefore, three cases to analyse under this optimization problem: i) if $f \leq \tilde{f}$, the Regulator will choose M and f to maximize its expected utility when the Compliant and Non-Compliant Firms contest; ii) if $\tilde{f} \leq f \leq \tilde{\tilde{f}}$, the Regulator will choose M and f to maximize its expected utility when the Compliant Firm contests and the Non-Compliant Firm pays; and iii) if $f \geq \tilde{\tilde{f}}$, the Regulator will choose M and f to maximize its expected utility when the Compliant and Non-Compliant Firms pay.

3.3.2.4.1.1 *Low first instance incentive ($f < \tilde{f}$)*

The optimization problem of the Regulator for the first range of f ($f < \tilde{f}$) is:[26]

$$\underset{M,f}{Max}\, \Phi\left[\delta_{NS}(\sigma_c)B(\theta_{NS}^{*}M) - (1+\gamma_A+\gamma_J)C_{NS}^{I}(M)\right] + (1-\Phi)[B(\theta_{NS}^{*}M) - (1+\gamma_A)CP(M)] \text{ s.t. } f \leq \tilde{f},\ M \geq 0 \text{ and } M \leq \bar{M} < \Pi_R$$

(Eq. 23)

Regarding the optimal sanction when there is no signal of regulatory violation, the marginal benefit derived from the Lagrangian of $\underset{M,f}{Max}\, \Phi\left[\delta_{NS}(\sigma_c)B(\theta_{NS}^{*}M) - (1+\gamma_A+\gamma_J)C_{NS}^{I}(M)\right] + (1-\Phi)[B(\theta_{NS}^{*}M) - (1+\gamma_A)CP(M)]$ s.t. $f \leq \tilde{f}$, $M \geq 0$ and $M \leq \bar{M} < \Pi_R$

(Eq. 23), equal to $[1-\Phi(1-\delta_{NS}(\sigma_c))]\frac{\tilde{\alpha}^2}{2}$, is reduced by the second instance incentive. For more details, see Appendix E.1.

[26] For the calculations, see Appendix E.1.

That is, the marginal benefit does not surpass the marginal cost, equivalent to $\Phi(1+\gamma_A+\gamma_J)\frac{\partial C_{NS}^{I}(M)}{\partial M}+(1-\Phi)(1+\gamma_A)\frac{\partial CP(M)}{\partial M}$. This leads to the decision of not sanctioning the firm, $M^*=0$.

3.3.2.4.1.2 *Medium first instance incentive* ($\tilde{f} \leq f \leq \tilde{\tilde{f}}$)

The optimization problem of the Regulator for the second range of f (if $\tilde{f} \leq f \leq \tilde{\tilde{f}}$) is:[27]

$$\underset{M,f}{Max}\,\Phi\big[\delta_{NS}(\sigma_c)B(\theta_{NS}^{*}M)-(1+\gamma_A+\gamma_J)C_{NS}^{I}(M)\big]+(1-\Phi)[B((1-f)M)-CP(M)-h(\sigma_c)(C_{NS}^{II}(M)+k)]\text{ s.t. } f\geq\tilde{f},\, f\leq\tilde{\tilde{f}}\; M\geq 0 \text{ and } M\leq\bar{M}<\Pi_R$$

(Eq. 24)

In this case, as stated in (Eq. 31), the difference between the marginal benefit and cost is $\Phi\delta_{NS}(\sigma_c)\left(\frac{1}{2}\tilde{\alpha}^2\right)+(1-\Phi)\beta(1-f)-\frac{1}{2}(1-f)^2-\Phi\big(1+\gamma_A+\gamma_J\big)\frac{\partial C_{NS}^{I}(M)}{\partial M}-(1-\Phi)\left[\frac{\partial CP(M)}{\partial M}+\mathrm{h}(\sigma_c)\frac{\partial C_{NS}^{II}(M)}{\partial M}\right]$, which is negative from the low utility derived from the second instance incentive, compared to the higher costs of both types I and II and the externality to the society CP.

Hence, the optimal decision is not to impose sanctions, and thus $M^*=0$.

3.3.2.4.1.3 *High first instance incentive* ($f \geq \tilde{\tilde{f}}$)

The optimization problem of the Regulator for the third range of f (if $f\geq\tilde{\tilde{f}}$) is:[28]

$$\underset{M,f}{Max}\,\Phi\big[B\big((1-f)M\big)-C_{NS}^{I}(M)+g(\sigma_c)(\Sigma-k)\big]+(1-\Phi)[B((1-f)M)-CP(M)-h(\sigma_c)(C_{NS}^{II}(M)+k)]\text{ s.t. } f\geq\bar{\bar{f}}\; M\geq 0 \text{ and } M\leq\bar{M}<\Pi_R$$

(Eq. 25)

For this range, the difference between marginal benefit and cost is $\left(\frac{1}{2}\beta^2\right)-\Phi\frac{\partial C_{NS}^{I}(M)}{\partial M}-(1-\Phi)\left[\frac{\partial CP(M)}{\partial M}+h(\sigma_c)\frac{\partial C_{NS}^{II}(M)}{\partial M}\right]$, if there is an interior solution, with $f^*>\tilde{\tilde{f}}$.

Note that, from the optimization condition, $f^*=1-\beta$. Hence, for a high value of the rebate, such as the case, there is a low related value for β. In this case, the solution is also $M^*=0$.

3.3.3 Regulation Compliance given low second instance incentive

Possible equilibria will be analysed based on the different first instance incentive strategies the Regulator may choose: low, medium, or high first instance incentives. Furthermore, policy implications are uncovered from the results.

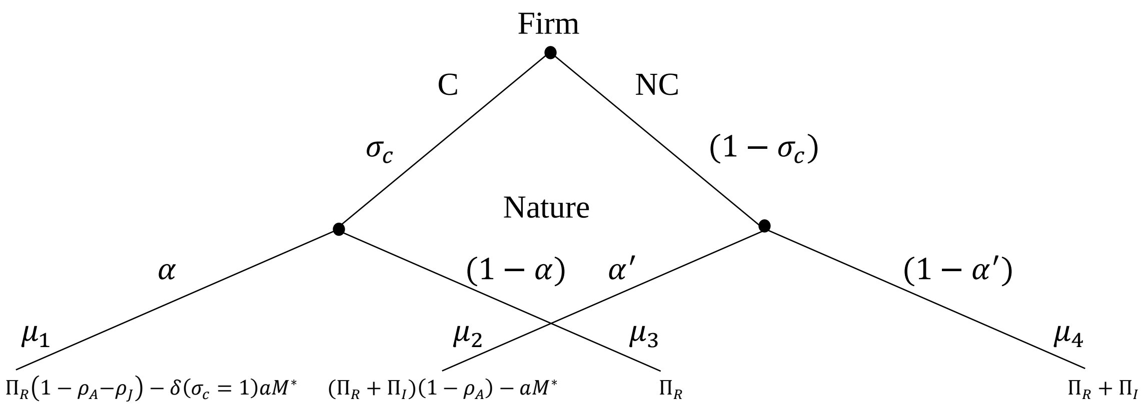
[27] For the calculations, see Appendix E.2.
[28] For the calculations, see Appendix E.3.

3.3.3.1 Low first instance incentive ($f^* < \bar{f}$)

In this case, if a positive signal is emitted, the Regulator chooses an optimal sanction M^*, a rebate that $f^* < \bar{f}$, and a reassessment of the sanction of the magnitude $\theta_S^* = a$.[29] If a negative signal is revealed, the Regulator chooses not to sanction, $M^* = 0$ (in this case, the factor reductor issue is redundant, this solution could also be denoted as $f^* = 1$), and a reassessment of the sanction of the magnitude $\theta_S^* = \tilde{a}$.

The Firm, on the other side, if there is a positive signal, will, in the first interaction with the Regulator, choose to appeal for a suspension despite having cooperated or not; additionally, in the second instance, it will judicialize if it has cooperated, in the first place, and will pay if it has not. Alternatively, if there is a negative signal, it will, in the first interaction with the Regulator, choose to file for suspension regardless having cooperated or not. Additionally, in the second instance, it will pay regardless of whether it has cooperated or not, in the first play.

Figure 13 Regulation Compliance under low first and second instance incentives



Source: Author's elaboration.

The Regulator possess the system of beliefs on finding itself in each one of the nodes of Figure 13 according to the probabilities μ_1, μ_2, μ_3 and μ_4.[30] Note that these probabilities are the ones utilized in its maximization problem under the optimal sanctioning set, with positive and negative signals. That is, the generic representation of the beliefs can be altered to the notation already used in the respective solutions: $\mu_1 = \psi$, $\mu_2 = 1 - \psi$, $\mu_3 = \Phi$, and $\mu_4 = 1 - \Phi$.

Utilizing Bayes' rule, these beliefs can be stated as:

$\psi = \frac{\alpha\sigma_c}{\alpha\sigma_c + \alpha'(1-\sigma_c)}$, $(1-\psi) = \frac{\alpha'(1-\sigma_c)}{\alpha\sigma_c + \alpha'(1-\sigma_c)}$, $\Phi = \frac{(1-\alpha)\sigma_c}{(1-\alpha)\sigma_c + (1-\alpha')(1-\sigma_c)}$,

and $(1-\Phi) = \frac{(1-\alpha')(1-\sigma_c)}{(1-\alpha)\sigma_c + (1-\alpha')(1-\sigma_c)}$.

Please note that all move decisions made by the Regulator stem from singleton information sets. Consequently, this player's belief assigns a probability of one to every

[29] Condition that $\lambda_{C.1.3} = \lambda_{C.1.2} = 0$. For exact formulas, see Appendix C.1.

[30] This generic form is applicable to all subsequent situations.

individual decision node. Hence, the beliefs from the optimal sanctioning and sanctioning reassessment are identical: $\psi = \mu$, and $\Phi = \eta$.

In this scenario, the firm will cooperate if $U^F(C) \geq U^F(NC)$:

$\Leftrightarrow \alpha\left[\Pi_R\left(1-\rho_A-\rho_J\right)-\delta(\sigma_c=1)aM^*\right]+(1-\alpha)\Pi_R \geq \alpha'[(\Pi_R+\Pi_I)(1-\rho_A)-aM^*]+(1-\alpha')(\Pi_R+\Pi_I)$

This equation can be summarised as:

$\rho_A[\Pi_R(\alpha'-\alpha)+\alpha'\Pi_I]-\alpha\rho_J\Pi_R+\left(\alpha'-\alpha\delta(\sigma_c=1)\right)aM^* \geq \Pi_I$

(Eq. 26)

Three possible situations may emerge in terms of the degree of asymmetry of information.

3.3.3.1.1 Perfectly Informative Signals

Consider that Nature's signals are perfectly informative: when there is no cooperation, the positive signal is emitted 100% of the cases – $\alpha' = 1$; conversely, when there is cooperation, no positive signal is ever revealed – $\alpha = 0$.

Therefore, considering that $\alpha' - \alpha = 1$, (Eq. 26) can be rewritten as:

$[\rho_A(\Pi_R+\Pi_I)]+[aM^*] \geq \Pi_I$

This first term in brackets is the administrative cost of the non-compliant firm while the second term in brackets is the sanction cost if it is resolved in second instance. If the administrative and sanction costs are higher than the excess profit derived from regulatory trespassing, the firm will cooperate, in the first place.

That is, there is a possible Nash Equilibrium in which the fine does not need to be greater than the excess profit Π_I. Note that if $a \to 1$, which is a reasonable proposition, given that, with a positive signal and administrative contestation, the resulting fine may be at least close to the first one imposed, and with a non-negligible administrative cost, indeed the firm may cooperate even if the net profit from regulatory infraction ($\Pi_I - M^*$) is positive.

This underscores the importance of establishing a systematic framework of procedural requirements for regulatory compliance, aiming to minimize information asymmetry and subjective decision-making processes.

Such procedural requisites serve as a framework for ensuring accountability, fairness, and transparency in the enforcement process. By delineating clear procedures for investigation, assessment of violations, and imposition of penalties, regulators can foster a culture of regulatory adherence within the industry. A systematic approach to compliance not only enhances the efficacy of enforcement actions but also installs confidence in stakeholders regarding the integrity and consistency of regulatory oversight.

Moreover, it promotes a level playing field among regulated entities, reducing the likelihood of selective enforcement or arbitrary sanctions. Therefore, investing in robust procedural requisites is paramount for regulatory authorities to uphold the rule of law, maintain market integrity, and safeguard public interests.

In sum, enforcement procedures based on rules rather discretion can encourage regulated firms to cooperate with regulation even with low f and non-maximal sanctions.

Note that the highest the regulator's response in the second instance a, the lowest the margin to explore the equilibrium with a low first instance incentive. Considering $a > 1$, it would lead to $\bar{f} = 0$. This scenario is not unimaginable since, given a positive signal, the regulator might have an incentive to reinforce the original sanction.

3.3.3.1.1.1 *Perfect Bayesian Equilibrium*

A Perfect Bayesian Equilibrium consists of strategy profiles and beliefs satisfying Requirements 1 to 4, specified at the beginning of the Game Solution Section.

The game has been resolved so far focusing on the strategies of players, now our attention is shifted to the beliefs of the Regulator, generically denoted $\psi, \mu, \Phi,$ and η. Note that all actions from the Firm occur in singleton information sets. Therefore, this player's belief assign probability one on every single decision node. Hence, the probability distribution of beliefs is well-defined, satisfying Requirement 1.

Sequential rationality requires each player's strategy to be payoff-maximising at each of his information sets, given his beliefs and the strategies of the others. Since the optimization problems of the players were constructed in a way optimality was achieved considering strategies of the others as restrictions, sequential rationality is obtained, satisfying Requirement 2.

For Requirement 3, it is necessary to update the beliefs probabilities according to the Bayes' rule, considering this scenario of perfect precision of signals, and check for consistency of the strategies adopted by the players.

According to the Bayes' rule, if $\alpha' - \alpha = 1$:

$$\psi = \frac{\alpha\sigma_C}{\alpha\sigma_C + \alpha'(1-\sigma_C)} = \frac{0}{0}$$

$$\Phi = \frac{\sigma_C(1-\alpha)}{\sigma_C(1-\alpha) + (1-\sigma_C)(1-\alpha')} = 1$$

That is, if the Regulated has cooperated, the Regulator perfectly knows it. So, the solution obtained in the optimal sanctioning problem of not sanctioning is indeed a consistent one with the fact that no positive signal has been received, now with certainty.

Conversely, if the Regulated has trespassed regulation, the Regulator perfectly knows it. Therefore, the solution obtained in the optimal sanctioning problem would count with a maximized marginal benefit of sanctioning, since $\frac{1}{2}\left(1 - \psi\big(1 - \delta(\sigma_C)\big)\right)\alpha^2 = \frac{\alpha^2}{2}$ and, once considered that $BMgM(M^*) = CMgM(M^*)$, if $\alpha' - \alpha = 1$, $BMgM(M) > CMgM(M)$, which means that $\lambda_{C.1.3} > 0$, $\lambda_{C.1.2} = 0 \Rightarrow M = \bar{M}$. In sum, the prior proposed strategy from the Regulator to set a fine in which the marginal benefit of the activity equal its marginal cost, in a scenario of perfect precision of signals, needs to be updated to a strategy of maximal sanctioning where $M = \bar{M}$.

After the sanctioning, the firm decides to pay in first instance or to ask for a suspension of the fine. If the former is chosen, the game ends; if the latter is preferred, the game proceeds to the reassessment of sanction. At this stage, the firm put unit probability on this single decision node, which implies that $\psi = \mu$ and $\Phi = \eta$.

Substituting the probabilities of μ in the reassessment problem of the Regulator, its solution remains as $\theta_S^* = a$, in the case of a positive signal; and $\theta_{NS}^* = \tilde{a}$, in case of a negative signal. Therefore, consistency is obtained for strategies of the sanction reassessment problem of the Regulator.

Passing to the final decision of the players, which is the one taken by the Regulated to either pay the fine in second instance or to judicialize it, the firm would always choose to pay the sanction in the administrative branch, since with $\theta_S^* = a$ the non-compliant firm pays, which is the case for the positive signal in this scenario, and with $\theta_{NS}^* = \tilde{a}$, both compliant and non-compliant firms pay in second instance.

Therefore, satisfying Requirement 3, all strategies are consistent, highlight the update of the optimal sanction from M^* to $\bar{M}$.

Since, for Requirement 4, at information sets that are off-the-equilibrium-path, beliefs are determined by Bayes' rule and the players' equilibrium strategies where possible, under perfect precision of signals and certainty of cooperation, the Regulator can update its belief for $\psi = 1$.

In conclusion, the strategy profiles and belief systems of the following form consist of a Perfect Bayesian Equilibrium:

$$\Omega_1 = \Big(\big(C, SE|(C,S), SE|(NC,S), J|(C,S), P2|(NC,S); M = \bar{M}\big|S, f < \bar{f}\big|S, M = 0|NS, f = 1|NS, \theta_S^* = a, \theta_{NS}^* = \tilde{a}\big), \psi = \mu = 1, \Phi = \eta = 1\Big).$$

In other words, if an institutional development is conducted towards informing society the true value of regulation, the precise route to its compliance, and the possible fixed penalty ($M = \bar{M}$), equilibrium can be achieved with a low f, $M < \Pi_I$ and no judicialization.

3.3.3.1.2 Imperfectly Informative Signals

Consider now that $\alpha = \alpha'$, that is, it can be seen as a situation in which the Regulator does not have resources to an adequate supervision or the regulation itself bring such subjectivity that positive or negative signals of regulatory infractions cannot be distinguished.

In this case, the firm will cooperate if:[31]

$$\alpha\big(1 - \delta(\sigma_c = 1)\big)aM^* \geq \Pi_I + \alpha(\rho_J \Pi_R - \rho_A \Pi_I)$$

The definition of cooperation or not is indefinite, because it depends on the magnitudes of M^* and Π_I.

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[31] Consider here the proposition made in (Eq. 30), in Appendix B.2, that $a\big(1 - \delta(\sigma_c = 1)\big) + \frac{1}{M}\big(\rho_A \Pi_I - \rho_J \Pi_R\big) > 0$.

3.3.3.1.3 Informative Signals

Consider now that $\frac{\alpha'}{\alpha} > 1$, that is, it can be seen as a situation in which there is a reasonable amount of information.

Here, (Eq. 26) can be descripted as:

$$\rho_A \Pi_R \left(\frac{\alpha'}{\alpha} - 1\right) + \left(\frac{\alpha'}{\alpha} - \delta(\sigma_c = 1)\right) a M^* \geq \Pi_I + \rho_J \Pi_R + \frac{\alpha'}{\alpha} \rho_A \Pi_I$$

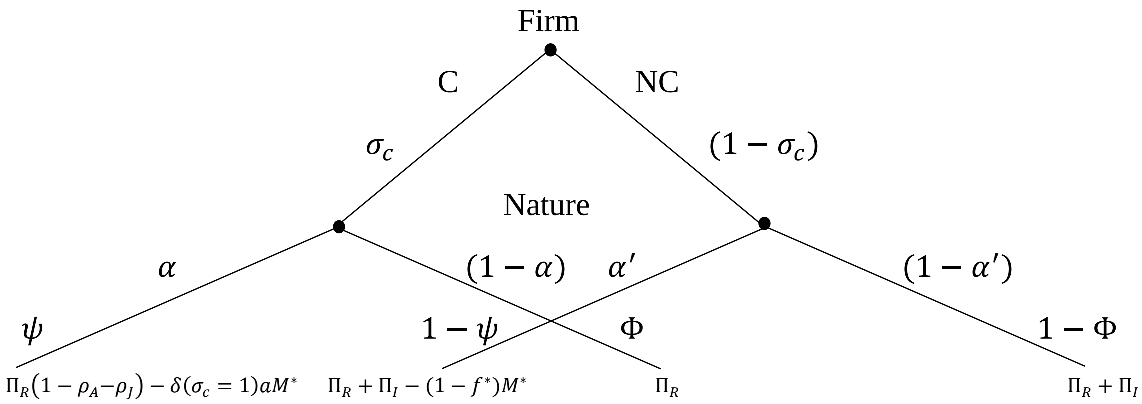
The decision of cooperation or not is indefinite, once it depends on the magnitudes of $\frac{M^*}{\Pi_I}$ and $\frac{\alpha'}{\alpha}$.

3.3.3.2 Medium first instance incentive ($\bar{f} \leq f^* \leq \bar{\bar{f}}$)

In this case, if a positive signal is emitted, the Regulator chooses an optimal sanction M^*, a factor reductor that $\bar{f} \leq f^* \leq \bar{\bar{f}}$, and a reassessment of the sanction of the magnitude $\theta_S^* = a$.[32] If a negative signal is revealed, the Regulator chooses not to sanction, $M^* = 0$ (in this case, the factor reductor issue is redundant, this solution could also be denoted as $f^* = 1$), and a reassessment of the sanction of the magnitude $\theta_S^* = \tilde{a}$.

The Firm, on the other side, if there is a positive signal, will, in the first interaction with the Regulator, choose to ask for a suspensive effect having cooperated and pay in first instance, if it has not. Additionally, in the second instance, it will take the case to court if it has cooperated, in the first place, and will pay if it hasn't. Alternatively, if there is a negative signal, it will, in the first interaction with the Regulator, choose to file for a suspension having cooperated, and pay in first instance, if it has not. Furthermore, in the second instance, it will pay regardless of whether it has cooperated or not, in the first round.

Figure 14 Regulation Compliance under medium first instance and low second instance incentives



Source: Author's elaboration.

The Firm's pure strategies will be analysed, the possible mixed strategies are outlined in Appendix F.2. The solutions derived from the optimal sanctioning of a medium first

[32] Condition that $\lambda_{C.2.4} = \lambda_{C.2.3} = 0$. For exact formulas, see Appendix C.2.

instance incentive are $f^* = \bar{\bar{f}}$ and $f^* = 1 - \beta_2^*$. The potential equilibriums obtained from both are below examined.

The firm will cooperate if:

$$\alpha\left[\Pi_R\left(1 - \rho_A - \rho_J\right) - \delta(\sigma_c = 1)aM^*\right] + (1 - \alpha)\Pi_R \geq \alpha'[\Pi_R + \Pi_I - (1 - f^*)M^*] + (1 - \alpha')(\Pi_R + \Pi_I)$$

(Eq. 27)

3.3.3.2.1 $f^* = \bar{\bar{f}}$

Substituting $f^* = \bar{\bar{f}}$ and assuming that $M^* = v\Pi_R$, the condition for cooperation equals to:

$$0 < \alpha\left[\Pi_R\left(\rho_A + \rho_J\right) + \delta(\sigma_c = 1)aM^*\right] \leq \Pi_I\left[v\alpha'\left(1 - \bar{\bar{f}}\right)\right]$$

$$\Leftrightarrow \Pi_I - \alpha'\left(\rho_A + \rho_J\right) > v\alpha'\delta(\sigma_c = 1)a\Pi_I$$

Since $\delta(\sigma_c = 1) \to 0$ and $\Pi_I - \left(\rho_A + \rho_J\right) > 0$, the first term is positive while the second converges to zero. Therefore, the conclusion is that the value of regulation, in excess of its administrative and judicial costs, outweighs the potential type I error from the judiciary.[33] In this scenario, the firm never cooperates, because the sanction now becomes a feasible price to be paid. Since the firm will never cooperate, there are no mixed strategies related.

3.3.3.2.1.1 *Perfect Bayesian Equilibrium*

A Perfect Bayesian Equilibrium consists of strategy profiles and belief systems satisfying Requirements 1 to 4, specified at the beginning of the Game Solution Section.

The game has been resolved so far focusing on the strategies of players, now attention is attended to the beliefs of the Regulator, generically denoted in the calculations as ψ, μ, Φ, and η. Note that all move decisions from the Regulated are from singleton information sets, therefore, this player's belief put probability one on every single decision node. Hence, the probability distribution of beliefs is defined, satisfying Requirement 1.

Sequential rationality requires each player's strategy to be payoff-maximising at each of his information sets, given his beliefs and the strategies of the others. Since the optimization problems of the players were constructed in a way that optimality was achieved considering strategies of the others as restrictions, sequential rationality is obtained, satisfying Requirement 2.

For Requirement 3, it is necessary to update the beliefs probabilities according the Bayes' rule, considering this proposed scenario that firms would never cooperate, and check for consistency of the strategies adopted by the players.

According to the Bayes' rule, if for a high first instance incentive, the decision of the firm would be always to not cooperate with regulation, then $\sigma_c = 0$, which implies that:

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[33] These relations are more broadly explained when $f^* = \bar{\bar{f}}$, in the subsequent section.

$\psi = \frac{\alpha\sigma_c}{\alpha\sigma_c + \alpha'(1-\sigma_c)} = 0$

$\Phi = \frac{\sigma_c(1-\alpha)}{\sigma_c(1-\alpha)+(1-\sigma_c)(1-\alpha')} = 0$

That is, the Regulator would believe that there would not be cooperation from the Regulated. However, the solution obtained in the optimal sanctioning problem of not sanctioning is not consistent with the fact that the Regulator now knows there is no cooperation.

Hence, this solution falls short in meeting Requirement 3, indicating that this potential outcome lacks the consistency necessary to qualify as a Perfect Bayesian Equilibrium.

In conclusion, the strategy profiles and belief systems of the following form does not consist of a Perfect Bayesian Equilibrium:

$\Omega_2 = \Big(\big(NC, SE|(C,S), SE|(NC,S), J|(C,S), P2|(NC,S); M = M^*\big|S, f^* = \bar{\bar{f}}\big|S, M = 0|NS, f^* = 1|NS, \theta_S^* = a, \theta_{NS}^* = \tilde{a}\big), \psi = \mu = 0, \Phi = \eta = 0\Big).$

3.3.3.2.2 $\boldsymbol{f^* = 1 - \beta_2^*}$

The firm will cooperate if:

$\alpha\big[\Pi_R\big(1-\rho_A-\rho_J\big) - \delta(\sigma_c)aM^*\big] + (1-\alpha)\Pi_R \geq \alpha'[\Pi_R + \Pi_I - (1-f^*)M^*] + (1-\alpha')(\Pi_R + \Pi_I)$

Substituting $f^* = 1 - \beta_2^*$ and conducting some manipulations:

$\Leftrightarrow \left[\frac{\alpha'}{\alpha}\beta_2^* - \delta(\sigma_c = 1)a\right]M^* \geq \frac{\Pi_R}{2} + (\rho_A + \rho_J)\Pi_R$

Recall from the medium first instance incentive problem solution (detailed in Appendix C.2) that $\beta_2^* = \sqrt{\frac{\alpha\sigma_c}{\alpha'(1-\sigma_c)}\left(\delta(\sigma_c)a^2 - 2\frac{\partial C_S^I(M^*)}{\partial M}\right) - 2\left(\frac{\partial CP(M^*)}{\partial M} + \mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M^*)}{\partial M}\right)}$

Note here that the solution was updated with Bayes' rule, considering the probabilities of cooperation σ_c and Nature's signals α and α'.

This optimal solution from the Regulator's viewpoint depends on the perceived probabilities of cooperation from the Regulated, which is unknown to the former. To this analysis, consider first the extreme cases of cooperation and non-cooperation.

If the firm is expected always to cooperate $\sigma_c = 1$, but since $\delta(\sigma_c = 1) \to 0$, β_2^* is not defined in real numbers, a very concerning feature for regulatory standardization purposes.

If the firm is expected never to cooperate $\sigma_c = 0$, but since the first term would be null and the second term negative, β_2^* would not be defined in real numbers neither.

The Regulator would then need to select a specific value of σ_c between the extremes to determine its optimal β_2^*. To do this, because the first term must return a positive value (for β_2^* not to be indefinite), one possible way would be to maximize $\beta_2^*(\sigma_c)$ and check its value[34].

The calculations indicate that the possible optimal β_2^* would demand a negative value for σ_c, which is an impossibility, which is automatically not consistent to any belief system.

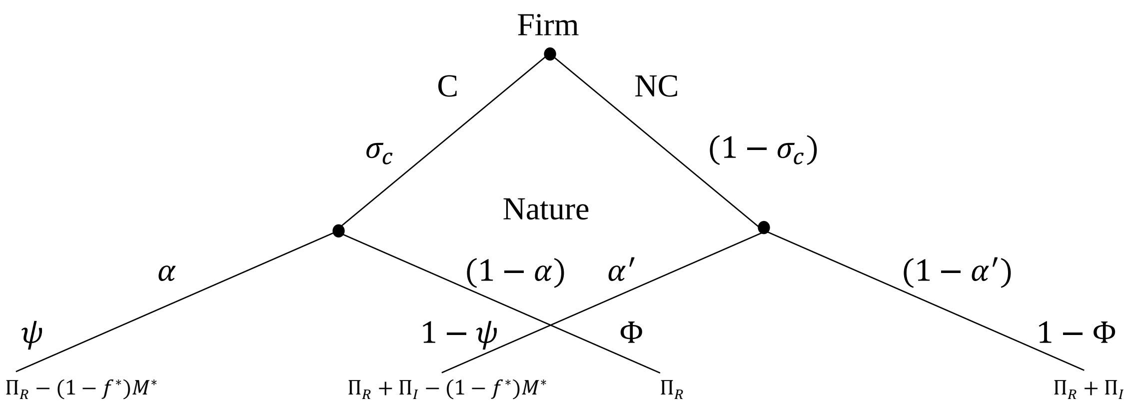
Therefore, no solution could be obtained for a medium first instance incentive mechanism.

3.3.3.3 High first instance incentive ($f^* \geq \bar{\bar{f}}$)

In this case, if a positive signal is emitted, the Regulator chooses an optimal sanction M^*, a factor reductor that $f^* \geq \bar{\bar{f}}$, and a reassessment of the sanction of the magnitude $\theta_S^* = a$.[35] If a negative signal is revealed, the Regulator chooses not to sanction, $M^* = 0$ (in this case, the factor reductor issue is redundant, this solution could also be denoted as $f^* = 1$), and a reassessment of the sanction of the magnitude $\theta_S^* = \tilde{a}$.

The Firm, on the other side, after a positive signal, will, in the second round, choose to pay in first instance regardless of its cooperation decision. Additionally, in the second instance, it will take the case to court if cooperation occurred in the first round and will pay if it hasn't. Alternatively, if there is a negative signal, it will, in the first instance, choose to pay regardless of its first-round cooperation decision. Furthermore, in the second instance, it will pay if it has cooperated or not, in the first place.

Figure 15 Regulation Compliance under high first instance and low second instance incentives



Source: Author's elaboration.

The solutions derived from the optimal sanctioning of a medium first instance incentive are $f^* = \bar{\bar{f}}$ and $f^* = 1 - \beta_4^*$. The potential equilibriums obtained from both are below examined.

The firm will cooperate if:

$$\alpha[\Pi_R - (1-f^*)M^*] + (1-\alpha)\Pi_R \geq \alpha'[\Pi_R + \Pi_I - (1-f^*)M^*] + (1-\alpha')[\Pi_R + \Pi_I]$$

$$\Leftrightarrow (\alpha' - \alpha)(1-f^*)M^* \geq \Pi_I$$

---

[34] The calculations was done in Appendix F.2.

[35] Condition that $\lambda_{C.3.4} = \lambda_{C.3.3} = 0$. For exact formulas, see Appendix C.3.

Consider first that $f^* = \bar{\bar{f}} = 1 - \frac{(\rho_A+\rho_J)\Pi_R}{M^*} - \delta(\sigma_c = 1)a$, substituting this value in the condition for cooperation, the firm will cooperate if the net present value is negative:

$$\Pi_I - (\alpha' - \alpha)\big[(\rho_A + \rho_J)\Pi_R + \delta(\sigma_c = 1)aM^*\big] \leq 0$$

The proposition that the private economic value of regulation Π_I outweighs its compliance costs $(\rho_A + \rho_J)\Pi_R$ is not heroic, quite on the contrary, it is rather a basic premise, once if the excess profits fail to exceed potential administrative or judicial costs, not to mention the sanctions themselves, then cooperation always emerges as the dominant strategy for the firm. This is not plausible, hence, $\Pi_I - (\rho_A + \rho_J)\Pi_R > 0$.

Furthermore, the function that reunites the probability of conviction and the factor of sanction reduction in the judiciary converges to zero if the firm has indeed cooperated $\delta(\sigma_c = 1) \to 0$. If this value would not converge to zero, it would mean the existence of a persistent Type I error also in the judiciary branch.

Therefore, it would be reasonable to assume that $\Pi_I - \big[(\rho_A + \rho_J)\Pi_R + \delta(\sigma_c = 1)aM^*\big] > 0$. Note that this condition does not even consider the multiplication of the factor $(\alpha' - \alpha)$ to the deduction factor in brackets, implying that the cooperation condition in this case does not hold.

In conclusion, the firm does not cooperate if the Regulator offers a high first instance incentive for payment $f^* = \bar{\bar{f}}$. In addition, if the result is valid for $f^* = \bar{\bar{f}}$, it is automatically valid for $f^* = 1 - \beta_4^* > \bar{\bar{f}}$, since the cooperation condition would not be met by a greater extend, in this case. Given this dominant pure strategy, there are no mixed strategies for this scenario.

3.3.3.3.1 Perfect Bayesian Equilibrium

A Perfect Bayesian Equilibrium consists of strategy profiles and belief systems satisfying Requirements 1 to 4, specified at the beginning of the Game Solution Section.

The game has been resolved so far focusing on the strategies of players, now attention is attended to the beliefs of the Regulator, generically denoted in the calculations as ψ, μ, Φ, and η. Note that all move decisions from the Regulated are from singleton information sets, therefore, this player's belief put probability one on every single decision node. Hence, the probability distribution of beliefs is defined, satisfying Requirement 1.

Sequential rationality requires each player's strategy to be payoff-maximising at each of his information sets, given his beliefs and the strategies of the others. Since the optimization problems of the players was constructed in a way that optimality was achieved considering strategies of the others as restrictions, sequential rationality is obtained, satisfying Requirement 2.

For Requirement 3, it is necessary to update the beliefs probabilities according the Bayes' rule, considering this proposed scenario that firms would never cooperate, and check for consistency of the strategies adopted by the players.

According to the Bayes' rule, if for a high first instance incentive, the decision of the firm would be always to not cooperate with regulation, then $\sigma_c = 0$, which implies that:

$\psi = \frac{\alpha\sigma_C}{\alpha\sigma_C + \alpha'(1-\sigma_C)} = 0$

$\Phi = \frac{\sigma_C(1-\alpha)}{\sigma_C(1-\alpha)+(1-\sigma_C)(1-\alpha')} = 0$

That is, the Regulator would believe that there would not be cooperation from the Regulated. However, the solution obtained in the optimal sanctioning problem of not sanctioning is not consistent with the fact that the Regulator now knows there is no cooperation.

Hence, this solution falls short in meeting Requirement 3, indicating that this potential outcome lacks the consistency necessary to qualify as a Perfect Bayesian Equilibrium.

In conclusion, the strategy profiles and belief systems of the following form does not consist of a Perfect Bayesian Equilibrium:

$$\Omega_3 = \Big(\big(NC, SE|(C,S), SE|(NC,S), J|(C,S), P2|(NC,S); M = M^*\big|S, f^* = \bar{\bar{f}}\big|S, M = 0|NS, f^* = 1|NS, \theta_S^* = a, \theta_{NS}^* = \tilde{a}\big), \psi = \mu = 0, \Phi = \eta = 0\Big).$$

Moreover, changing financial incentives can have other, possibly unintended consequences. For instance, if (some) drivers essentially perceive speeding fines as the cost of driving fast, the introduction of an early payment discount effectively makes speeding cheaper, leading drivers to “purchase” more speed, ultimately resulting in more traffic violations and fatalities on the roads (PLESSIS et al., 2020). In an experiment conducted by Gneezy and Rustichini (2000), a financial penalty was introduced for parents who were late to pick up their children from school; as a result, the number of tardy parents surprisingly increased significantly.

These cases challenge the Deterrence Theory, which posits that the introduction of penalties will lead to a reduction in abusive behaviour, as the penalty can be seen as a feasible price to be paid. In the realm of telecommunications, the penalty can also be regarded as a feasible price to be paid, either after a lengthy administrative/legal process, wherein its financial impact may not be felt by the actors responsible for the infractions, or, alternatively, mitigated through a reasonably high discounted payment, such as in the case of $f^* = \bar{\bar{\bar{f}}}$.

3.3.4 Regulation Compliance given high second instance incentive

Possible equilibriums will be analysed based on the varying first instance incentive strategies the Regulator may choose: low, medium, or high first instance incentives. Furthermore, policy implications are uncovered from the results.

3.3.4.1 Low first instance incentive ($f^* < \breve{f}$)

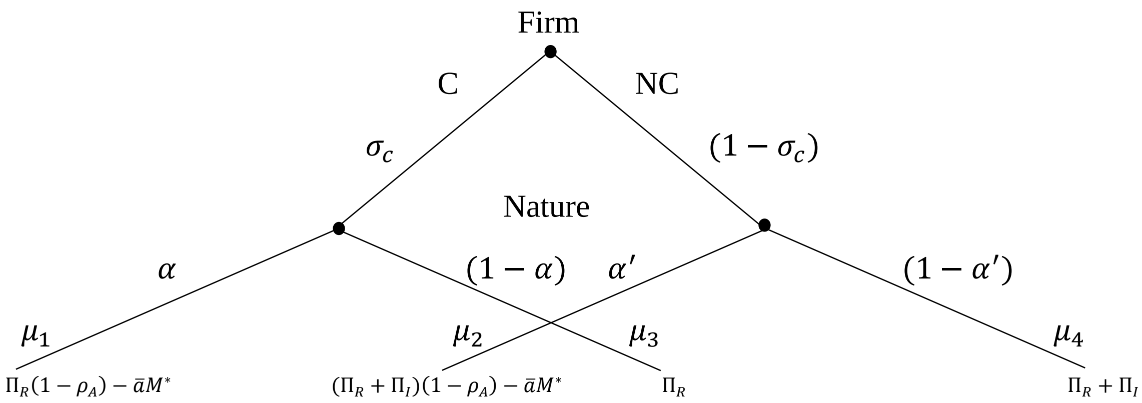
In this case, if a positive signal is emitted, the Regulator chooses an optimal sanction M^*, a factor reductor that $f^* < \breve{f}$, and a reassessment of the sanction of the magnitude $\theta_S^* = \bar{a}$.[36] If a negative sign is issued, the Regulator chooses not to sanction, $M^* = 0$ (in this

[36] Condition that $\lambda_{C.4.3} = \lambda_{C.4.2} = 0$. For exact formulas, see Appendix C.4.

case, the factor reductor issue is redundant, this solution could also be denoted as $f^* = 1$), and a reassessment of the sanction of the magnitude $\theta_S^* = \tilde{\alpha}$.

The Regulated, on the other side, if there is a positive sign, will, in the first interaction with the Regulator, choose to ask for a suspensive effect despite having cooperated or not; additionally, in the second instance, it will always pay the fine, not judicializing. Alternatively, if there is a negative sign, it will, in the first interaction with the Regulator, choose to ask for a suspensive effect despite having cooperated or not; additionally, in the second instance, it will pay if it has cooperated or not, in the first place.

Figure 16 Regulation Compliance under low first instance and high second instance incentives



Source: Author's elaboration.

The Regulator possess the system of beliefs on finding itself in each one of the nodes of Figure 13 according to the probabilities μ_1, μ_2, μ_3 and μ_4. Note that these probabilities are the ones utilized in its maximization problem under the optimal sanctioning set, with positive and negative signals. That is, the generic representation of the beliefs can be altered to the notation already used in the respective solutions: $\mu_1 = \psi$, $\mu_2 = 1-\psi$, $\mu_3 = \Phi$, and $\mu_4 = 1 - \Phi$.

Utilizing Bayes' rule, these beliefs can be stated as:

$\psi = \frac{\alpha\sigma_C}{\alpha\sigma_C + \alpha'(1-\sigma_C)}$,$(1-\psi) = \frac{\alpha'(1-\sigma_C)}{\alpha\sigma_C+\alpha'(1-\sigma_C)}$, $\Phi = \frac{(1-\alpha)\sigma_C}{(1-\alpha)\sigma_C + (1-\alpha')(1-\sigma_C)}$,

and $(1-\Phi) = \frac{(1-\alpha')(1-\sigma_C)}{(1-\alpha)\sigma_C+(1-\alpha')(1-\sigma_C)}$.

Please note that all move decisions made by the Regulator stem from singleton information sets. Consequently, this player's belief assigns a probability of one to every individual decision node. Hence, the beliefs from the optimal sanctioning and sanctioning reassessment are identical: $\psi = \mu$, and $\Phi = \eta$.

In this scenario, the firm will cooperate if $U^F(C) \geq U^F(NC)$[37]

$\Leftrightarrow \alpha[\Pi_R(1-\rho_A) - \bar{a}M^*] + (1-\alpha)\Pi_R \geq \alpha'[(\Pi_R + \Pi_I)(1-\rho_A) - \bar{a}M^*] + (1 - \alpha)(\Pi_R + \Pi_I)$

[37] Calculations are in Appendix F.4.

This equation can be summarised as:

$$\rho_A[\Pi_R(\alpha' - \alpha) + \alpha'\Pi_I] + (\alpha' - \alpha)\bar{a}M^* \geq \Pi_I$$

(Eq. 28)

Three possible situations may emerge in terms of the degree of asymmetry of information.

3.3.4.1.1 Perfectly Informative Signals

Consider that Nature's signals are perfectly precise: when there is no cooperation, the positive signal is emitted 100% of the cases – $\alpha' = 1$; conversely, when there is cooperation, no positive signal is ever issued – $\alpha = 0$.

Therefore, considering that $\alpha' - \alpha = 1$, (Eq. 28) can be rewritten as:

$$[\rho_A(\Pi_R + \Pi_I)] + [\bar{a}M^*] \geq \Pi_I$$

This first term in brackets is the administrative cost of the non-compliant firm while the second term in brackets is the sanction cost if it is resolved in second instance. If the administrative and sanction costs are higher than the excess profit derived from regulatory trespassing, the firm will cooperate, in the first place.

Given $\bar{a} < \bar{\theta}_S = \frac{\rho_J \Pi_R}{M(1-\delta(\sigma_C=1))}$, consider $\bar{a} = w\frac{\rho_J \Pi_R}{M(1-\delta(\sigma_C=1))}$, with $w < 1$.

The cooperation condition can be restated as:

$$[\rho_A(\Pi_R + \Pi_I)] + \left[w\frac{\rho_J \Pi_R}{(1-\delta(\sigma_C=1))}\right] \geq \Pi_I$$

But note that the excess profits surpass the administrative and judicial cost, which means that the firm does not cooperate in this scenario.

3.3.4.1.2 Imperfectly Informative Signals

Consider now that $\alpha = \alpha'$, that is, it can be seen as a situation in which the Regulator does not have resources to an adequate supervision or the regulation itself bring such subjectivity that positive or negative signals of regulatory infractions cannot be distinguished.

In this case, the firm will never cooperate given the above impossibility:

$$0 \geq \Pi_I(1 - \alpha'\rho_A) > 0$$

3.3.4.1.3 Informative Signals

Consider now that $\frac{\alpha'}{\alpha} > 1$, that is, it can be seen as a situation in which there is a reasonable amount of information.

Here, (Eq. 28) can be descripted as:

$$[\alpha' - \alpha]\left[w\frac{\rho_J \Pi_R}{(1-\delta(\sigma_C=1))} + \rho_A(\Pi_R + \Pi_I)\right] + \alpha\rho_A\Pi_I \geq \Pi_I$$

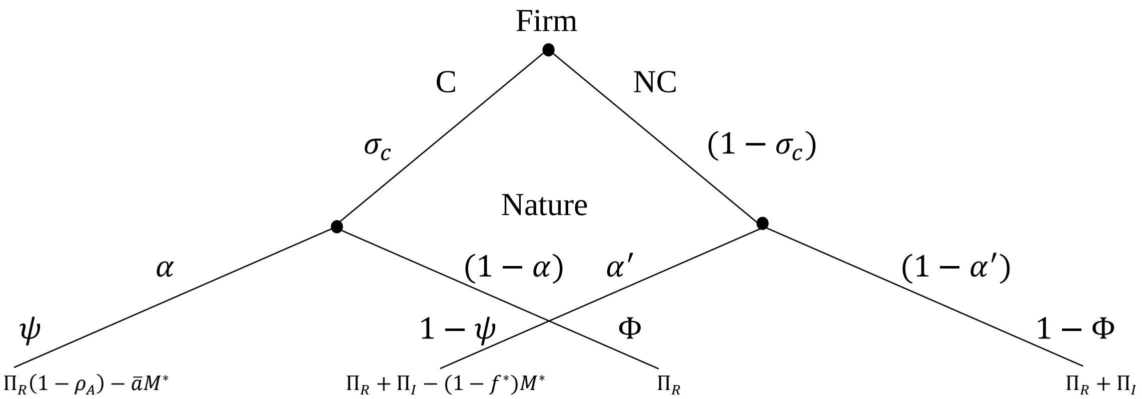
Given the regulation vs compliance proposition and the multiplication for a value less the unity in the left side of the equation, it can be said that the firm does not cooperate.

3.3.4.2 Medium first instance incentive ($\breve{f} \leq f^{*} \leq \breve{\breve{f}}$)

In this case, if a positive signal is emitted, the Regulator chooses an optimal sanction M^{*}, a factor reductor that $\breve{f} \leq f^{*} \leq \breve{\breve{f}}$, and a reassessment of the sanction of the magnitude $\theta_S^{*} = \bar{\alpha}$.[38] If a negative signal is issued, the Regulator chooses not to sanction, $M^{*} = 0$ (in this case, the factor reductor issue is redundant, this solution could also be denoted as $f^{*} = 1$), and a reassessment of the sanction of the magnitude $\theta_S^{*} = \tilde{\alpha}$.

The Regulated, on the other side, if there is a positive sign, will, in the first interaction with the Regulator, choose to ask for a suspensive effect having cooperated and pay in first instance, if it has not; additionally, in the second instance, it will pay despite cooperation. Alternatively, if there is a negative sign, it will, in the first interaction with the Regulator, choose to ask for a suspensive effect having cooperated and pay in first instance, if it has not; additionally, in the second instance, it will pay if it has cooperated or not, in the first place.

Figure 17 Regulation Compliance under medium first instance and high second instance incentives



Source: Author's elaboration.

The solutions derived from the optimal sanctioning of a medium first instance incentive are $f^{*} = \breve{\breve{f}}$ and $f^{*} = 1 - \beta_6^{*}$. The potential equilibriums obtained from both are below examined.

The firm will cooperate if:

$$\alpha[\Pi_R(1-\rho_A) - \bar{\alpha}M^{*}] + (1-\alpha)\Pi_R \geq \alpha'[\Pi_R + \Pi_I - (1-f^{*})M^{*}] + (1-\alpha')(\Pi_R + \Pi_I)$$

$$\Leftrightarrow f^{*} \leq 1 - \frac{\alpha}{\alpha'}\left(\frac{\rho_A \Pi_R}{M^{*}} + \bar{\alpha}\right) - \frac{\Pi_I}{M^{*}}$$

(Eq. 29)

3.3.4.2.1 $f^{*} = \breve{\breve{f}}$

Substituting $f^{*} = \breve{\breve{f}}$ and assuming that $\bar{\alpha} = w\frac{\rho_J \Pi_R}{M(1-\delta(\sigma_c=1))}$, the condition for cooperation equals to:

[38] Condition that $\lambda_{C.5.4} = \lambda_{C.5.3} = 0$. For exact formulas, see Appendix C.5.

$$\left(1-\frac{\alpha}{\alpha'}\right)\left(\rho_A\Pi_R+w\frac{\rho_J\Pi_R}{(1-\delta(\sigma_c=1))}\right)\geq\Pi_I$$

3.3.4.2.1.1 *Perfectly Informative Signals*

If $\alpha'-\alpha=1$, the cooperation condition would back to the regulation vs compliance proposition, which results in no cooperation.

3.3.4.2.1.2 *Imperfectly Informative Signals*

Consider now that $\alpha=\alpha'$. In this case, the cooperation condition would require:

$0\geq\frac{\Pi_I}{M^*}>0$

An impossibility, therefore, the firm does not cooperate.

3.3.4.2.1.3 *Informative Signals*

Consider now that $\frac{\alpha'}{\alpha}>1$, that is, it can be seen as a situation in which there is a reasonable amount of information, the cooperation condition would also back to the regulation vs compliance proposition, which results in no cooperation.

The point is that the firm does not cooperate in any scenario, but would it still be a second-best policy to pursue this possible equilibrium?

3.3.4.2.1.3.1 Perfect Bayesian Equilibrium

A Perfect Bayesian Equilibrium consists of strategy profiles and belief systems satisfying Requirements 1 to 4, specified at the beginning of the Game Solution Section.

The game has been resolved so far focusing on the strategies of players, now attention is attended to the beliefs of the Regulator, generically denoted in the calculations as $\psi, \mu, \Phi,$ and η. Note that all move decisions from the Regulated are from singleton information sets, therefore, this player's belief put probability one on every single decision node. Hence, the probability distribution of beliefs is defined, satisfying Requirement 1.

Sequential rationality requires each player's strategy to be payoff-maximising at each of his information sets, given his beliefs and the strategies of the others. Since the optimization problems of the players was constructed in a way that optimality was achieved considering strategies of the others as restrictions, sequential rationality is obtained, satisfying Requirement 2.

For Requirement 3, it is necessary to update the beliefs probabilities according to the Bayes' rule, considering this proposed scenario that firms would never cooperate, and check for consistency of the strategies adopted by the players.

According to the Bayes' rule, if for a medium first instance incentive, the decision of the firm would be always to not cooperate with regulation, then $\sigma_c=0$, which implies that:

$\psi=\frac{\alpha\sigma_c}{\alpha\sigma_c+\alpha'(1-\sigma_c)}=0$

$\Phi=\frac{\sigma_c(1-\alpha)}{\sigma_c(1-\alpha)+(1-\sigma_c)(1-\alpha')}=0$

That is, the Regulator would believe that there would not be cooperation from the Regulated. However, the solution obtained in the optimal sanctioning problem of not sanctioning is not consistent with the fact that the Regulator now knows there is no cooperation. Therefore, to maintain consistency, the Regulator would need to update its optimal sanction to $M = M^*$, even in the absence of a signal.

Hence, with this adjustment, the solution meets Requirement 3, indicating that this potential outcome attends the consistency necessary to qualify as a Perfect Bayesian Equilibrium.

In conclusion, the strategy profiles and belief systems of the following form does consist of a Perfect Bayesian Equilibrium:

$$\Omega_4 = \Big(\Big(NC, SE|(C,S), P1|(NC,S), P2|(C,S), P2|(NC,S); M = M^* \Big|S, f^* = \breve{\breve{f}}\Big| S, M = M^* \Big|NS, f^* = \breve{\breve{f}}\Big| NS, \theta_S^* = \bar{a}, \theta_{NS}^* = \tilde{a}\Big), \psi = \mu = 0, \Phi = \eta = 0\Big).$$

This would constitute a scenario of persistent sanctioning from the regulator, despite having positive or negative signals, once now it needs to cover the possibility of non-cooperation. The result is then an equilibrium in which administrative payment is indeed observed by the Regulated, if not in the first, certainly in second instance, with positive or negative signal.

Doubts arise, however, on the quality and dynamic system of incentives of this equilibrium, once the extended discount $\breve{\breve{f}}$ may leave regulated more prone to explore net gains of regulation trespassing, even though being in line with the resulting sanctioning process conducted by the regulator.

In essence, the prevailing institutional framework exists on the primary premise of safeguarding adherence to the approved regulation, the compliance itself present in this equilibrium not necessarily would drive society towards its social goals translated into the norms.

3.3.4.2.2 $f^* = 1 - \beta_6^*$

Recall from the medium first instance incentive problem solution (detailed in Appendix C.5) that $\beta_6^* = \sqrt{\frac{\alpha\sigma_c}{\alpha'(1-\sigma_c)}\left(\bar{a}^2 - 2\frac{\partial C_S^I(M^*)}{\partial M}\right) - 2\left(\frac{\partial CP(M^*)}{\partial M} + \mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M^*)}{\partial M}\right)}$

Substituting $f^* = 1 - \beta_6^*$ in (Eq. 29) and conducting some manipulations:

$$\Leftrightarrow \sqrt{\frac{\alpha\sigma_c}{\alpha'(1-\sigma_c)}\left(\bar{a}^2 - 2\frac{\partial C_S^I(M^*)}{\partial M}\right) - 2\left(\frac{\partial CP(M^*)}{\partial M} + \mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M^*)}{\partial M}\right)} \geq \frac{\alpha}{\alpha'}\left(\frac{\rho_A\Pi_R}{M^*} + \bar{a}\right) + \frac{\Pi_I}{M^*}$$

Contrary to the Regulation Compliance case under low second instance incentive, where there is no viable solution, here, there is a solution, but, strategy-wise, the firm never cooperates, despite the magnitudes of Nature's signals.

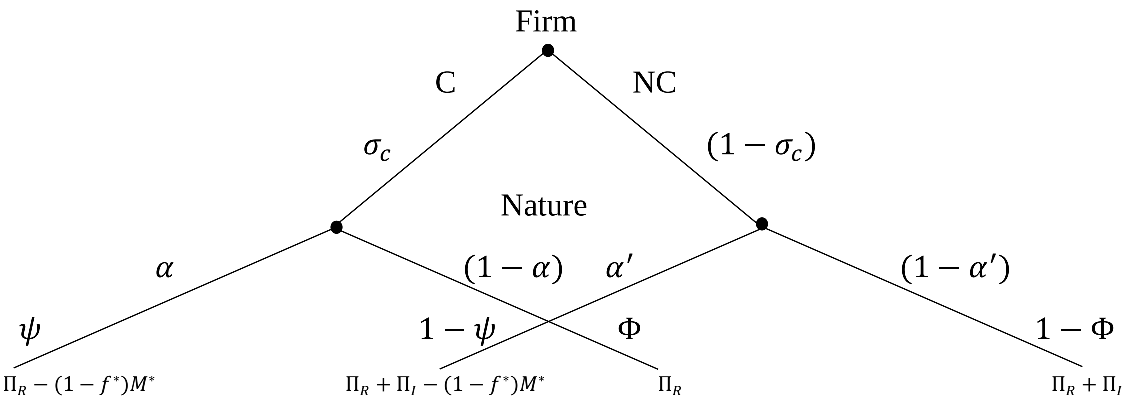
The consistency analysis is the same from the $f^* = \breve{\breve{f}}$ case, that is, administrative payment can be induced, but at the cost of higher infringements, once the equilibrium consists of non-cooperation in the first place.

3.3.4.3 High first instance incentive ($f^* \geq \breve{\breve{f}}$)

In this case, if a positive signal is emitted, the Regulator chooses an optimal sanction M^*, a factor reductor that $f^* \geq \breve{\breve{f}}$, and a reassessment of the sanction of the magnitude $\theta_S^* = \bar{\alpha}$.[39] If a negative sign is issued, the Regulator chooses not to sanction, $M^* = 0$ (in this case, the factor reductor issue is redundant, this solution could also be denoted as $f^* = 1$), and a reassessment of the sanction of the magnitude $\theta_S^* = \tilde{\alpha}$.

The Regulated, on the other side, if there is a positive sign, will, in the first interaction with the Regulator, choose to pay in first instance despite having cooperated or not; additionally, in the second instance, it will pay either wise. Alternatively, if there is a negative sign, it will, in the first interaction with the Regulator, choose to pay in first instance despite having cooperated or not; additionally, in the second instance, it will pay if it has cooperated or not, in the first place.

Figure 18 Regulation Compliance under high first and second instance incentives



Source: Author's elaboration.

The firm will cooperate if:

$$\alpha[\Pi_R - (1-f^*)M^*] + (1-\alpha)\Pi_R \geq \alpha'[\Pi_R + \Pi_I - (1-f^*)M^*] + (1-\alpha')[\Pi_R + \Pi_I]$$

$$\Leftrightarrow (\alpha' - \alpha)(1-f^*)M^* \geq \Pi_I$$

Consider first that $f^* = \breve{\breve{f}} = 1 - \frac{\rho_A \Pi_R}{M^*} - \bar{\alpha}$, substituting this value in the condition for cooperation, the firm will cooperate if the net present value of regulation compliance is negative:

$$\Pi_I - (\alpha' - \alpha)[\rho_A \Pi_R + \bar{\alpha} M^*] \leq 0$$

Considering $\bar{\alpha} = w \frac{\rho_J \Pi_R}{M(1-\delta(\sigma_c = 1))}$, it could also be stated as:

$$\Pi_I - (\alpha' - \alpha)\left[\rho_A \Pi_R + w \frac{\rho_J \Pi_R}{(1-\delta(\sigma_c = 1))}\right] \leq 0$$

[39] Condition that $\lambda_{C.6.4} = \lambda_{C.6.3} = 0$. For exact formulas, see Appendix C.6.

This equation will always be positive, so regulated firms never comply with regulation. Here, not even the compliant firm asks for a suspensive effect in first instance, since, it would be economically interesting to finish the process faster with the high discount.

In conclusion, the firm does not cooperate if the Regulator offers a high first instance incentive for payment $f^* = \breve{\breve{f}}$. In addition, if the result is valid for $f^* = \breve{\breve{f}}$, it is automatically valid for $f^* = 1 - \beta_8^* > \breve{\breve{f}}$, since the cooperation condition would not be met by a greater extend, in this case. Given this dominant pure strategy, there are no mixed strategies for this scenario.

3.3.4.3.1 Perfect Bayesian Equilibrium

A Perfect Bayesian Equilibrium consists of strategy profiles and belief systems satisfying Requirements 1 to 4, specified at the beginning of the Game Solution Section.

The game has been resolved so far focusing on the strategies of players, now attention is attended to the beliefs of the Regulator, generically denoted in the calculations as $\psi, \mu, \Phi,$ and η. Note that all move decisions from the Regulated are from singleton information sets, therefore, this player's belief put probability one on every single decision node. Hence, the probability distribution of beliefs is defined, satisfying Requirement 1.

Sequential rationality requires each player's strategy to be payoff-maximising at each of his information sets, given his beliefs and the strategies of the others. Since the optimization problems of the players was constructed in a way that optimality was achieved considering strategies of the others as restrictions, sequential rationality is obtained, satisfying Requirement 2.

For Requirement 3, it is necessary to update the beliefs probabilities according the Bayes' rule, considering this proposed scenario that firms would never cooperate, and check for consistency of the strategies adopted by the players.

According to the Bayes' rule, if for a high first instance incentive, the decision of the firm would be always to not cooperate with regulation, then $\sigma_c = 0$, which implies that:

$$\psi = \frac{\alpha\sigma_c}{\alpha\sigma_c + \alpha'(1-\sigma_c)} = 0$$

$$\Phi = \frac{\sigma_c(1-\alpha)}{\sigma_c(1-\alpha) + (1-\sigma_c)(1-\alpha')} = 0$$

That is, the Regulator would believe that there would not be cooperation from the Regulated. However, the solution obtained in the optimal sanctioning problem of not sanctioning is not consistent with the fact that the Regulator now knows there is no cooperation.

Hence, this solution falls short in meeting Requirement 3, indicating that this potential outcome lacks the consistency necessary to qualify as a Perfect Bayesian Equilibrium.

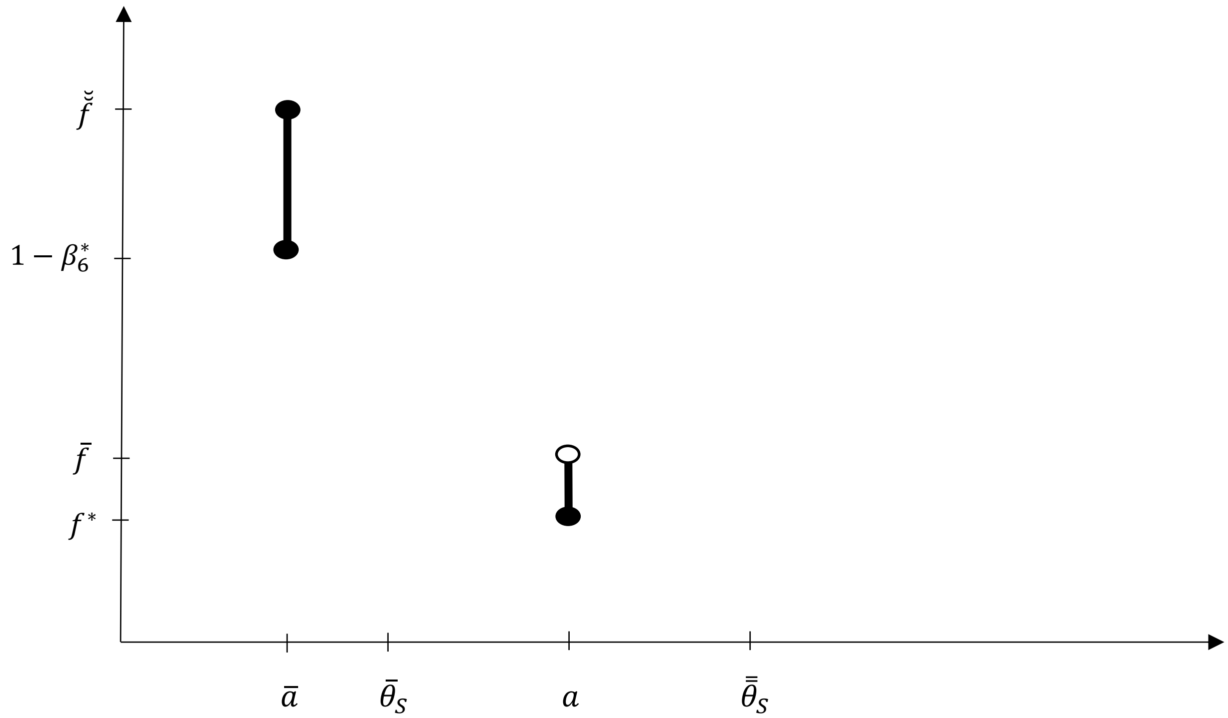
In conclusion, the strategy profiles and belief systems of the following form does not consist of a Perfect Bayesian Equilibrium:

$\Omega_5 = \Big(\Big(NC, P1|(C,S), P1|(NC,S), P2|(C,S), P2|(NC,S); M = M^* \Big| S, f^* = \breve{\breve{f}} \Big| S, M = 0|NS, f^* = 1|NS, \theta_S^* = \bar{a}, \theta_{NS}^* = \tilde{a}\Big), \psi = \mu = 0, \Phi = \eta = 0\Big).$

4 POLICY IMPLICATIONS

The guideline to the Regulator is then first to understand its true utility function $\hat{a}$, calculate its $\bar{\theta}_S$, $\bar{\bar{\theta}}_S$, $\bar{f}$ and $\bar{\bar{f}}$, based on cost modelling techniques, and to estimate the median value of θ_S^* and β_6^*. Today, the standardized value for f is $\frac{1}{4}$.

Figure 19 Second-best policy



Source: Author's elaboration.

If $\frac{1}{4} > \breve{\breve{f}}$ and $\hat{a} = \bar{a}$, then the Regulator should reduce f to $1 - \beta_6^*$ (choosing the minimum possible value for discount).

If $\frac{1}{4} < \bar{f}$ and $\hat{a} = a$, then the Regulator should opt for a *status quo* policy.

In either case, there might not be a scenario for leveraging f over a higher value of $\frac{1}{4}$.

However, if $\bar{f} < \frac{1}{4} < 1 - \beta_6^*$, then the decision would be to level up f to $1 - \beta_6^*$ or to reduce it down to below $\bar{f}$, which certainly depends on the magnitude of the estimated numbers, limiting the prospective analysis of this recommendation.

5 CONCLUSIONS

This study elaborated a game theory model in which the strategic actions of telecommunication Regulator and Regulated Firms are put into context: the Regulated decides to cooperate with regulation initially, while the Regulator, after receiving signals of possible infringements, responds ex-post with sanctioning processes. There are then two incentive mechanisms between Regulator and Regulated, in the first one, the latter can pay the fine with a discount f, while in the second one, it will receive a new penalty multiplied by a factor θ. Lastly, it bounds to the Regulated to pay the sanction in the administrative branch or to extend the plea to the judiciary.

The payoffs of both players were adjusted based on generic considerations of their respective welfare utility and profit functions. The game was then resolved with backward induction techniques, starting with the final decision of the firm whether or not to judicialize the process, which will depend on the varying assumptions on the sanction reassessment θ. This approach of identifying, case by case, possible separating and pooling equilibriums is convenient for posterior analysis on sequential rationality.

Afterwards, the sanction reassessment problem of the regulator was resolved, considering all possible ($\hat{a}$ varying) formulas of the utility function of the Regulator from the sanctioning process and its final objective function for this optimization problem. Two possible solutions were found for the case of a positive signal from Nature ($\theta_S^* = \bar{a}$, $\theta_S^* = a$), while for the negative signal, $\theta_{NS}^* = \tilde{a}$.

The decision of first instance payment from the Firm's perspective then consists of analysing its possible outcomes relying on the values of f and θ given its own cooperation decision. Those possible pooling and separating equilibrium were also examined.

The game then converges to the optimal sanctioning problem of the Regulator, which is how to respond on the magnitude of the penalty M and on the level of the discount f, utilized to encourage administrative processes. Note that, in this stage, all possible ranges of f and θ were considered, which means that the study was conducted in a way that the Regulator could set three types of first incentive mechanisms: low, medium, and high incentive policies. The optimality condition for sanctioning implies that there is an optimal fine M^* considering each process. Then, given the optimal fine M^*, belief consistency was checked.

After resolving for the positive and negative signals in each branch of the game, regulation compliance is explored, from the firm's viewpoint, considering all possible policy scenarios and different Nature's characteristics, confirming the existence or not of Bayesian equilibriums.

The first result (Perfect Bayesian Equilibrium Ω_1) worth of presenting relates to a first-best solution in which the Regulator tackles the main source of possible under deterrence: information asymmetry. This equilibrium, obtained with low first ($f^* < \bar{f}$) and second incentive policies ($\theta_S^* = a$), consists of the one in which there is regulation cooperation, under perfectly precision of Nature's signals. For consistency sake, the optimal sanction would have to take the form of the maximal sanction $\bar{M}$, that is, the penalty would take the form of a standardized previously published fine, not leaving space for the Regulator to fine-tune the magnitude of the sanction.

This highlights the importance of establishing a system of incentives for regulatory compliance. Such a framework aims to minimize information asymmetry and subjective decision-making processes.

By delineating clear procedures for investigation, assessment of violations, and imposition of penalties, regulators can foster a culture of regulatory adherence within the industry. A systematic approach to compliance not only enhances the efficacy of enforcement actions, but also installs confidence in stakeholders regarding the integrity and consistency of regulatory oversight. Therefore, investing in robust procedural requirements is paramount for regulatory authorities to uphold the rule of law, maintain market integrity, and safeguard public interests.

In summary, enforcement procedures based on rules rather than discretion can encourage regulated firms to cooperate with regulation, even with low discount schemes and sanctions below the excess profits of the firms.

In other words, achieving a state in which the signals of Nature are perfectly informative is the same as investing on institutional quality of the processes, converging into an improved relationship between Regulator and Regulated, under which regulation goals and possible sanctions are of common knowledge.

All remaining solutions found for any $f^* > \bar{f}$ ($\Omega_2, \Omega_2, \Omega_3, \Omega_4,$ and Ω_5) are based on pure strategies of non-cooperation from the firm's viewpoint. For $\Omega_2, \Omega_3,$ and Ω_5 no equilibriums were found. For the Perfect Bayesian Equilibrium Ω_4, an equilibrium in which the Regulator sets a medium first instance incentive in line with a high second instance incentive, the Regulator resets the optimal fine to M^* even without signalling.

This scenario presents a situation where the regulator persistently imposes sanctions, regardless of whether there are positive or negative signals, in order to account for the possibility of non-cooperation. As a result, an equilibrium is reached where regulated entities pay for the administrative penalties, either in the first instance or certainly in the second, regardless of the signal received.

However, doubts arise regarding the quality and dynamic nature of the incentives system in this equilibrium. The extended discount factor may make regulated entities more inclined to exploit net gains by violating regulations, even though they are in line with the resulting sanctioning process conducted by the regulator.

Essentially, the prevailing institutional framework is based on the primary premise of ensuring adherence to approved regulations. However, compliance within this equilibrium does not necessarily drive society towards its social goals as outlined in the norms.

In sum, once the incentive systems must me aligned, second-best policies should take the form of either low first and second instances incentives, or medium first and high second instances incentives. As results express, equilibrium might not exist in the former. This latter one, however, is already constructed based on the assumption of possible regulatory infringements, hence, second-best policies may be explored through this equilibrium.

In conclusion, the Regulator can improve its interaction with sector members and regulatory outcomes by focusing on two key areas: first, pursuing a Paretian approach of working with Nature by his side; and second, assessing its policy parameters to ensure they are consistent with theoretical principles.

6 APPENDIX

6.1 APPENDIX A

6.1.1 APPENDIX A.1

This constitutes the optimization problem for the Agency to determine θ_S constrained by the condition $\theta_S \leq \bar{\theta}_S$:

$$\max_{\theta_S} \mu[B(\theta_S M) - C_S^I(M)] + (1-\mu)[B(\theta_S M) - (1+\gamma_A)CP(M)] \quad \text{s.t.} \quad \theta_S \leq \frac{\rho_J \Pi_R}{M(1-\delta(\sigma_C=1))} = \bar{\theta}_S$$

Its Lagrangian is:

$$\mathcal{L} = B(\theta_S M) - \mu C_S^I(M) + (1-\mu)(1+\gamma_A)CP(M) - \lambda_{A.1.1}(\theta_S - \bar{\theta}_S)$$

The Kuhn-Tucker Conditions are:

1. $\frac{\partial \mathcal{L}}{\partial \theta_S} = 0$
   1.1. $\Leftrightarrow \frac{\partial B(\theta_S M)}{\partial(\theta_S M)} \frac{\partial(\theta_S M)}{\partial \theta_S} - \lambda_{A.1.1} = 0$
2. $\lambda_{A.1.1}(\theta_S - \bar{\theta}_S) = 0$
3. $\theta_S \leq \bar{\theta}_S$
4. $\lambda_{A.1.1} \geq 0$

There are three scenarios, if the maximum value for $B(\theta_S M)$ occurs before $\bar{\theta}_S$, between $\bar{\theta}_S$ and $\overline{\overline{\theta_S}}$, and after $\overline{\overline{\theta_S}}$. The three of them are below considered.

The generic formula of $B(\theta_S M)$ is $B(\theta_S M) = \left(\hat{a} - \frac{1}{2}\theta_S\right)\theta_S M$. As it shall be below defined, $\hat{a}$ can take the form of either $\bar{a}, a$ or $\bar{\bar{a}}$.

1.1 If $\arg\max_{\theta_S} B(\theta_S M) = \bar{a} < \bar{\theta}_S$

Consider, in this case, that $B(\theta_S M) = \left(\bar{a} - \frac{1}{2}\theta_S\right)\theta_S M$.

1.1.1 If $\lambda_{A.1.1} = 0$, $\frac{\partial B(\theta_S M)}{\partial(\theta_S M)} \frac{\partial(\theta_S M)}{\partial \theta_S} = 0 \Leftrightarrow \theta_S^* = \bar{a} < \bar{\theta}_S$

1.1.2 If $\lambda_{A.1.1} > 0 \Longrightarrow^2 \theta_S^* = \bar{\theta}_S$, but from Condition 1, with $\theta_S^* = \bar{\theta}_S$, it takes a negative value, which contradicts Condition 4 $\lambda_{A.1.1} = \frac{\partial B(\theta_S M)}{\partial(\theta_S M)} \frac{\partial(\theta_S M)}{\partial \theta_S} < 0$.

1.2 If $\bar{\theta}_S < \arg\max_{\theta_S} B(\theta_S M) = a < \overline{\overline{\theta_S}}$

Consider, in this case, that $B(\theta_S M) = \left(a - \frac{1}{2}\theta_S\right)\theta_S M$.

1.2.1 If $\lambda_{A.1.1} = 0, \frac{\partial B(\theta_S M)}{\partial(\theta_S M)} \frac{\partial(\theta_S M)}{\partial \theta_S} = 0 \Leftrightarrow \theta_S^* > \bar{\theta}_S$, which contradicts Condition 3.

1.2.2 If $\lambda_{A.1.1} > 0 \Longrightarrow^2 \theta_S^* = \bar{\theta}_S$, and from Condition 1, with $\theta_S^* = \bar{\theta}_S$, $\lambda_{A.1.1} = \frac{\partial B(\theta_S M)}{\partial(\theta_S M)} \frac{\partial(\theta_S M)}{\partial \theta_S} > 0$.

$$\lambda_{A.1.1}^* = M^2(a - \bar{\theta}_S)$$

1.3 If $\arg\max_{\theta_S} B(\theta_S M) = \bar{\bar{a}} > \overline{\overline{\theta_S}}$

Consider, in this case, that $B(\theta_S M) = \left(\bar{\bar{a}} - \frac{1}{2}\theta_S\right)\theta_S M$.

1.3.1 If $\lambda_{A.1.1} = 0, \frac{\partial B(\theta_S M)}{\partial(\theta_S M)} \frac{\partial(\theta_S M)}{\partial \theta_S} = 0 \Leftrightarrow \theta_S^* > \bar{\theta}_S$, which contradicts Condition 3.

1.3.2 If $\lambda_{A.1.1} > 0 \Longrightarrow^2 \theta_S^* = \bar{\theta}_S$, and from Condition 1, with $\theta_S^* = \bar{\theta}_S$, $\lambda_{A.1.1} = \frac{\partial B(\theta_S M)}{\partial(\theta_S M)} \frac{\partial(\theta_S M)}{\partial \theta_S} > 0$.

$$\lambda_{A.1.1}^* = M^2(\bar{\bar{a}} - \bar{\theta}_S)$$

Therefore, considering all possible scenarios, $\theta_S^* = min\{\hat{a}, \bar{\theta}_S\}$.

6.1.2 APPENDIX A.2

This constitutes the optimization problem for the Agency to determine θ_S constrained by the condition $\bar{\theta}_S \leq \theta_S \leq \overline{\overline{\theta_S}}$:

$$\max_{\theta_S} \mu[\delta(\sigma_c)B(\theta_S M) - C_S^I(M)] + (1-\mu)[B(\theta_S M) - (1+\gamma_A)CP(M)] \quad \text{s.t.} \quad \theta_S \geq \frac{\rho_J \Pi_R}{M(1-\delta(\sigma_c=1))} = \bar{\theta}_S \text{ and } \theta_S \leq \frac{\rho_J(\Pi_R + \Pi_I)}{M(1-\delta(\sigma_c=0))} = \overline{\overline{\theta_S}}$$

Its Lagrangian is:

$$\mathcal{L} = B(\theta_S M)[1 - \mu(1-\delta(\sigma_c))] - \mu C_S^I(M) - (1-\mu)(1+\gamma_A)CP(M) - \lambda_{A.2.1}\left(\theta_S - \overline{\overline{\theta_S}}\right) - \lambda_{A.2.2}(\bar{\theta}_S - \theta_S)$$

The Kuhn-Tucker Conditions are:

1. $\frac{\partial \mathcal{L}}{\partial \theta_S} = 0$
   1.1. $\Leftrightarrow \frac{\partial B(\theta_S M)}{\partial(\theta_S M)} \frac{\partial(\theta_S M)}{\partial \theta_S}\left[1 - \mu\left(1 - \delta(\sigma_c)\right)\right] - \lambda_{A.2.1} + \lambda_{A.2.2} = 0$
2. $\lambda_{A.2.1}(\theta_S - \overline{\overline{\theta_S}}) = 0$
3. $\lambda_{A.2.2}(\bar{\theta}_S - \theta_S) = 0$
4. $\theta_S \leq \overline{\overline{\theta_S}}$
5. $\theta_S \geq \bar{\theta}_S$
6. $\lambda_{A.2.1}, \lambda_{A.2.2} \geq 0$

There are three scenarios, if the maximum value for $B(\theta_S M)$ occurs before $\bar{\theta}_S$, between $\bar{\theta}_S$ and $\overline{\overline{\theta_S}}$, and after $\overline{\overline{\theta_S}}$. The three of them are below considered.

The generic formula of $B(\theta_S M)$ is $B(\theta_S M) = \left(\hat{a} - \frac{1}{2}\theta_S\right)\theta_S M$. As it shall be below defined, $\hat{a}$ can take the form of either $\bar{a}, a$ or $\bar{\bar{a}}$.

1.4 If $\underset{\theta_S}{arg\,max}\, B(\theta_S M) = \bar{a} < \bar{\theta}_S$

Consider, in this case, that $B(\theta_S M) = \left(\bar{a} - \frac{1}{2}\theta_S\right)\theta_S M$.

1.4.1 If $\lambda_{A.2.1} = \lambda_{A.2.2} = 0, \frac{\partial B(\theta_S M)}{\partial(\theta_S M)}\frac{\partial(\theta_S M)}{\partial\theta_S} = 0 \Leftrightarrow \theta_S^* = \bar{a} < \bar{\theta}_S$, which contradicts Condition 5.

1.4.2 If $\lambda_{A.2.1} = 0, \lambda_{A.2.2} > 0 \Longrightarrow^3 \theta_S^* = \bar{\theta}_S$, and from Condition 1, with $\theta_S^* = \bar{\theta}_S$, it takes a positive value, since $\bar{a} - \bar{\theta}_S < 0$, which is in line with Condition 6: $\lambda_{A.2.2}^* = -\frac{\partial B(\theta_S M)}{\partial(\theta_S M)}\frac{\partial(\theta_S M)}{\partial\theta_S}\left[1 - \mu\left(1 - \delta(\sigma_c)\right)\right] = -M(\bar{a} - \bar{\theta}_S)\left[1 - \mu\left(1 - \delta(\sigma_c)\right)\right] > 0$.

1.4.3 If $\lambda_{A.2.2} = 0, \lambda_{A.2.1} > 0 \Longrightarrow^2 \theta_S^* = \overline{\overline{\theta_S}}$, and from Condition 1, with $\theta_S^* = \overline{\overline{\theta_S}}$, it takes a positive value, since $\bar{a} - \bar{\theta}_S < 0$, which contradicts Condition 6: $\lambda_{A.2.1} = \frac{\partial B(\theta_S M)}{\partial(\theta_S M)}\frac{\partial(\theta_S M)}{\partial\theta_S}\left[1 - \mu\left(1 - \delta(\sigma_c)\right)\right] = M(\bar{a} - \bar{\theta}_S)\left[1 - \mu\left(1 - \delta(\sigma_c)\right)\right] < 0$.

1.4.4 If $\lambda_{A.2.1}, \lambda_{A.2.2} > 0, \Longrightarrow^{3,4} \theta_S^* = \bar{\theta}_S = \overline{\overline{\theta_S}}$, which is an impossibility, given $\bar{\theta}_S \ll \overline{\overline{\theta_S}}$.

Therefore, $\theta_S^* = \bar{\theta}_S$.

1.5 If $\bar{\theta}_S < \underset{\theta_S}{arg\,max}\, B(\theta_S M) = a < \overline{\overline{\theta_S}}$

Consider, in this case, that $B(\theta_S M) = \left(a - \frac{1}{2}\theta_S\right)\theta_S M$.

1.5.1 If $\lambda_{A.2.1} = \lambda_{A.2.2} = 0, \frac{\partial B(\theta_S M)}{\partial(\theta_S M)}\frac{\partial(\theta_S M)}{\partial\theta_S} = 0 \Leftrightarrow \theta_S^* = a$, which is in line with all Conditions. This is the internal optimal solution.

1.5.2 If $\lambda_{A.2.1} = 0, \lambda_{A.2.2} > 0 \Longrightarrow^3 \theta_S^* = \bar{\theta}_S$, and from Condition 1, with $\theta_S^* = \bar{\theta}_S$, $\lambda_{A.2.2}$ takes a negative value, since $a - \bar{\theta}_S > 0$, which contradicts Condition 6: $\lambda_{A.2.2} = -\frac{\partial B(\theta_S M)}{\partial(\theta_S M)}\frac{\partial(\theta_S M)}{\partial\theta_S}\left[1 - \mu\left(1 - \delta(\sigma_c)\right)\right] = -M(a - \bar{\theta}_S)\left[1 - \mu\left(1 - \delta(\sigma_c)\right)\right] < 0$.

1.5.3 If $\lambda_{A.2.2} = 0, \lambda_{A.2.1} > 0 \Longrightarrow^2 \theta_S^* = \overline{\overline{\theta_S}}$, and from Condition 1, with $\theta_S^* = \overline{\overline{\theta_S}}$, $\lambda_{A.2.1}$ takes a negative value, since $a - \overline{\overline{\theta_S}} < 0$, which contradicts Condition 6: $\lambda_{A.2.1} = \frac{\partial B(\theta_S M)}{\partial(\theta_S M)}\frac{\partial(\theta_S M)}{\partial\theta_S}\left[1 - \mu\left(1 - \delta(\sigma_c)\right)\right] = M\left(a - \overline{\overline{\theta_S}}\right)\left[1 - \mu\left(1 - \delta(\sigma_c)\right)\right] < 0$.

1.5.4 If $\lambda_{A.2.1}, \lambda_{A.2.2} > 0, \Longrightarrow^{3,4} \theta_S^* = \bar{\theta}_S = \overline{\overline{\theta_S}}$, which is an impossibility, given $\bar{\theta}_S \ll \overline{\overline{\theta_S}}$.

Therefore, $\theta_S^* = a$

1.6 If $\underset{\theta_S}{arg\,max}\, B(\theta_S M) = \bar{\bar{a}} > \overline{\overline{\theta_S}}$

Consider, in this case, that $B(\theta_S M) = \left(\bar{\bar{a}} - \frac{1}{2}\theta_S\right)\theta_S M$.

1.6.1 If $\lambda_{A.2.1} = \lambda_{A.2.2} = 0, \frac{\partial B(\theta_S M)}{\partial(\theta_S M)}\frac{\partial(\theta_S M)}{\partial\theta_S} = 0 \Leftrightarrow \theta_S^* = \bar{a} > \overline{\overline{\theta_S}}$, which contradicts Condition 4.

1.6.2 If $\lambda_{A.2.1} = 0, \lambda_{A.2.2} > 0 \Longrightarrow^{3} \theta_S^* = \bar{\theta}_S$, and from Condition 1, with $\theta_S^* = \bar{\theta}_S$, $\lambda_{A.2.2}$ takes a negative value, since $\bar{\bar{a}} - \bar{\theta}_S > 0$, which contradicts Condition 6: $\lambda_{A.2.2} = -\frac{\partial B(\theta_S M)}{\partial(\theta_S M)} \frac{\partial(\theta_S M)}{\partial \theta_S}\left[1 - \mu\left(1 - \delta(\sigma_c)\right)\right] = -M(\bar{\bar{a}} - \bar{\theta}_S)\left[1 - \mu\left(1 - \delta(\sigma_c)\right)\right] < 0.$

1.6.3 If $\lambda_{A.2.2} = 0, \lambda_{A.2.1} > 0 \Longrightarrow^{2} \theta_S^* = \overline{\overline{\theta_S}}$, and from Condition 1, with $\theta_S^* = \overline{\overline{\theta_S}}$, it represents the optimal corner solution, since $\bar{\bar{a}} - \overline{\overline{\theta_S}} > 0$, which is in line with all conditions: $\lambda_{A.2.1}^* = \frac{\partial B(\theta_S M)}{\partial(\theta_S M)} \frac{\partial(\theta_S M)}{\partial \theta_S}\left[1 - \mu\left(1 - \delta(\sigma_c)\right)\right] = M\left(\bar{\bar{a}} - \overline{\overline{\theta_S}}\right)\left[1 - \mu\left(1 - \delta(\sigma_c)\right)\right] > 0.$

1.6.4 If $\lambda_{A.2.1}, \lambda_{A.2.2} > 0, \Longrightarrow^{3,4} \theta_S^* = \bar{\theta}_S = \overline{\overline{\theta_S}}$, which is an impossibility, given $\bar{\theta}_S \ll \overline{\overline{\theta_S}}$.

Therefore, $\theta_S^* = \overline{\overline{\theta_S}}$.

Therefore, considering all possible scenarios,

$$\theta_S^* = \begin{cases} max\{\bar{a}, \bar{\theta}_S\}, & if\ arg \max_{\theta_S} B(\theta_S M) = \bar{a} < \bar{\theta}_S \\ a, & if\ \bar{\theta}_S < arg \max_{\theta_S} B(\theta_S M) = a < \overline{\overline{\theta_S}} \\ min\{\bar{\bar{a}}, \overline{\overline{\theta_S}}\}, & if\ arg \max_{\theta_S} B(\theta_S M) = \bar{\bar{a}} > \overline{\overline{\theta_S}} \end{cases}$$

6.1.3 APPENDIX A.3

This constitutes the optimization problem for the Agency to determine θ_S constrained by the condition $\theta_S \geq \overline{\overline{\theta_S}}$:

$$\max_{\theta_S} \mu[\delta(\sigma_c)B(\theta_S M) - C_S^I(M)] + (1-\mu)\left[\delta(\sigma_c)B(\theta_S M) - (1 + \gamma_A + \gamma_J)CP(M)\right] \quad \text{s.t.}$$

$$\theta_S \geq \frac{\rho_J(\Pi_R + \Pi_I)}{M(1 - \delta(\sigma_c = 0))} = \overline{\overline{\theta_S}}$$

Its Lagrangian is:

$$\mathcal{L} = \delta(\sigma_c)B(\theta_S M) - \mu C_S^I(M) - (1-\mu)(1 + \gamma_A + \gamma_J)CP(M) - \lambda_{A.3.1}(\overline{\overline{\theta_S}} - \theta_S)$$

The Kuhn-Tucker Conditions are:

5. $\frac{\partial \mathcal{L}}{\partial \theta_S} = 0$
   5.1. $\Leftrightarrow \delta(\sigma_c)\frac{\partial B(\theta_S M)}{\partial(\theta_S M)} \frac{\partial(\theta_S M)}{\partial \theta_S} + \lambda_{A.3.1} = 0$
6. $\lambda_{A.3.1}(\overline{\overline{\theta_S}} - \theta_S) = 0$
7. $\theta_S \geq \overline{\overline{\theta_S}}$
8. $\lambda_{A.3.1} \geq 0$

There are three scenarios, if the maximum value for $B(\theta_S M)$ occurs before $\bar{\theta}_S$, between $\bar{\theta}_S$ and $\overline{\overline{\theta_S}}$, and after $\overline{\overline{\theta_S}}$. The three of them are below considered.

The generic formula of $B(\theta_S M)$ is $B(\theta_S M) = \left(\hat{a} - \frac{1}{2}\theta_S\right)\theta_S M$. As it shall be below defined, $\hat{a}$ can take the form of either $\bar{a}, a$ or $\bar{\bar{a}}$.

1.7 If $arg \max_{\theta_S} B(\theta_S M) = \bar{a} < \bar{\theta}_S$

Consider, in this case, that $B(\theta_S M) = \left(\bar{a} - \frac{1}{2}\theta_S\right)\theta_S M$.

1.7.1 If $\lambda_{A.3.1} = 0, \frac{\partial B(\theta_S M)}{\partial(\theta_S M)}\frac{\partial(\theta_S M)}{\partial\theta_S} = 0 \Leftrightarrow \theta_S^* = \bar{a} < \overline{\overline{\theta_S}}$, violating Condition 3.

1.7.2 If $\lambda_{A.3.1} > 0 \Longrightarrow^2 \theta_S^* = \overline{\overline{\theta_S}}$, and from Condition 1, with $\theta_S^* = \overline{\overline{\theta_S}}$, it represents the optimal corner solution, since $\bar{a} - \overline{\overline{\theta_S}} < 0$, which is in line with all conditions: $\lambda_{A.3.1}^* = -\delta(\sigma_c)\frac{\partial B(\theta_S M)}{\partial(\theta_S M)}\frac{\partial(\theta_S M)}{\partial\theta_S} = -\delta(\sigma_c)M\left(\bar{a} - \overline{\overline{\theta_S}}\right) > 0$.

Therefore, $\theta_S^* = \overline{\overline{\theta_S}}$.

1.8 If $\bar{\theta}_S < arg\max_{\theta_S} B(\theta_S M) = a < \overline{\overline{\theta_S}}$

Consider, in this case, that $B(\theta_S M) = \left(a - \frac{1}{2}\theta_S\right)\theta_S M$.

1.8.1 If $\lambda_{A.3.1} = 0, \frac{\partial B(\theta_S M)}{\partial(\theta_S M)}\frac{\partial(\theta_S M)}{\partial\theta_S} = 0 \Leftrightarrow \theta_S^* = a < \overline{\overline{\theta_S}}$, violating Condition 3.

1.8.2 If $\lambda_{A.3.1} > 0 \Longrightarrow^2 \theta_S^* = \overline{\overline{\theta_S}}$, and from Condition 1, with $\theta_S^* = \overline{\overline{\theta_S}}$, it represents the optimal corner solution, since $a - \overline{\overline{\theta_S}} < 0$, which is in line with all conditions: $\lambda_{A.3.1}^* = \delta(\sigma_c)\frac{\partial B(\theta_S M)}{\partial(\theta_S M)}\frac{\partial(\theta_S M)}{\partial\theta_S} = -\delta(\sigma_c)M\left(a - \overline{\overline{\theta_S}}\right) > 0$.

Therefore, $\theta_S^* = \overline{\overline{\theta_S}}$.

1.9 If $arg\max_{\theta_S} B(\theta_S M) = \bar{\bar{a}} > \overline{\overline{\theta_S}}$

Consider, in this case, that $B(\theta_S M) = \left(\bar{\bar{a}} - \frac{1}{2}\theta_S\right)\theta_S M$.

1.9.1 If $\lambda_{A.3.1} = 0, \frac{\partial B(\theta_S M)}{\partial(\theta_S M)}\frac{\partial(\theta_S M)}{\partial\theta_S} = 0 \Leftrightarrow \theta_S^* = \bar{\bar{a}}$, which is in line with all Conditions. This is the internal optimal solution.

1.9.2 If $\lambda_{A.3.1} > 0 \Longrightarrow^2 \theta_S^* = \overline{\overline{\theta_S}}$, and from Condition 1, with $\theta_S^* = \bar{\theta}_S$, $\lambda_{A.3.1} = -\delta(\sigma_c)\frac{\partial B(\theta_S M)}{\partial(\theta_S M)}\frac{\partial(\theta_S M)}{\partial\theta_S} = -\delta(\sigma_c)M\left(\bar{\bar{a}} - \overline{\overline{\theta_S}}\right) < 0$, which violates Condition 4.

Therefore, $\theta_S^* = \bar{\bar{a}}$.

Therefore, considering all possible scenarios,

$$\theta_S^* = \begin{cases} \overline{\overline{\theta_S}}, & if\ arg\max_{\theta_S} B(\theta_S M) = \bar{a} < \bar{\theta}_S \\ \overline{\overline{\theta_S}}, & if\ \bar{\theta}_S < arg\max_{\theta_S} B(\theta_S M) = a < \overline{\overline{\theta_S}} \\ \bar{\bar{a}}, & if\ arg\max_{\theta_S} B(\theta_S M) = \bar{\bar{a}} > \overline{\overline{\theta_S}} \end{cases}$$

6.1.4 APPENDIX A.4

The regulator’s utility in the internal and corner optimal solutions are, respectively:

$$U^A(\bar{\bar{a}}) = \delta(\sigma_c)\frac{\bar{\bar{a}}^2}{2}M - \mu C_S^I(M) - (1-\mu)(1+\gamma_A+\gamma_J)CP(M)$$

$$U^A\left(\overline{\overline{\theta_S}}\right) = \left(\bar{\bar{a}} - \frac{\overline{\overline{\theta_S}}}{2}\right)\overline{\overline{\theta_S}}M[1-\mu(1-\delta(\sigma_c))] - \mu C_S^I(M) - (1-\mu)(1+\gamma_A)CP(M)$$

Therefore, the difference between these terms is:

$U^A(\bar{\bar{a}}) - U^A(\overline{\overline{\theta_S}}) = \delta(\sigma_c)\frac{\bar{\bar{a}}^2}{2}M - \left(\bar{\bar{a}} - \frac{\overline{\overline{\theta_S}}}{2}\right)\overline{\overline{\theta_S}}M\left[1 - \mu\left(1 - \delta(\sigma_c)\right)\right] - (1 -$
$\mu)\gamma_J CP(M)$

6.2 APPENDIX B

6.2.1 APPENDIX B.1

$B((1-f)M)$ is different from $B(\theta_i M)$ due to the distinct level of incentives. For the decision node in which the Agency receives the positive signal, f must be the maximum value that makes the firm to pay in first instance, in case it hasn't cooperated in the first place, but the minimum value that drives it to cooperate.

If $B((1-f)M) = B(\theta_S M) \Longrightarrow f^* = 1 - \theta_S^*$, always.

That is, if the function $B((1-f)M)$ is exogenous, there is no space for optimization regarding the variable f as its optimal value would be given by θ_S^*. This would go against the objective of the study, which is to determine optimal paths in relation to the first instance incentive. Therefore, the modelling of the function $B(.)$ is endogenous on first instance and exogenous in the second instance.

6.2.2 APPENDIX B.2

In the analysis of the decision of the firm either to pay in first instance or to contest, the premise implied that the threshold $\bar{\bar{f}}$ for the Compliant Firm would be greater than the one $\bar{f}$ for the Non-Compliant.

Before reaching to this conclusion, let's check these threshold limits.

Consider that the Non-Compliant Firm will choose to pay in first instance if:

$\Pi_R + \Pi_I - (1-f)M > (\Pi_R + \Pi_I)(1-\rho_A) - aM$

Initially, let's call this threshold as f^B:

$\Leftrightarrow f > 1 - \frac{\rho_A(\Pi_R + \Pi_I)}{M} - a = f^B$

Alternatively, the Compliant Firm will choose to pay in first instance if:

$\Pi_R - (1-f)M > \Pi_R\left(1 - \rho_A - \rho_J\right) - \delta(\sigma_c = 1)aM$

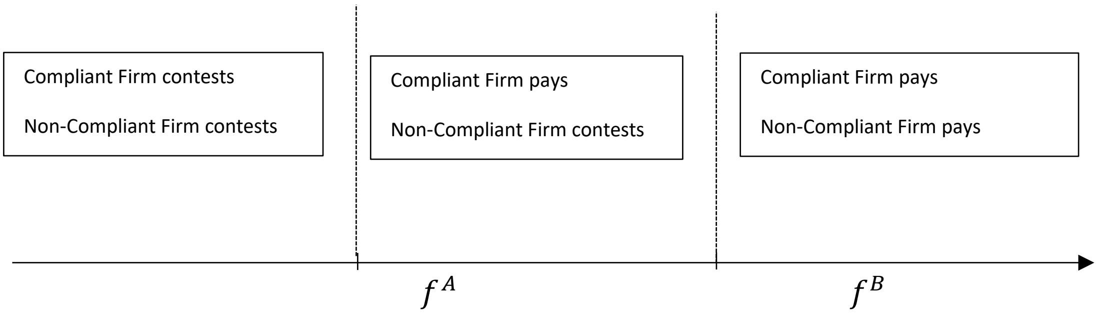
This threshold is defined firstly as f^A:

$\Leftrightarrow f > 1 - \frac{(\rho_A + \rho_J)\Pi_R}{M} - \delta(\sigma_c = 1)a = f^A$

The question is then if $f^A \lesseqgtr f^B$. This question can be answered logically and arithmetically. Let's consider first that $f^B > f^A$. In this situation, as outlined in

Figure *20*, analysing the spectrum of f, the Compliant Firm would pay the fine in first instance and the Non-Compliant Firm would contest, which is not consistent, strategically-wise.

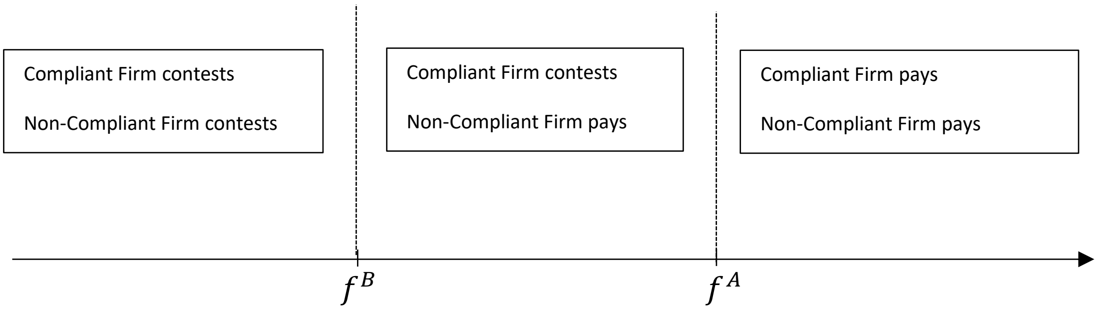
Figure 20 Spectrum of f if $f^B > f^A$



Source: Author’s elaboration.

However, if $f^A > f^B$, the spectrum analysis would be:

Figure 21 Spectrum of f if $f^A > f^B$



Source: Author’s elaboration.

Arithmetically, the difference $f^A - f^B$ between these values is:

$$f^A - f^B = a\big(1 - \delta(\sigma_c = 1)\big) + \frac{1}{M}\big(\rho_A \Pi_I - \rho_J \Pi_R\big)$$

(Eq. 30)

Considering a low value of type I Error from the judiciary system, after the process has passed through the administrative instances, the first term of $fA - f B = a\big(1 - \delta(\sigma_c = 1)\big) + \frac{1}{M}\big(\rho_A \Pi_I - \rho_J \Pi_R\big)$

(Eq. 30) converges to a, which is close to zero once it's the one associated with low second instance incentive – the Regulator possesses a $B(\theta_S M)$ function with its peak between $\overline{\theta_S}$ and $\overline{\overline{\theta_S}}$. The second term of the equation, on the other hand, despite being negative even if it is not assumed that legal costs are higher in the judiciary system than in the regulatory instance ($\rho_J > \rho_A$), because the standard operation of the telecommunication services Π_R surpasses by a considerable extent the excess profits derived from infringement of the sectoral legislation Π_I, that is $\Pi_R \gg \Pi_I$, it can be argue that the difference $\rho_A \Pi_I - \rho_J \Pi_R$ relative to the fine is considerable less than the unit. Therefore, it can be stated that $f^A - f^B > 0$.

6.3 APPENDIX C

6.3.1 APPENDIX C.1

This is the optimization problem of the Agency as to define M and f in the scenario of a low second instance incentive ($\theta_S^* = a$) and low first instance incentives ($f \leq \bar{f}$):

$$\underset{M,f}{\text{Max}}\, \psi[\delta(\sigma_c)B(\theta_S^* M) - C_S^I(M)] + (1-\psi)[B(\theta_S^* M) - (1+\gamma_A)CP(M)] \quad \text{s.t.} \quad f \leq \bar{f} \;,$$
$M \geq 0$ and $M \leq \bar{M} < \Pi_R$

Its Lagrangian is:

$$\mathcal{L} = \Big(1 - \psi\big(1 - \delta(\sigma_c)\big)\Big) B(\theta_S^* M) - \psi C_S^I(M) - (1-\psi)(1+\gamma_A)CP(M) - \lambda_{C.1.1}\big(f - \bar{f}\big) + \lambda_{C.1.2} M - \lambda_{C.1.3}(M - \bar{M})$$

The Kuhn-Tucker Conditions are:

1. $\frac{\partial \mathcal{L}}{\partial M} = 0$
    1.1. $\Leftrightarrow \Big(1 - \psi\big(1 - \delta(\sigma_c)\big)\Big) \frac{\partial B(\theta_S^* M)}{\partial(\theta_S^* M)} \frac{\partial(\theta_S^* M)}{\partial M} - \frac{\psi \partial C_S^I(M)}{\partial M} - (1-\psi)(1+\gamma_A)\frac{\partial CP(M)}{\partial M} + \lambda_{C.1.2} - \lambda_{C.1.3} = 0$
2. $\frac{\partial \mathcal{L}}{\partial f} = 0$
    2.1. $\Leftrightarrow -\lambda_{C.1.1} = 0$
3. $\lambda_{C.1.1}\big(f - \bar{f}\big) = 0$
4. $\lambda_{C.1.2} M = 0$
5. $\lambda_{C.1.3}(M - \bar{M})$=0
6. $f \leq \bar{f}$
7. $M \geq 0$
8. $M \leq \bar{M}$
9. $\lambda_{C.1.1}, \lambda_{C.1.2}, \lambda_{C.1.3} \geq 0$

From Condition 1:

$$\Big(1 - \psi\big(1 - \delta(\sigma_c)\big)\Big)\left(a - \frac{1}{2}\theta_S^*\right)\theta_S^* - \frac{\psi \partial C_S^I(M)}{\partial M} - (1-\psi)(1+\gamma_A)\frac{\partial CP(M)}{\partial M} = \lambda_{C.1.3} - \lambda_{C.1.2}$$

Since $\theta_S^* = a$,

$$\frac{1}{2}\left(1-\psi\left(1-\delta(\sigma_c)\right)\right)a^2-\frac{\psi\partial C_S^I(M)}{\partial M}-(1-\psi)(1+\gamma_A)\frac{\partial CP(M)}{\partial M}=\lambda_{C.1.3}-\lambda_{C.1.2}$$

It can be stated that $\frac{1}{2}\left(1-\psi\left(1-\delta(\sigma_c)\right)\right)a^2$ is the marginal benefit of sanctioning $BMgM$ the Regulated while $\frac{\psi\partial C_S^I(M)}{\partial M}+(1-\psi)(1+\gamma_A)\frac{\partial CP(M)}{\partial M}$ can be seen as the marginal cost of the sanction $CMgM$. In sum,

$$BMgM-CMgM=\lambda_{C.1.3}-\lambda_{C.1.2}$$

If $BMgM<CMgM$, then $\lambda_{C.1.2}>0\Rightarrow M=0$, from Condition 4.

If $BMgM=CMgM$, then $\lambda_{C.1.3}=\lambda_{C.1.2}=0\Rightarrow 0<M=M^*<\bar{M}$, from Conditions 1, 4, 5, 9.

If $BMgM>CMgM$, then $\lambda_{C.1.3}>0$, $\lambda_{C.1.2}=0\Rightarrow M=\bar{M}$

In sum, the optimal sanction depends a cost-benefit analysis: if the gains obtained from the process of sanctioning does not compensate for its costs, then the fine is null; if there is an optimal value for which the Regulator can set a “price” for the externality to be internalized, then there is an internal optimal solution M^*; but if the benefits for the society of sanctioning always surpasses the associated costs, then the fine is maximal $\bar{M}$, generally exogenously imposed by legislation.

From Condition 2, $\lambda_{C.1.1}=0\Longrightarrow f<\bar{f}$, from Condition 3.

Note that the function $\mathcal{L}$ does not count with the policy variable f.

6.3.2 APPENDIX C.2

This is the optimization problem of the Agency as to define M and f in the scenario of a low second instance incentive ($\theta_S^*=a$) and medium first instance incentives ($\bar{f}\leq f\leq\bar{\bar{f}}$):

$$\max_{M,f}\psi[\delta(\sigma_c)B(\theta_S^*M)-C_S^I(M)]+(1-\psi)[B((1-f)M)-CP(M)-$$
$$\mathrm{h}(\sigma_c)(q(k)C_S^{II}(M)+k)] \text{ s.t. } f\leq\bar{\bar{f}}, f\geq\bar{f}, M\geq 0 \text{ and } M\leq\bar{M}<\Pi_R$$

Its Lagrangian is:

$$\mathcal{L}=\max_{M,f}\psi[\delta(\sigma_c)B(\theta_S^*M)-C_S^I(M)]+(1-\psi)[B((1-f))-CP(M)-$$
$$\mathrm{h}(\sigma_c)(C_S^{II}(M)+k)]-\lambda_{C.2.1}\left(f-\bar{\bar{f}}\right)-\lambda_{C.2.2}\left(\bar{f}-f\right)+\lambda_{C.2.3}M-\lambda_{C.2.4}(M-\bar{M})$$

The Kuhn-Tucker Conditions are:

1. $\frac{\partial\mathcal{L}}{\partial M}=0$
   1.1. $\Leftrightarrow\psi\delta(\sigma_c)\frac{\partial B(\theta_S^*M)}{\partial(\theta_S^*M)}\frac{\partial(\theta_S^*M)}{\partial M}-\frac{\psi\partial C_S^I(M)}{\partial M}+(1-\psi)\left[\frac{\partial B((1-f)M)}{\partial((1-f)M)}\frac{\partial((1-f)M)}{\partial M}-\frac{\partial CP(M)}{\partial M}-\mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M)}{\partial M}\right]+\lambda_{C.2.3}-\lambda_{C.2.4}=0$
2. $\frac{\partial\mathcal{L}}{\partial f}=0$
   2.1. $\Leftrightarrow(1-\psi)\frac{\partial B\left((1-f)M\right)}{\partial(f)}-\lambda_{C.2.1}+\lambda_{C.2.2}=0$

3. $\lambda_{C.2.1}(f - \bar{\bar{f}}) = 0$
4. $-\lambda_{C.2.2}(\bar{f} - f) = 0$
5. $\lambda_{C.2.3}M = 0$
6. $\lambda_{C.2.4}(M - \bar{M}) = 0$
7. $f \leq \bar{\bar{f}}$
8. $f \geq \bar{f}$
9. $M \geq 0$
10. $M \leq \bar{M}$
11. $\lambda_{C.2.1}, \lambda_{C.2.2}, \lambda_{C.2.3}, \lambda_{C.2.4} \geq 0$

From Condition 1:

$$\psi\, \delta(\sigma_c)\left(a\theta_S^* - \frac{1}{2}\theta_S^{*2}\right) - \frac{\psi \partial C_S^I(M)}{\partial M} + (1-\psi)\left[\beta(1-f) - \frac{1}{2}(1-f)^2 - \frac{\partial CP(M)}{\partial M} - \mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M)}{\partial M}\right] + \lambda_{C.2.3} - \lambda_{C.2.4} = 0$$

$$\Leftrightarrow \psi\, \delta(\sigma_c)\left(\frac{1}{2}a^2\right) - \frac{\psi \partial C_S^I(M)}{\partial M} + (1-\psi)\left[\beta(1-f) - \frac{1}{2}(1-f)^2 - \frac{\partial CP(M)}{\partial M} - \mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M)}{\partial M}\right] = \lambda_{C.2.4} - \lambda_{C.2.3}$$

Following the same logic as Appendix C.1,

$BMgM - CMgM = \lambda_{C.2.4} - \lambda_{C.2.3}$

If $BMgM < CMgM$, then $\lambda_{C.2.3} > 0 \Rightarrow M = 0$, from Condition 4.

If $BMgM = CMgM$, then $\lambda_{C.2.4} = \lambda_{C.2.3} = 0 \Rightarrow 0 < M = M^* < \bar{M}$, from Conditions 1, 5, 6, 11.

If $BMgM > CMgM$, then $\lambda_{C.2.4} > 0$, $\lambda_{C.1.2} = 0 \Rightarrow M = \bar{M}$

In sum, the optimal sanction depends a cost-benefit analysis: if the gains obtained from the process of sanctioning does not compensate for its costs, then the fine is null; if there is an optimal value for which the Regulator can set a "price" for the externality to be internalized, then there is an internal optimal solution M^*; but if the benefits for the society of sanctioning always surpasses the associated costs, then the fine is maximal $\bar{M}$, generally exogenously imposed by legislation.

From Condition 2,

$(1-\psi)(-\beta + 1 - f)M = \lambda_{C.2.1} - \lambda_{C.2.2}$

$\Leftrightarrow f^* = 1 - \beta - \frac{(\lambda_{C.2.1} - \lambda_{C.2.2})}{(1-\psi)M}$

2.1 Considering $\lambda_{C.2.1} > 0 \implies^3 f^* = \bar{\bar{f}} \implies^4 \lambda_{C.2.2} = 0 \implies^2 f^* = 1 - \beta - \frac{\lambda_{C.2.1}}{(1-\psi)M}$ and $\lambda_{C.2.3} = \lambda_{C.2.4} = 0$, that is, there is an internal optimal solution for M ($M = M^*$).

From Condition 1:

$\psi\,\delta(\sigma_c)\left(\frac{1}{2}a^2\right)+(1-\psi)\left[\beta(1-f)-\frac{1}{2}(1-f)^2\right]-\frac{\psi\partial C_S^I(M^*)}{\partial M}-(1-\psi)\left[\frac{\partial CP(M^*)}{\partial M}+\right.$
$\left.\mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M^*)}{\partial M}\right]=0$

For simplicity on carrying on the calculations, consider:

$CMg_1=\frac{\partial C_S^I(M^*)}{\partial M}$ and

$CMg_2=\frac{\partial CP(M^*)}{\partial M}+\mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M^*)}{\partial M}$

Then the problem of an optimal factor reduction f converges to a quadratic equation of the form:

$f^2-2(1-\beta)f-(2\beta-1)-\frac{\psi}{1-\psi}\delta(\sigma_c)(a^2)+2\frac{\psi}{1-\psi}CMg_1+2CMg_2=0$

Which leads to an impossible first solution (given Condition 11) and the possible solution, given Δ:

$f=1-\beta\mp\frac{\sqrt{\Delta}}{2}$

$f=1-\beta+\frac{\sqrt{\Delta}}{2}\Longrightarrow\lambda_{C.2.1}=-(1-\psi)M\frac{\sqrt{\Delta}}{2}<0$ Impossible solution

$f=1-\beta-\frac{\sqrt{\Delta}}{2}\Longrightarrow\lambda_{C.2.1}=(1-\psi)M\frac{\sqrt{\Delta}}{2}>0$ Possible solution

$\Delta=4\left[\beta^2+2\left(\frac{\psi}{1-\psi}\left(CMg_1-\delta(\sigma_c)\left(\frac{1}{2}a^2\right)\right)+CMg_2\right)\right]$

Consider $l=\frac{\psi}{1-\psi}\left(CMg_1-\delta(\sigma_c)\left(\frac{1}{2}a^2\right)\right)+CMg_2>0$, then:

$\Delta=4(\beta^2+2l)\Leftrightarrow\sqrt{\Delta}\approx 2\beta+j$

Substituting the possible solution for f, under optimality:

$f^*\approx 1-2\beta-\frac{j}{2}$

Considering this approximation (note that there is no equality in the previous equation), and substituting the value for $\bar{\bar{f}}$, then the corner solution exists when $\beta=\beta_1^*$:

$\beta_1^*=\frac{1}{2}\left[\delta(\sigma_c=1)a+\frac{(\rho_A+\rho_J)\Pi_R}{M^*}-\frac{j}{2}\right]$

2.2 Now consider $\lambda_{C.2.2}>0\ \Longrightarrow^4\ f^*=\bar{f}\Longrightarrow^3\lambda_{C.2.1}=0$ and $M=M^*$

From Condition 2:

$f^*=1-\beta+\frac{\lambda_{C.2.2}}{(1-\psi)M}$

The calculations are analogous to the 2.1 section, in which:

$f=1-\beta+\frac{\sqrt{\Delta}}{2}$ and $\Delta=4(\beta^2+2l)$

$\bar{f} = f^* \approx 1 - \beta + \beta + \frac{j}{2} \approx 1 + \frac{j}{2} > 1 > \bar{f}$ Contradiction

Therefore, there is no corner solution when $f^* = \bar{f}$.

2.3 Consider now $\lambda_{C.2.1} = \lambda_{C.2.2} = 0 \Longrightarrow^5 f > \bar{f} \Longrightarrow^6 f < \bar{\bar{f}}$ and $M = M^*$

From Condition 2:

$f^* = 1 - \beta$

From Condition 1:

$\Delta = 0$

$\Leftrightarrow \ \beta_2^* = \sqrt{\frac{\psi}{1-\psi}(\delta(\sigma_c)a^2 - 2CMg_1) - 2CMg_2}$

Therefore, if $\beta = \beta_2^*$, there is an optimal internal solution[40] with $\bar{f} < f < \bar{\bar{f}}$.

6.3.3 APPENDIX C.3

This is the optimization problem of the Agency as to define M and f in the scenario of a low second instance incentive ($\theta_S^* = a$) and high first instance incentives ($f \geq \bar{\bar{f}}$):

$\underset{M,f}{\text{Max}}\, \psi\big[B\big((1-f)M\big) - C_S^I(M) + \mathrm{g}(\sigma_c)(\Sigma - k)\big] + (1-\psi)[B((1-f)M) - CP(M) - \mathrm{h}(\sigma_c)(C_S^{II}(M) + k)]$ s.t. $f \geq \bar{\bar{f}}$, $M \geq 0$ and $M \leq \bar{M} < \Pi_R$

Its Lagrangian is:

$\mathcal{L} = \underset{M,f}{\text{Max}}\, B\big((1-f)M\big) - \psi[C_S^I(M) - \mathrm{g}(\sigma_c)(\Sigma - k)] - (1-\psi)[CP(M) + \mathrm{h}(\sigma_c)(C_S^{II}(M) + k)] - \lambda_{C.3.1}\big(\bar{\bar{f}} - f\big) + \lambda_{C.3.2}M - \lambda_{C.3.3}(M - \bar{M})$

The Kuhn-Tucker Conditions are:

7. $\frac{\partial \mathcal{L}}{\partial M} = 0$
   7.1. $\Leftrightarrow \frac{\partial B((1-f)M)}{\partial((1-f)M)}\frac{\partial((1-f)M)}{\partial M} - \frac{\psi \partial C_S^I(M)}{\partial M} - (1-\psi)\frac{\partial CP(M)}{\partial M} - (1-\psi)\mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M)}{\partial M} + \lambda_{C.3.2} - \lambda_{C.3.3} = 0$
8. $\frac{\partial \mathcal{L}}{\partial f} = 0$
   8.1. $\Leftrightarrow \frac{\partial B((1-f)M)}{\partial(f)} + \lambda_{C.3.1} = 0$
9. $-\lambda_{C.3.1}\big(\bar{\bar{f}} - f\big) = 0$
10. $\lambda_{C.3.2}M = 0$
11. $\lambda_{C.3.3}(M - \bar{M}) = 0$
12. $f \geq \bar{\bar{f}}$
13. $M \geq 0$

[40] Note that the value of probability ψ still needs to be updated accordingly with Bayes's rule, which shall be done in the Regulation Compliance Section.

14. $M \leq \bar{M}$
15. $\lambda_{C.3.1}, \lambda_{C.3.2}, \lambda_{C.3.3} \geq 0$

From Condition 1:

$$\beta(1-f) - \frac{1}{2}(1-f)^2 - \frac{\psi \partial C_S^I(M)}{\partial M} - (1-\psi)\left[\frac{\partial CP(M)}{\partial M} + \mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M)}{\partial M}\right] + \lambda_{C.3.2} - \lambda_{C.3.3} = 0$$

Following the same logic as Appendix C.1 and C.2,

$$BMgM - CMgM = \lambda_{C.3.3} - \lambda_{C.3.2}$$

If $BMgM < CMgM$, then $\lambda_{C.3.2} > 0 \Rightarrow M = 0$, from Condition 4.

If $BMgM = CMgM$, then $\lambda_{C.3.3} = \lambda_{C.3.2} = 0 \Rightarrow 0 < M = M^* < \bar{M}$, from Conditions 1, 4, 5, 9.

If $BMgM > CMgM$, then $\lambda_{C.3.3} > 0$, $\lambda_{C.3.2} = 0 \Rightarrow M = \bar{M}$

In sum, the optimal sanction depends a cost-benefit analysis: if the gains obtained from the process of sanctioning does not compensate for its costs, then the fine is null; if there is an optimal value for which the Regulator can set a "price" for the externality to be internalized, then there is an internal optimal solution M^*; but if the benefits for the society of sanctioning always surpasses the associated costs, then the fine is maximal $\bar{M}$, generally exogenously imposed by legislation.

From Condition 2,

$$(-\beta + 1 - f)M = -\lambda_{C.3.1}$$

$$\Leftrightarrow f^* = 1 - \beta + \frac{\lambda_{C.3.1}}{M}$$

2.1 Considering $\lambda_{C.3.1} > 0 \implies$[3] $f^* = \bar{\bar{f}}$ and $\lambda_{C.3.2} = \lambda_{C.3.3} = 0$, that is, there is an internal optimal solution for M ($M = M^*$).

From Condition 1:

$$\beta(1-f) - \frac{1}{2}(1-f)^2 = \frac{\psi \partial C_S^I(M^*)}{\partial M} + (1-\psi)\left[\frac{\partial CP(M)}{\partial M} + \mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M^*)}{\partial M}\right]$$

For simplicity on carrying on the calculations, consider:

$$\frac{\psi \partial C_S^I(M^*)}{\partial M} + (1-\psi)\left[\frac{\partial CP(M)}{\partial M} + \mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M^*)}{\partial M}\right] = d$$

Then the problem of an optimal factor reduction f converges to a quadratic equation of the form:

$$f^2 - 2(1-\beta)f + (1-2\beta) - d = 0$$

Which leads to an impossible first solution (given Condition 9) and the possible solution, given Δ:

$$f = 1 - \beta \mp \frac{\sqrt{\Delta}}{2}$$

$f = 1 - \beta + \frac{\sqrt{\Delta}}{2} \Longrightarrow \lambda_{C.3.1} = -M\frac{\sqrt{\Delta}}{2} < 0$ Impossible solution

$f = 1 - \beta - \frac{\sqrt{\Delta}}{2} \Longrightarrow \lambda_{C.3.1} = M\frac{\sqrt{\Delta}}{2} > 0$ Possible solution

$\Delta = 4[\beta^2 - \psi CMg_1 - (1-\psi)CMg_2]$

Consider:

$\Delta = 4(\beta^2 - v) \Leftrightarrow \sqrt{\Delta} \approx 2\beta - m$

Substituting the possible solution for f, under optimality:

$f^* \approx 1 - \frac{m}{2}$

Considering this approximation (note that there is no equality in the previous equation), and substituting the value for $\bar{\bar{f}}$, then the corner solution exists when:

$f^* \approx 1 - \frac{m}{2} = \bar{\bar{f}} = 1 - \frac{(\rho_A + \rho_J)\Pi_R}{M^*} - \delta(\sigma_c = 1)a$

$2\left[\frac{(\rho_A + \rho_J)\Pi_R}{M^*} + \delta(\sigma_c = 1)a\right] = m = s(CMg_1, CMg_2, \psi)$, equation which determines β_3^*.

2.2 Now consider $\lambda_{C.3.1} = 0 \Longrightarrow^3 f^* > \bar{\bar{f}}$ and $M = M^*$

From Condition 2:

$f^* = 1 - \beta_4^* \Longrightarrow^1 \Delta = 0 \Leftrightarrow \beta^2 = \psi CMg_1 + (1-\psi)CMg_2$

$\Leftrightarrow \beta_4^* = \sqrt{\psi\frac{\partial C_S^I(M^*)}{\partial M} + (1-\psi)\left(\frac{\partial CP(M)}{\partial M} + \mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M^*)}{\partial M}\right)}$

Therefore, if $\beta = \beta_4^*$, there is an optimal internal solution with $f > \bar{\bar{f}}$.

6.3.4 APPENDIX C.4

This is the optimization problem of the Agency as to define M and f in the scenario of a high second instance incentive ($\theta_S^* = \bar{a}$) and low first instance incentives ($f \leq \breve{f}$):

$\underset{M,f}{\mathrm{Max}}\, \psi[B(\theta_S^* M) - C_S^I(M)] + (1-\psi)[B(\theta_S^* M) - (1+\gamma_A)CP(M)]$ s.t. $f \leq \breve{f}$, $M \geq 0$ and $M \leq \bar{M} < \Pi_R$

Its Lagrangian is:

$\mathcal{L} = B(\theta_S^* M) - \psi C_S^I(M) - (1-\psi)(1+\gamma_A)CP(M) - \lambda_{C.4.1}(f - \bar{f}) + \lambda_{C.4.2}M - \lambda_{C.4.3}(M - \bar{M})$

The Kuhn-Tucker Conditions are:

10. $\frac{\partial \mathcal{L}}{\partial M} = 0$

10.1. $\Leftrightarrow \frac{\partial B(\theta_S^* M)}{\partial(\theta_S^* M)}\frac{\partial(\theta_S^* M)}{\partial M} - \frac{\psi \partial C_S^I(M)}{\partial M} - (1-\psi)(1+\gamma_A)\frac{\partial CP(M)}{\partial M} + \lambda_{C.4.2} - \lambda_{C.4.3} = 0$

11. $\frac{\partial \mathcal{L}}{\partial f} = 0$
    11.1. $\Leftrightarrow -\lambda_{C.4.1} = 0$
12. $\lambda_{C.4.1}\left(f - \bar{f}\right) = 0$
13. $\lambda_{C.4.2}M = 0$
14. $\lambda_{C.4.3}(M - \bar{M})$=0
15. $f \leq \breve{f}$
16. $M \geq 0$
17. $M \leq \bar{M}$
18. $\lambda_{C.4.1}, \lambda_{C.4.2}, \lambda_{C.4.3} \geq 0$

From Condition 1:

$$\left(\bar{a} - \frac{1}{2}\theta_S^*\right)\theta_S^* - \frac{\psi \partial C_S^I(M)}{\partial M} - (1-\psi)(1+\gamma_A)\frac{\partial CP(M)}{\partial M} = \lambda_{C.4.3} - \lambda_{C.4.2}$$

Since $\theta_S^* = \bar{a}$,

$$\frac{\bar{a}^2}{2} - \frac{\psi \partial C_S^I(M)}{\partial M} - (1-\psi)(1+\gamma_A)\frac{\partial CP(M)}{\partial M} = \lambda_{C.4.3} - \lambda_{C.4.2}$$

It can be stated that $\frac{\bar{a}^2}{2}$ is the marginal benefit of sanctioning $BMgM$ the Regulated, while $\frac{\psi \partial C_S^I(M)}{\partial M} + (1-\psi)(1+\gamma_A)\frac{\partial CP(M)}{\partial M}$ can be seen as the marginal cost of the sanction $CMgM$. In sum,

$$BMgM - CMgM = \lambda_{C.4.3} - \lambda_{C.4.2}$$

If $BMgM < CMgM$, then $\lambda_{C.4.2} > 0 \Rightarrow M = 0$, from Condition 4.

If $BMgM = CMgM$, then $\lambda_{C.4.3} = \lambda_{C.4.2} = 0 \Rightarrow 0 < M = M^* < \bar{M}$, from Conditions 1, 4, 5, 9.

If $BMgM > CMgM$, then $\lambda_{C.4.3} > 0$, $\lambda_{C.4.2} = 0 \Rightarrow M = \bar{M}$

In sum, the optimal sanction depends a cost-benefit analysis: if the gains obtained from the process of sanctioning does not compensate for its costs, then the fine is null; if there is an optimal value for which the Regulator can set a "price" for the externality to be internalized, then there is an internal optimal solution M^*; but if the benefits for the society of sanctioning always surpasses the associated costs, then the fine is maximal $\bar{M}$, generally exogenously imposed by legislation.

From Condition 2, $\lambda_{C.4.1} = 0 \Longrightarrow f < \breve{f}$, from Condition 3.

Note that the function $\mathcal{L}$ does not count with the policy variable f.

6.3.5 APPENDIX C.5

This is the optimization problem of the Agency as to define M and f in the scenario of a high second instance incentive ($\theta_S^* = \bar{a}$) and medium first instance incentives ($\breve{f} \leq f \leq \breve{\breve{f}}$):

$\underset{M,f}{\text{Max}}\,\psi[B(\theta_S^* M) - C_S^I(M)] + (1-\psi)[B((1-f)M) - CP(M) - \mathrm{h}(\sigma_c)(C_S^{II}(M) + k)]$

s.t. $f \leq \breve{\breve{f}}, f \geq \breve{f}, M \geq 0$ and $M \leq \bar{M} < \Pi_R$

Its Lagrangian is:

$\mathcal{L} = \underset{M,f}{\text{Max}}\,\psi[B(\theta_S^* M) - C_S^I(M)] + (1-\psi)[B((1-f)) - CP(M) - \mathrm{h}(\sigma_c)(C_S^{II}(M) + k)] - \lambda_{C.5.1}\left(f - \breve{\breve{f}}\right) - \lambda_{C.5.2}\left(\breve{f} - f\right) + \lambda_{C.5.3}M - \lambda_{C.5.4}(M - \bar{M})$

The Kuhn-Tucker Conditions are:

12. $\frac{\partial\mathcal{L}}{\partial M} = 0$
    12.1. $\Leftrightarrow \psi\frac{\partial B(\theta_S^* M)}{\partial(\theta_S^* M)}\frac{\partial(\theta_S^* M)}{\partial M} - \frac{\psi\partial C_S^I(M)}{\partial M} + (1-\psi)\left[\frac{\partial B((1-f)M)}{\partial((1-f)M)}\frac{\partial((1-f)M)}{\partial M} - \frac{\partial CP(M)}{\partial M} - \mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M)}{\partial M}\right] + \lambda_{C.5.3} - \lambda_{C.5.4} = 0$
13. $\frac{\partial\mathcal{L}}{\partial f} = 0$
    13.1. $\Leftrightarrow (1-\psi)\frac{\partial B((1-f)M)}{\partial(f)} - \lambda_{C.5.1} + \lambda_{C.5.2} = 0$
14. $\lambda_{C.5.1}\left(f - \breve{\breve{f}}\right) = 0$
15. $-\lambda_{C.5.2}\left(\breve{f} - f\right) = 0$
16. $\lambda_{C.5.3}M = 0$
17. $\lambda_{C.5.4}(M - \bar{M}) = 0$
18. $f \leq \breve{\breve{f}}$
19. $f \geq \breve{f}$
20. $M \geq 0$
21. $M \leq \bar{M}$
22. $\lambda_{C.5.1}, \lambda_{C.5.2}, \lambda_{C.5.3}, \lambda_{C.5.4} \geq 0$

From Condition 1:

$\psi\,\left(\bar{a}\theta_S^* - \frac{1}{2}\theta_S^{*2}\right) - \frac{\psi\partial C_S^I(M)}{\partial M} + (1-\psi)\left[\beta(1-f) - \frac{1}{2}(1-f)^2 - \frac{\partial CP(M)}{\partial M} - \mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M)}{\partial M}\right] + \lambda_{C.5.3} - \lambda_{C.5.4} = 0$

$\Leftrightarrow \psi\,\left(\frac{1}{2}\bar{a}^2\right) - \frac{\psi\partial C_S^I(M)}{\partial M} + (1-\psi)\left[\beta(1-f) - \frac{1}{2}(1-f)^2 - \frac{\partial CP(M)}{\partial M} - \mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M)}{\partial M}\right] = \lambda_{C.5.4} - \lambda_{C.5.3}$

Following the same logic as Appendix C.4,

$BMgM - CMgM = \lambda_{C.5.4} - \lambda_{C.5.3}$

If $BMgM < CMgM$, then $\lambda_{C.5.3} > 0 \Rightarrow M = 0$, from Condition 4.

If $BMgM = CMgM$, then $\lambda_{C.5.4} = \lambda_{C.5.3} = 0 \Rightarrow 0 < M = M^* < \bar{M}$, from Conditions 1, 5, 6, 11.

If $BMgM > CMgM$, then $\lambda_{C.5.4} > 0, \lambda_{C.5.2} = 0 \Rightarrow M = \bar{M}$

In sum, the optimal sanction depends a cost-benefit analysis: if the gains obtained from the process of sanctioning does not compensate for its costs, then the fine is null; if there is an optimal value for which the Regulator can set a "price" for the externality to be internalized, then there is an internal optimal solution M^*; but if the benefits for the society of sanctioning always surpasses the associated costs, then the fine is maximal $\overline{M}$, generally exogenously imposed by legislation.

From Condition 2,

$$(1-\psi)(-\beta+1-f)M = \lambda_{C.5.1} - \lambda_{C.5.2}$$

$$\Leftrightarrow f^* = 1-\beta-\frac{(\lambda_{C.5.1}-\lambda_{C.5.2})}{(1-\psi)M}$$

2.1 Considering $\lambda_{C.5.1} > 0 \Longrightarrow^3 f^* = \breve{\breve{f}} \Longrightarrow^4 \lambda_{C.5.2} = 0 \Longrightarrow^2 f^* = 1-\beta-\frac{\lambda_{C.5.1}}{(1-\psi)M}$ and $\lambda_{C.5.3} = \lambda_{C.5.4} = 0$, that is, there is an internal optimal solution for M ($M = M^*$).

From Condition 1:

$$\psi\left(\frac{1}{2}\bar{a}^2\right) + (1-\psi)\left[\beta(1-f) - \frac{1}{2}(1-f)^2\right] - \frac{\psi \partial C_S^I(M^*)}{\partial M} - (1-\psi)\left[\frac{\partial CP(M^*)}{\partial M} + \mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M^*)}{\partial M}\right] = 0$$

For simplicity on carrying on the calculations, consider:

$CMg_1 = \frac{\partial C_S^I(M^*)}{\partial M}$ and

$CMg_2 = \frac{\partial CP(M^*)}{\partial M} + \mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M^*)}{\partial M}$

Then the problem of an optimal factor reduction f converges to a quadratic equation of the form:

$$f^2 - 2(1-\beta)f - (2\beta-1) - \frac{\psi}{1-\psi}(\bar{a}^2) + 2\frac{\psi}{1-\psi}CMg_1 + 2CMg_2 = 0$$

Which leads to an impossible first solution (given Condition 11) and the possible solution, given Δ:

$$f = 1-\beta \mp \frac{\sqrt{\Delta}}{2}$$

$f = 1-\beta+\frac{\sqrt{\Delta}}{2} \Longrightarrow \lambda_{C.2.1} = -(1-\psi)M\frac{\sqrt{\Delta}}{2} < 0$ Impossible solution

$f = 1-\beta-\frac{\sqrt{\Delta}}{2} \Longrightarrow \lambda_{C.2.1} = (1-\psi)M\frac{\sqrt{\Delta}}{2} > 0$ Possible solution

$$\Delta = 4\left[\beta^2 + 2\left(\frac{\psi}{1-\psi}\left(CMg_1 - \left(\frac{1}{2}\bar{a}^2\right)\right) + CMg_2\right)\right]$$

Consider $\breve{l} = \frac{\psi}{1-\psi}\left(CMg_1 - \left(\frac{1}{2}\bar{a}^2\right)\right) + CMg_2 > 0$, then:

$$\Delta = 4\left(\beta^2 + 2\breve{l}\right) \Leftrightarrow \sqrt{\Delta} \approx 2\beta + \breve{j}$$

Substituting the possible solution for f, under optimality:

$f^* \approx 1 - 2\beta - \frac{\breve{J}}{2}$

Considering this approximation (note that there is no equality in the previous equation), and substituting the value for $\breve{\breve{f}}$, then the corner solution exists when $\beta = \beta_5^*$:

$\beta_5^* = \frac{1}{2}\left[\bar{a} + \frac{\rho_A \Pi_R}{M^*} - \frac{\breve{J}}{2}\right]$

2.2 Now consider $\lambda_{C.5.2} > 0 \implies^4 f^* = \breve{f} \implies^3 \lambda_{C.5.1} = 0$ and $M = M^*$

From Condition 2:

$f^* = 1 - \beta + \frac{\lambda_{C.5.2}}{(1-\psi)M}$

The calculations are analogous to the 2.1 section, in which:

$f = 1 - \beta + \frac{\sqrt{\Delta}}{2}$ and $\Delta = 4(\beta^2 + 2l)$

$\breve{f} = f^* \approx 1 - \beta + \beta + \frac{\breve{J}}{2} \approx 1 + \frac{\breve{J}}{2} > 1 > \breve{f}$ Contradiction

Therefore, there is no corner solution when $f^* = \breve{f}$.

2.3 Consider now $\lambda_{C.5.1} = \lambda_{C.5.2} = 0 \implies^5 f > \breve{f} \implies^6 f < \breve{\breve{f}}$ and $M = M^*$

From Condition 2:

$f^* = 1 - \beta$

From Condition 1:

$\Delta = 0$

$\Leftrightarrow \beta_6^* = \sqrt{\frac{\psi}{1-\psi}(\bar{a}^2 - 2CMg_1) - 2CMg_2}$

Therefore, if $\beta = \beta_6^*$, there is an optimal internal solution[41] with $\breve{f} < f < \breve{\breve{f}}$.

6.3.6 APPENDIX C.6

This is the optimization problem of the Agency as to define M and f in the scenario of a high second instance incentive ($\theta_S^* = \bar{a}$) and high first instance incentives ($f \geq \breve{\breve{f}}$):

$\underset{M,f}{\text{Max}}\, \psi\big[B\big((1-f)M\big) - C_S^I(M) + \mathrm{g}(\sigma_c)(\Sigma - k)\big] + (1-\psi)[B((1-f)M) - CP(M) - \mathrm{h}(\sigma_c)(C_S^{II}(M) + k)]$ s.t. $f \geq \breve{\breve{f}}$, $M \geq 0$ and $M \leq \bar{M} < \Pi_R$

Its Lagrangian is:

[41] Note that the value of probability ψ still needs to be updated accordingly with Bayes's rule, which shall be done in the Regulation Compliance Section.

$$\mathcal{L} = \underset{M,f}{\operatorname{Max}} B\big((1-f)M\big) - \psi[C_S^I(M) - \mathrm{g}(\sigma_c)(\Sigma - k)] - (1-\psi)[CP(M) + \mathrm{h}(\sigma_c)(C_S^{II}(M) + k)] - \lambda_{C.6.1}\left(\breve{\breve{f}} - f\right) + \lambda_{C.6.2}M - \lambda_{C.6.3}(M - \bar{M})$$

The Kuhn-Tucker Conditions are:

16. $\frac{\partial \mathcal{L}}{\partial M} = 0$

    16.1. $\Leftrightarrow \frac{\partial B\left((1-f)M\right)}{\partial\left((1-f)M\right)} \frac{\partial\left((1-f)M\right)}{\partial M} - \frac{\psi \partial C_S^I(M)}{\partial M} - (1-\psi)\frac{\partial CP(M)}{\partial M} - (1-\psi)\mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M)}{\partial M} + \lambda_{C.6.2} - \lambda_{C.6.3} = 0$

17. $\frac{\partial \mathcal{L}}{\partial f} = 0$

    17.1. $\Leftrightarrow \frac{\partial B\left((1-f)M\right)}{\partial(f)} + \lambda_{C.6.1} = 0$

18. $-\lambda_{C.6.1}\left(\breve{\breve{f}} - f\right) = 0$
19. $\lambda_{C.6.2}M = 0$
20. $\lambda_{C.6.3}(M - \bar{M}) = 0$
21. $f \geq \breve{\breve{f}}$
22. $M \geq 0$
23. $M \leq \bar{M}$
24. $\lambda_{C.6.1}, \lambda_{C.6.2}, \lambda_{C.6.3} \geq 0$

From Condition 1:

$$\beta(1-f) - \frac{1}{2}(1-f)^2 - \frac{\psi \partial C_S^I(M)}{\partial M} - (1-\psi)\left[\frac{\partial CP(M)}{\partial M} + \mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M)}{\partial M}\right] + \lambda_{C.6.2} - \lambda_{C.6.3} = 0$$

Following the same logic as Appendix C.4 and C.5,

$$BMgM - CMgM = \lambda_{C.6.3} - \lambda_{C.6.2}$$

If $BMgM < CMgM$, then $\lambda_{C.6.2} > 0 \Rightarrow M = 0$, from Condition 4.

If $BMgM = CMgM$, then $\lambda_{C.6.3} = \lambda_{C.6.2} = 0 \Rightarrow 0 < M = M^* < \bar{M}$, from Conditions 1, 4, 5, 9.

If $BMgM > CMgM$, then $\lambda_{C.6.3} > 0$, $\lambda_{C.6.2} = 0 \Rightarrow M = \bar{M}$

In sum, the optimal sanction depends a cost-benefit analysis: if the gains obtained from the process of sanctioning does not compensate for its costs, then the fine is null; if there is an optimal value for which the Regulator can set a “price” for the externality to be internalized, then there is an internal optimal solution M^*; but if the benefits for the society of sanctioning always surpasses the associated costs, then the fine is maximal $\bar{M}$, generally exogenously imposed by legislation.

From Condition 2,

$$(-\beta + 1 - f)M = -\lambda_{C.6.1}$$

$$\Leftrightarrow f^* = 1 - \beta + \frac{\lambda_{C.6.1}}{M}$$

2.1 Considering $\lambda_{C.6.1} > 0 \Longrightarrow^{3} f^{*} = \breve{\breve{f}}$ and $\lambda_{C.6.2} = \lambda_{C.6.3} = 0$, that is, there is an internal optimal solution for M ($M = M^{*}$).

From Condition 1:

$$\beta(1-f) - \frac{1}{2}(1-f)^2 = \frac{\psi \partial C_S^I(M^*)}{\partial M} + (1-\psi)\left[\frac{\partial CP(M)}{\partial M} + \mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M^*)}{\partial M}\right]$$

For simplicity on carrying on the calculations, consider:

$$\frac{\psi \partial C_S^I(M^*)}{\partial M} + (1-\psi)\left[\frac{\partial CP(M)}{\partial M} + \mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M^*)}{\partial M}\right] = d$$

Then the problem of an optimal factor reduction f converges to a quadratic equation of the form:

$$f^2 - 2(1-\beta)f + (1-2\beta) - d = 0$$

Which leads to an impossible first solution (given Condition 9) and the possible solution, given Δ:

$$f = 1 - \beta \mp \frac{\sqrt{\Delta}}{2}$$

$f = 1 - \beta + \frac{\sqrt{\Delta}}{2} \Longrightarrow \lambda_{C.6.1} = -M\frac{\sqrt{\Delta}}{2} < 0$ Impossible solution

$f = 1 - \beta - \frac{\sqrt{\Delta}}{2} \Longrightarrow \lambda_{C.6.1} = M\frac{\sqrt{\Delta}}{2} > 0$ Possible solution

$$\Delta = 4[\beta^2 - \psi CMg_1 - (1-\psi)CMg_2]$$

Consider:

$$\Delta = 4(\beta^2 - v) \Leftrightarrow \sqrt{\Delta} \approx 2\beta - m$$

Substituting the possible solution for f, under optimality:

$$f^* \approx 1 - \frac{m}{2}$$

Considering this approximation (note that there is no equality in the previous equation), and substituting the value for $\breve{\breve{f}}$, then the corner solution exists when:

$$f^* \approx 1 - \frac{m}{2} = \breve{\breve{f}} = 1 - \frac{\rho_A \Pi_R}{M^*} - \bar{a}$$

$2\left[\frac{\rho_A \Pi_R}{M^*} + \bar{a}\right] = \breve{m} = \check{s}(CMg_1, CMg_2, \psi)$, equation which determines β_7^*.

2.2 Now consider $\lambda_{C.6.1} = 0 \Longrightarrow^{3} f^* > \breve{\breve{f}}$ and $M = M^*$

From Condition 2:

$$f^* = 1 - \beta_8^* \Longrightarrow^{1} \Delta = 0 \Leftrightarrow \beta^2 = \psi CMg_1 + (1-\psi)CMg_2$$

$$\Leftrightarrow \beta_8^* = \sqrt{\psi\frac{\partial C_S^I(M^*)}{\partial M} + (1-\psi)\left(\frac{\partial CP(M)}{\partial M} + \mathrm{h}(\sigma_c)\frac{\partial C_S^{II}(M^*)}{\partial M}\right)}$$

Therefore, if $\beta = \beta_8^*$, there is an optimal internal solution with $f > \breve{\breve{f}}$.

6.4 APPENDIX D

6.4.1 APPENDIX D.1

This constitutes the optimization problem for the Agency to determine θ_{NS} constrained by the condition $\theta_{NS} \leq \bar{\theta}_{NS}$:

$$\underset{\theta_{NS}}{Max}\, \eta[B(\theta_{NS}M) - C_{NS}^{I}(M)] + (1-\eta)[B(\theta_{NS}M) - (1+\gamma_A)CP(M)]$$

s.t. $\theta_{NS} \leq \frac{\rho_J \Pi_R}{M(1-\delta_{NS}(\sigma_c=1))} = \bar{\theta}_{NS}$

Its Lagrangian is:

$$\mathcal{L} = B(\theta_{NS}M) - \eta C_{NS}^{ErroI}(M) - (1-\eta)(1+\gamma_A)CP(M) - \lambda_{D.1.1}(\theta_{NS} - \bar{\theta}_{NS})$$

The Kuhn-Tucker Conditions are:

1. $\frac{\partial \mathcal{L}}{\partial \theta_{NS}} = 0$
   1.1. $\Leftrightarrow \frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)} \frac{\partial(\theta_{NS}M)}{\partial \theta_{NS}} - \lambda_{D.1.1} = 0$
2. $\lambda_{D.1.1}(\theta_{NS} - \bar{\theta}_{NS}) = 0$
3. $\theta_{NS} \leq \bar{\theta}_{NS}$
4. $\lambda_{D.1.1} \geq 0$

There are three scenarios, if the maximum value for $B(\theta_{NS}M)$ occurs before $\bar{\theta}_{NS}$, between $\bar{\theta}_{NS}$ and $\bar{\bar{\theta}}_{NS}$, and after $\bar{\bar{\theta}}_{NS}$. The three of them are below considered.

The generic formula of $B(\theta_{NS}M)$ is $B(\theta_{NS}M) = \left(\hat{a} - \frac{1}{2}\theta_{NS}\right)\theta_{NS}M$. As it shall be below defined, $\hat{a}$ can take the form of either $\tilde{a}, \dot{a}$ or $\tilde{\tilde{a}}$.

1.1 If $arg\,\underset{\theta_{NS}}{max}\, B(\theta_{NS}M) = \tilde{a} < \bar{\theta}_{NS}$

Consider, in this case, that $B(\theta_{NS}M) = \left(\tilde{a} - \frac{1}{2}\theta_{NS}\right)\theta_{NS}M$.

1.1.1 If $\lambda_{D.1.1} = 0$, $\frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)} \frac{\partial(\theta_{NS}M)}{\partial \theta_{NS}} = 0 \Leftrightarrow \theta_{NS}^* = \tilde{a} < \bar{\theta}_{NS}$

1.1.2 If $\lambda_{D.1.1} > 0 \Longrightarrow^2 \theta_{NS}^* = \bar{\theta}_S$, but from Condition 1, with $\theta_{NS}^* = \bar{\theta}_{NS}$, it takes a negative value, which contradicts Condition 4 $\lambda_{D.1.1} = \frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)} \frac{\partial(\theta_{NS}M)}{\partial \theta_{NS}} < 0$.

1.2 If $\bar{\theta}_{NS} < arg\,\underset{\theta_{NS}}{max}\, B(\theta_{NS}M) = \dot{a} < \bar{\bar{\theta}}_{NS}$

Consider, in this case, that $B(\theta_{NS}M) = \left(\dot{a} - \frac{1}{2}\theta_{NS}\right)\theta_{NS}M$.

1.2.1 If $\lambda_{D.1.1} = 0$, $\frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)} \frac{\partial(\theta_{NS}M)}{\partial \theta_{NS}} = 0 \Leftrightarrow \theta_{NS}^* > \bar{\theta}_{NS}$, which contradicts Condition 3.

1.2.2 If $\lambda_{D.1.1} > 0 \Longrightarrow^2 \theta_{NS}^* = \bar{\theta}_{NS}$, and from Condition 1, with $\theta_{NS}^* = \bar{\theta}_{NS}$, $\lambda_{D.1.1} = \frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)} \frac{\partial(\theta_{NS}M)}{\partial \theta_{NS}} > 0$.

$\lambda^*_{D.1.1} = M^2(\dot{a} - \bar{\theta}_{NS})$

1.3 If $\underset{\theta_{NS}}{arg\,max}\, B(\theta_{NS}M) = \tilde{\tilde{a}} > \bar{\bar{\theta}}_{NS}$

Consider, in this case, that $B(\theta_{NS}M) = \left(\tilde{\tilde{a}} - \frac{1}{2}\theta_{NS}\right)\theta_{NS}M$.

1.3.1 If $\lambda_{D.1.1} = 0$, $\frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_{NS}M)}{\partial\theta_{NS}} = 0 \Leftrightarrow \theta^*_{NS} > \bar{\theta}_{NS}$, which contradicts Condition 3.

1.3.2 If $\lambda_{D.1.1} > 0 \Longrightarrow^2 \theta^*_{NS} = \bar{\theta}_{NS}$, and from Condition 1, with $\theta^*_{NS} = \bar{\theta}_{NS}$, $\lambda_{D.1.1} = \frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_{NS}M)}{\partial\theta_{NS}} > 0$.

$\lambda^*_{D.1.1} = M^2(\tilde{\tilde{a}} - \bar{\theta}_{NS})$

Therefore, considering all possible scenarios, $\theta^*_{NS} = min\{\hat{a}, \bar{\theta}_{NS}\}$.

6.4.2 APPENDIX D.2

This constitutes the optimization problem for the Agency to determine θ_{NS} constrained by the condition $\bar{\theta}_{NS} \leq \theta_{NS} \leq \bar{\bar{\theta}}_{NS}$:

$\underset{\theta_{NS}}{Max}\, \eta\left[\delta_{NS}(\sigma_c)B(\theta_{NS}M) - (1+\gamma_A+\gamma_J)C^{ErroI}_{NS}(M)\right] + (1-\eta)[B(\theta_{NS}M) - (1+\gamma_A)CP(M)]$ s.t. $\theta_{NS} \geq \frac{\rho_J\Pi_R}{M(1-\delta_{NS}(\sigma_c=1))} = \bar{\theta}_{NS}$ and $\theta_{NS} \leq \frac{\rho_J(\Pi_R+\Pi_I)}{M(1-\delta_{NS}(\sigma_c=0))} = \bar{\bar{\theta}}_{NS}$

Its Lagrangian is:

$\mathcal{L} = B(\theta_{NS}M)[1-\eta(1-\delta_{NS}(\sigma_c))] - \eta(1+\gamma_A+\gamma_J)C^{ErroI}_{NS}(M) - (1-\eta)(1+\gamma_A)CP(M) - \lambda_{D.2.1}\left(\theta_{NS} - \bar{\bar{\theta}}_{NS}\right) - \lambda_{D.2.2}(\bar{\theta}_{NS} - \theta_S)$

The Kuhn-Tucker Conditions are:

1. $\frac{\partial\mathcal{L}}{\partial\theta_{NS}} = 0$
   1.1. $\Leftrightarrow \frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_{NS}M)}{\partial\theta_{NS}}\left[1-\eta\left(1-\delta_{NS}(\sigma_c)\right)\right] - \lambda_{D.2.1} + \lambda_{D.2.2} = 0$
2. $\lambda_{D.2.1}(\theta_{NS} - \bar{\bar{\theta}}_{NS}) = 0$
3. $\lambda_{D.2.2}(\bar{\theta}_{NS} - \theta_{NS}) = 0$
4. $\theta_{NS} \leq \bar{\bar{\theta}}_{NS}$
5. $\theta_{NS} \geq \bar{\theta}_{NS}$
6. $\lambda_{D.2.1}, \lambda_{D.2.2} \geq 0$

There are three scenarios, if the maximum value for $B(\theta_{NS}M)$ occurs before $\bar{\theta}_{NS}$, between $\bar{\theta}_{NS}$ and $\bar{\bar{\theta}}_{NS}$, and after $\bar{\bar{\theta}}_{NS}$. The three of them are below considered.

The generic formula of $B(\theta_{NS}M)$ is $B(\theta_{NS}M) = \left(\hat{a} - \frac{1}{2}\theta_{NS}\right)\theta_{NS}M$. As it shall be below defined, $\hat{a}$ can take the form of either $\tilde{a}, \dot{a}$ or $\tilde{\tilde{a}}$.

1.4 If $\underset{\theta_{NS}}{arg\,max}\, B(\theta_{NS}M) = \tilde{a} < \bar{\theta}_{NS}$

Consider, in this case, that $B(\theta_{NS}M) = \left(\tilde{a} - \frac{1}{2}\theta_{NS}\right)\theta_{NS}M$.

1.4.1 If $\lambda_{D.2.1} = \lambda_{D.2.2} = 0, \frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_{NS}M)}{\partial\theta_{NS}} = 0 \Leftrightarrow \theta_{NS}^* = \tilde{a} < \bar{\theta}_{NS}$, which contradicts Condition 5.

1.4.2 If $\lambda_{D.2.1} = 0, \lambda_{D.2.2} > 0 \Longrightarrow^3 \theta_{NS}^* = \bar{\theta}_{NS}$, and from Condition 1, with $\theta_{NS}^* = \bar{\theta}_{NS}$, it takes a positive value, since $\bar{a} - \bar{\theta}_{NS} < 0$, which is in line with Condition 6: $\lambda_{D.2.2}^* = -\frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_{NS}M)}{\partial\theta_{NS}}\left[1 - \eta\left(1 - \delta_{NS}(\sigma_c)\right)\right] = -M(\tilde{a} - \bar{\theta}_{NS})\left[1 - \eta\left(1 - \delta_{NS}(\sigma_c)\right)\right] > 0$.

1.4.3 If $\lambda_{D.2.2} = 0, \lambda_{D.2.1} > 0 \Longrightarrow^2 \theta_S^* = \bar{\bar{\theta}}_{NS}$, and from Condition 1, with $\theta_{NS}^* = \bar{\bar{\theta}}_{NS}$, it takes a positive value, since $\tilde{a} - \bar{\theta}_{NS} < 0$, which contradicts Condition 6: $\lambda_{D.2.1} = \frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_{NS}M)}{\partial\theta_{NS}}\left[1 - \eta\left(1 - \delta_{NS}(\sigma_c)\right)\right] = M(\tilde{a} - \bar{\theta}_{NS})\left[1 - \eta\left(1 - \delta_{NS}(\sigma_c)\right)\right] < 0$.

1.4.4 If $\lambda_{D.2.1}, \lambda_{D.2.2} > 0, \Longrightarrow^{3,4} \theta_{NS}^* = \bar{\theta}_{NS} = \bar{\bar{\theta}}_{NS}$, which is an impossibility, as $\bar{\theta}_{NS} \ll \bar{\bar{\theta}}_{NS}$.

Therefore, $\theta_{NS}^* = \bar{\theta}_{NS}$.

1.5 If $\bar{\theta}_{NS} < \underset{\theta_{NS}}{arg\,max}\, B(\theta_{NS}M) = \dot{a} < \bar{\bar{\theta}}_{NS}$

Consider, in this case, that $B(\theta_{NS}M) = \left(\dot{a} - \frac{1}{2}\theta_{NS}\right)\theta_{NS}M$.

1.5.1 If $\lambda_{D.2.1} = \lambda_{D.2.2} = 0, \frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_{NS}M)}{\partial\theta_{NS}} = 0 \Leftrightarrow \theta_{NS}^* = \dot{a}$, which is in line with all Conditions. This is the internal optimal solution.

1.5.2 If $\lambda_{D.2.1} = 0, \lambda_{D.2.2} > 0 \Longrightarrow^3 \theta_{NS}^* = \bar{\theta}_{NS}$, and from Condition 1, with $\theta_{NS}^* = \bar{\theta}_{NS}$, $\lambda_{D.2.2}$ takes a negative value, since $a - \bar{\theta}_{NS} > 0$, which contradicts Condition 6: $\lambda_{A.2.2} = -\frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_{NS}M)}{\partial\theta_{NS}}\left[1 - \mu\left(1 - \delta(\sigma_c)\right)\right] = -M(a - \bar{\theta}_S)\left[1 - \mu\left(1 - \delta(\sigma_c)\right)\right] < 0$.

1.5.3 If $\lambda_{A.2.2} = 0, \lambda_{A.2.1} > 0 \Longrightarrow^2 \theta_S^* = \bar{\bar{\theta}}_{NS}$, and from Condition 1, with $\theta_{NS}^* = \bar{\bar{\theta}}_{NS}$, $\lambda_{D.2.1}$ takes a negative value, since $\dot{a} - \bar{\bar{\theta}}_{NS} < 0$, which contradicts Condition 6: $\lambda_{D.2.1} = \frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_{NS}M)}{\partial\theta_{NS}}\left[1 - \eta\left(1 - \delta_{NS}(\sigma_c)\right)\right] = M\left(\dot{a} - \bar{\bar{\theta}}_{NS}\right)\left[1 - \eta\left(1 - \delta_{NS}(\sigma_c)\right)\right] < 0$.

1.5.4 If $\lambda_{D.2.1}, \lambda_{D.2.2} > 0, \Longrightarrow^{3,4} \theta_{NS}^* = \bar{\theta}_{NS} = \bar{\bar{\theta}}_{NS}$, which is an impossibility, as $\bar{\theta}_{NS} \ll \bar{\bar{\theta}}_{NS}$.

Therefore, $\theta_{NS}^* = \dot{a}$.

1.6 If $\underset{\theta_{NS}}{arg\,max}\, B(\theta_{NS}M) = \tilde{\tilde{a}} > \bar{\bar{\theta}}_{NS}$

Consider, in this case, that $B(\theta_{NS}M) = \left(\tilde{\tilde{a}} - \frac{1}{2}\theta_{NS}\right)\theta_{NS}M$.

1.6.1 If $\lambda_{D.2.1} = \lambda_{D.2.2} = 0, \frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_{NS}M)}{\partial\theta_{NS}} = 0 \Leftrightarrow \theta_{NS}^* = \tilde{a} > \bar{\bar{\theta}}_{NS}$, which contradicts Condition 4.

1.6.2 If $\lambda_{D.2.1} = 0, \lambda_{D.2.2} > 0 \Longrightarrow^3 \theta_{NS}^* = \bar{\theta}_{NS}$, and from Condition 1, with $\theta_{NS}^* = \bar{\theta}_{NS}$, $\lambda_{D.2.2}$ takes a negative value, since $\tilde{\tilde{a}} - \bar{\theta}_S > 0$, which contradicts Condition 6:

$\lambda_{D.2.2} = -\frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_{NS}M)}{\partial\theta_{NS}}\left[1-\eta\left(1-\delta_{NS}(\sigma_c)\right)\right] = -M\left(\tilde{\tilde{a}}-\bar{\theta}_{NS}\right)\left[1-\eta\left(1-\delta_{NS}(\sigma_c)\right)\right] < 0.$

1.6.3 If $\lambda_{D.2.2} = 0, \lambda_{D.2.1} > 0 \Longrightarrow^2 \theta^*_{NS} = \bar{\bar{\theta}}_{NS}$, and from Condition 1, with $\theta^*_{NS} = \bar{\bar{\theta}}_{NS}$, it represents the optimal corner solution, since $\tilde{\tilde{a}} - \bar{\bar{\theta}}_{NS} > 0$, which is in line with all conditions: $\lambda^*_{D.2.1} = \frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_{NS}M)}{\partial\theta_{NS}}\left[1-\eta\left(1-\delta_{NS}(\sigma_c)\right)\right] = M\left(\tilde{\tilde{a}}-\bar{\bar{\theta}}_{NS}\right)\left[1-\eta\left(1-\delta_{NS}(\sigma_c)\right)\right] > 0.$

1.6.4 If $\lambda_{D.2.1}, \lambda_{D.2.2} > 0, \Longrightarrow^{3,4} \theta^*_{NS} = \bar{\theta}_{NS} = \bar{\bar{\theta}}_{NS}$, which is an impossibility, as $\bar{\theta}_{NS} \ll \bar{\bar{\theta}}_{NS}$.

Therefore, $\theta^*_{NS} = \bar{\bar{\theta}}_{NS}$.

Therefore, considering all possible scenarios,

$$\theta^*_{NS} = \begin{cases} max\{\tilde{a}, \bar{\theta}_{NS}\}, & if\ arg\max_{\theta_{NS}} B(\theta_{NS}M) = \tilde{a} < \bar{\theta}_{NS} \\ \dot{a}, & if\ \bar{\theta}_{NS} < arg\max_{\theta_{NS}} B(\theta_{NS}M) = \dot{a} < \bar{\bar{\theta}}_{NS} \\ min\{\tilde{\tilde{a}}, \bar{\bar{\theta}}_{NS}\}, & if\ arg\max_{\theta_{NS}} B(\theta_{NS}M) = \tilde{\tilde{a}} > \bar{\bar{\theta}}_{NS} \end{cases}$$

6.4.3 APPENDIX D.3

This constitutes the optimization problem for the Agency to determine θ_{NS} constrained by the condition $\theta_{NS} \geq \bar{\bar{\theta}}_{NS}$:

$$\underset{\theta_{NS}}{Max}\,\eta\left[\delta_{NS}(\sigma_c)B(\theta_{NS}M) - (1+\gamma_A+\gamma_J)C^I_{NS}(M)\right] + (1-\eta)\left[\delta_{NS}(\sigma_c)B(\theta_{NS}M) - (1+\gamma_A+\gamma_J)CP(M)\right] \text{ s.t. } \theta_{NS} \geq \frac{\rho_J(\Pi_R+\Pi_I)}{M(1-\delta_{NS}(\sigma_c=0))} = \bar{\bar{\theta}}_{NS}$$

Its Lagrangian is:

$$\mathcal{L} = \delta_{NS}(\sigma_c)B(\theta_{NS}M) - \eta(1+\gamma_A+\gamma_J)C^{ErroI}_{NS}(M) - (1-\eta)(1+\gamma_A+\gamma_J)CP(M) - \lambda_{D.3.1}(\bar{\bar{\theta}}_{NS} - \theta_S)$$

The Kuhn-Tucker Conditions are:

5. $\frac{\partial\mathcal{L}}{\partial\theta_{NS}} = 0$
   5.1. $\Leftrightarrow \delta_{NS}(\sigma_c)\frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_{NS}M)}{\partial\theta_{NS}} + \lambda_{D.3.1} = 0$
6. $\lambda_{D.3.1}(\bar{\bar{\theta}}_{NS} - \theta_{NS}) = 0$
7. $\theta_{NS} \geq \bar{\bar{\theta}}_{NS}$
8. $\lambda_{D.3.1} \geq 0$

There are three scenarios, if the maximum value for $B(\theta_{NS}M)$ occurs before $\bar{\theta}_{NS}$, between $\bar{\theta}_{NS}$ and $\bar{\bar{\theta}}_{NS}$, and after $\bar{\bar{\theta}}_{NS}$. The three of them are below considered.

The generic formula of $B(\theta_{NS}M)$ is $B(\theta_{NS}M) = \left(\hat{a} - \frac{1}{2}\theta_{NS}\right)\theta_{NS}M$. As it shall be below defined, $\hat{a}$ can take the form of either $\tilde{a}, \dot{a}$ or $\tilde{\tilde{a}}$.

1.7 If $arg\max_{\theta_{NS}} B(\theta_{NS}M) = \tilde{a} < \bar{\theta}_{NS}$

Consider, in this case, that $B(\theta_{NS}M) = \left(\tilde{a} - \frac{1}{2}\theta_{NS}\right)\theta_{NS}M$.

1.7.1 If $\lambda_{D.3.1} = 0, \frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_{NS}M)}{\partial\theta_{NS}} = 0 \Leftrightarrow \theta^*_{NS} = \tilde{a} < \bar{\bar{\theta}}_{NS}$, violating Condition 3.

1.7.2 If $\lambda_{D.3.1} > 0 \Longrightarrow^2 \theta^*_{NS} = \bar{\bar{\theta}}_{NS}$, and from Condition 1, with $\theta^*_{NS} = \bar{\bar{\theta}}_{NS}$, it represents the optimal corner solution, since $\bar{a} - \bar{\bar{\theta}}_{NS} < 0$, which is in line with all conditions: $\lambda^*_{D.3.1} = -\delta_{ns}(\sigma_c)\frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_S M)}{\partial\theta_{NS}} = -\delta_{NS}(\sigma_c)M\left(\tilde{a} - \bar{\bar{\theta}}_{NS}\right) > 0$.

Therefore, $\theta^*_{NS} = \bar{\bar{\theta}}_{NS}$.

1.8 If $\bar{\theta}_{NS} < arg\max_{\theta_{NS}} B(\theta_{NS}M) = \dot{a} < \bar{\bar{\theta}}_{NS}$

Consider, in this case, that $B(\theta_{NS}M) = \left(\dot{a} - \frac{1}{2}\theta_{NS}\right)\theta_{NS}M$.

1.8.1 If $\lambda_{D.3.1} = 0, \frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_S M)}{\partial\theta_{NS}} = 0 \Leftrightarrow \theta^*_{NS} = \dot{a} < \bar{\bar{\theta}}_{NS}$, violating Condition 3.

1.8.2 If $\lambda_{D.3.1} > 0 \Longrightarrow^2 \theta^*_{NS} = \bar{\bar{\theta}}_{NS}$, and from Condition 1, with $\theta^*_{NS} = \bar{\bar{\theta}}_{NS}$, it represents the optimal corner solution, since $\dot{a} - \overline{\overline{\theta_S}} < 0$, which is in line with all conditions: $\lambda^*_{D.3.1} = \delta_{NS}(\sigma_c)\frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_{NS}M)}{\partial\theta_{NS}} = -\delta_{NS}(\sigma_c)M\left(\dot{a} - \bar{\bar{\theta}}_{NS}\right) > 0$.

Therefore, $\theta^*_{NS} = \bar{\bar{\theta}}_{NS}$.

1.9 If $arg\max_{\theta_{NS}} B(\theta_{NS}M) = \tilde{\tilde{a}} > \bar{\bar{\theta}}_{NS}$

Consider, in this case, that $B(\theta_{NS}M) = \left(\tilde{\tilde{a}} - \frac{1}{2}\theta_{NS}\right)\theta_{NS}M$.

1.9.1 If $\lambda_{D.3.1} = 0, \frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_{NS}M)}{\partial\theta_{NS}} = 0 \Leftrightarrow \theta^*_{NS} = \tilde{\tilde{a}}$, which is in line with all Conditions. This is the internal optimal solution.

1.9.2 If $\lambda_{D.3.1} > 0 \Longrightarrow^2 \theta^*_{NS} = \bar{\bar{\theta}}_{NS}$, and from Condition 1, with $\theta^*_{NS} = \bar{\theta}_{NS}$, $\lambda_{D.3.1} = -\delta_{NS}(\sigma_c)\frac{\partial B(\theta_{NS}M)}{\partial(\theta_{NS}M)}\frac{\partial(\theta_{NS}M)}{\partial\theta_{NS}} = -\delta_{NS}(\sigma_c)M\left(\tilde{\tilde{a}} - \bar{\bar{\theta}}_{NS}\right) < 0$, which violates Condition 4.

Therefore, $\theta^*_{NS} = \tilde{\tilde{a}}$.

Therefore, considering all possible scenarios,

$$\theta^*_{NS} = \begin{cases} \bar{\bar{\theta}}_{NS}, \ \ if\ arg\max_{\theta_{NS}} B(\theta_{NS}M) = \tilde{a} < \bar{\theta}_{NS} \\ \bar{\bar{\theta}}_{NS}, \ \ if\ \bar{\theta}_{NS} < arg\max_{\theta_{NS}} B(\theta_{NS}M) = \dot{a} < \bar{\bar{\theta}}_{NS} \\ \tilde{\tilde{a}}, \ \ if\ arg\max_{\theta_{NS}} B(\theta_{NS}M) = \tilde{\tilde{a}} > \bar{\bar{\theta}}_{NS} \end{cases}$$

6.4.4 APPENDIX D.4

For the analysis when $\hat{a} = \dot{a}$, the regulator's utility in the internal and corner optimal solutions are, respectively:

$$U^A(\dot{a}) = \delta_{NS}(\sigma_c)\frac{\dot{a}^2}{2}[1 - \eta(1 - \delta_{NS}(\sigma_c))]M - \eta(1 + \gamma_A + \gamma_J)C^I_{NS}(M) - (1 - \eta)(1 + \gamma_A)CP(M)$$

$$U^A(\bar{\theta}_{NS}) = \left(\dot{a} - \frac{\bar{\theta}_{NS}}{2}\right)\bar{\theta}_{NS}M - \eta C^I_{NS}(M) - (1 - \eta)(1 + \gamma_A)CP(M)$$

Therefore, the difference between these terms is:

$$U^A(\dot{a}) - U^A(\bar{\theta}_{NS}) = \delta_{NS}(\sigma_c)\frac{\dot{a}^2}{2}\left[1 - \eta\left(1 - \delta_{NS}(\sigma_c)\right)\right]M - \left(\dot{a} - \frac{\bar{\theta}_{NS}}{2}\right)\bar{\theta}_{NS}M - \eta\gamma_J C_{NS}^{ErroI}(M) < 0$$

For the analysis when $\hat{a} = \tilde{\tilde{a}}$, the regulator's utility in the internal and corner optimal solutions are, respectively:

$$U^A(\bar{\theta}_{NS}) = \left(\tilde{\tilde{a}} - \frac{\bar{\theta}_{NS}}{2}\right)\bar{\theta}_{NS}M - \eta(1 + \gamma_A)C_{NS}^I(M) - (1 - \eta)(1 + \gamma_A)CP(M)$$

$$U^A\left(\tilde{\tilde{a}}\right) = \delta_{NS}(\sigma_c)\frac{\tilde{\tilde{a}}^2}{2}M - \eta(1 + \gamma_A + \gamma_J)C_{NS}^I(M) - (1 - \eta)(1 + \gamma_A + \gamma_J)CP(M)$$

Therefore, the difference between these terms is:

$$U^A(\bar{\theta}_{NS}) - U^A\left(\tilde{\tilde{a}}\right) = M\tilde{\tilde{a}}\left(\bar{\theta}_{NS} - \delta_{NS}(\sigma_c)\frac{\tilde{\tilde{a}}}{2}\right) - \frac{\bar{\theta}_{NS}{}^2}{2}M + \gamma_J[\eta C_{NS}^I(M) + (1 - \eta)CP(M)] > 0$$

6.5 APPENDIX E

6.5.1 APPENDIX E.1

This is the optimization problem of the Agency as to define M and f in the scenario of a high second instance incentive ($\theta_{NS}^* = \tilde{a}$) and low first instance incentives ($f \leq \tilde{f}$):

$$\underset{M,f}{\text{Max}}\, \Phi\left[\delta_{NS}(\sigma_c)B(\theta_{NS}^*M) - (1 + \gamma_A + \gamma_J)C_{NS}^I(M)\right] + (1 - \Phi)[B(\theta_{NS}^*M) - (1 + \gamma_A)CP(M)]$$ s.t. $f \leq \tilde{f}$, $M \geq 0$ and $M \leq \bar{M} < \Pi_R$

Its Lagrangian is:

$$\mathcal{L} = \left(1 - \Phi\left(1 - \delta_{NS}(\sigma_c)\right)\right)B(\theta_{NS}^*M) - \Phi(1 + \gamma_A + \gamma_J)C_{NS}^I(M) - (1 - \Phi)(1 + \gamma_A)CP(M) - \lambda_{E.1.1}\left(f - \bar{f}\right) + \lambda_{E.1.2}M - \lambda_{E.1.3}(M - \bar{M})$$

The Kuhn-Tucker Conditions are:

19. $\frac{\partial \mathcal{L}}{\partial M} = 0$
    19.1. $\Leftrightarrow \left(1 - \Phi\left(1 - \delta_{NS}(\sigma_c)\right)\right)\frac{\partial B(\theta_{NS}^*M)}{\partial(\theta_{NS}^*M)}\frac{\partial(\theta_{NS}^*M)}{\partial M} - \frac{\Phi(1+\gamma_A+\gamma_J)\partial C_{NS}^I(M)}{\partial M} - (1 - \Phi)(1 + \gamma_A)\frac{\partial CP(M)}{\partial M} + \lambda_{E.1.2} - \lambda_{E.1.3} = 0$
20. $\frac{\partial \mathcal{L}}{\partial f} = 0$
    20.1. $\Leftrightarrow -\lambda_{E.1.1} = 0$
21. $\lambda_{E.1.1}\left(f - \bar{f}\right) = 0$
22. $\lambda_{E.1.2}M = 0$
23. $\lambda_{E.1.3}(M - \bar{M})$=0
24. $f \leq \bar{f}$
25. $M \geq 0$
26. $M \leq \bar{M}$
27. $\lambda_{E.1.1}, \lambda_{E.1.2}, \lambda_{E.1.3} \geq 0$

From Condition 1:

$\left(1-\Phi\left(1-\delta_{NS}(\sigma_c)\right)\right)\left(\tilde{a}-\frac{1}{2}\theta_{NS}^*\right)\theta_{NS}^*-\frac{\Phi(1+\gamma_A+\gamma_J)\partial C_{NS}^I(M)}{\partial M}-(1-\Phi)(1+\gamma_A)\frac{\partial CP(M)}{\partial M}=\lambda_{E.1.3}-\lambda_{E.1.2}$

Since $\theta_{NS}^*=\tilde{a}$,

$\frac{1}{2}\left(1-\Phi\left(1-\delta_{NS}(\sigma_c)\right)\right)\tilde{a}^2-\frac{\Phi(1+\gamma_A+\gamma_J)\partial C_{NS}^I(M)}{\partial M}-(1-\Phi)(1+\gamma_A)\frac{\partial CP(M)}{\partial M}=\lambda_{E.1.3}-\lambda_{E.1.2}$

It can be stated that $\frac{1}{2}\left(1-\Phi\left(1-\delta_{NS}(\sigma_c)\right)\right)\tilde{a}^2$ is the marginal benefit of sanctioning $BMgM$ the Regulated while $\frac{\Phi(1+\gamma_A+\gamma_J)\partial C_{NS}^I(M)}{\partial M}+(1-\Phi)(1+\gamma_A)\frac{\partial CP(M)}{\partial M}$ can be seen as the marginal cost of the sanction $CMgM$. In sum,

$BMgM-CMgM=\lambda_{E.1.3}-\lambda_{E.1.2}$

If $BMgM<CMgM$, then $\lambda_{C.1.2}>0\Rightarrow M=0$, from Condition 4.

If $BMgM=CMgM$, then $\lambda_{E.1.3}=\lambda_{E.1.2}=0\Rightarrow 0<M=M^*<\bar{M}$, from Conditions 1, 4, 5, 9.

If $BMgM>CMgM$, then $\lambda_{E.1.3}>0$, $\lambda_{E.1.2}=0\Rightarrow M=\bar{M}$

In sum, the optimal sanction depends a cost-benefit analysis: if the gains obtained from the process of sanctioning does not compensate for its costs, then the fine is null; if there is an optimal value for which the Regulator can set a “price” for the externality to be internalized, then there is an internal optimal solution M^*; but if the benefits for the society of sanctioning always surpasses the associated costs, then the fine is maximal $\bar{M}$, generally exogenously imposed by legislation.

Note that, when there is no signal, the Type I Error cost is higher, which leads the decision of no sanction.

From Condition 2, $\lambda_{E.1.1}=0\Longrightarrow f<\tilde{f}$, from Condition 3.

Note that the function $\mathcal{L}$ does not count with the policy variable f.

6.5.2 APPENDIX E.2

This is the optimization problem of the Agency as to define M and f in the scenario of a low second instance incentive ($\theta_{NS}^*=\tilde{a}$) and medium first instance incentives ($\tilde{f}\leq f\leq\tilde{\tilde{f}}$):

$\underset{M,f}{\text{Max}}\,\Phi\left[\delta_{NS}(\sigma_c)B(\theta_{NS}^*M)-(1+\gamma_A+\gamma_J)C_{NS}^I(M)\right]+(1-\Phi)[B((1-f)M)-CP(M)-\mathrm{h}_{NS}(\sigma_c)(C_{NS}^{II}(M)+k)]$ s.t. $f\leq\tilde{\tilde{f}}$, $f\geq\tilde{f}, M\geq 0$ and $M\leq\bar{M}<\Pi_R$

Its Lagrangian is:

$\mathcal{L} = \underset{M,f}{\text{Max}}\, \Phi\left[\delta_{NS}(\sigma_c)B(\theta_{NS}^{*}M) - (1+\gamma_A+\gamma_J)C_{NS}^{I}(M)\right] + (1-\Phi)[B((1-f)) - CP(M) - \text{h}_{NS}(\sigma_c)(C_{NS}^{II}(M)+k)] - \lambda_{E.2.1}\left(f - \tilde{\tilde{f}}\right) - \lambda_{E.2.2}\left(\tilde{f} - f\right) + \lambda_{E.2.3}M - \lambda_{E.2.4}(M - \overline{M})$

The Kuhn-Tucker Conditions are:

23. $\frac{\partial \mathcal{L}}{\partial M} = 0$

 23.1. $\Leftrightarrow \Phi\delta_{NS}(\sigma_c)\frac{\partial B(\theta_{NS}^{*}M)}{\partial(\theta_{NS}^{*}M)}\frac{\partial(\theta_{NS}^{*}M)}{\partial M} - \frac{\Phi(1+\gamma_A+\gamma_J)\partial C_{NS}^{I}(M)}{\partial M} + (1 - \Phi)\left[\frac{\partial B((1-f)M)}{\partial((1-f)M)}\frac{\partial((1-f)M)}{\partial M} - \frac{\partial CP(M)}{\partial M} - \text{h}_{NS}(\sigma_c)\frac{\partial C_{S}^{II}(M)}{\partial M}\right] + \lambda_{E.2.3} - \lambda_{E.2.4} = 0$

24. $\frac{\partial \mathcal{L}}{\partial f} = 0$

 24.1. $\Leftrightarrow (1-\Phi)\frac{\partial B\left((1-f)M\right)}{\partial(f)} - \lambda_{E.2.1} + \lambda_{E.2.2} = 0$

25. $\lambda_{E.2.1}\left(f - \tilde{\tilde{f}}\right) = 0$
26. $-\lambda_{E.2.2}\left(\tilde{f} - f\right) = 0$
27. $\lambda_{E.2.3}M = 0$
28. $\lambda_{E.2.4}(M - \overline{M}) = 0$
29. $f \leq \tilde{\tilde{f}}$
30. $f \geq \tilde{f}$
31. $M \geq 0$
32. $M \leq \overline{M}$
33. $\lambda_{E.2.1}, \lambda_{E.2.2}, \lambda_{E.2.3}, \lambda_{E.2.4} \geq 0$

From Condition 1:

$$\Phi\, \delta_{NS}(\sigma_c)\left(\tilde{a}\theta_{NS}^{*} - \frac{1}{2}{\theta_{NS}^{*}}^{2}\right) - \frac{\Phi(1+\gamma_A+\gamma_J)\partial C_{NS}^{I}(M)}{\partial M} + (1-\Phi)\left[\beta(1-f) - \frac{1}{2}(1-f)^2 - \frac{\partial CP(M)}{\partial M} - \text{h}_{NS}(\sigma_c)\frac{\partial C_{NS}^{II}(M)}{\partial M}\right] + \lambda_{E.2.3} - \lambda_{E.2.4} = 0$$

$$\Leftrightarrow \Phi\, \delta_{NS}(\sigma_c)\left(\frac{1}{2}\tilde{a}^2\right) - \frac{\Phi(1+\gamma_A+\gamma_J)\partial C_{NS}^{I}(M)}{\partial M} + (1-\Phi)\left[\beta(1-f) - \frac{1}{2}(1-f)^2 - \frac{\partial CP(M)}{\partial M} - \text{h}_{NS}(\sigma_c)\frac{\partial C_{NS}^{II}(M)}{\partial M}\right] = \lambda_{E.2.4} - \lambda_{E.2.3}$$

Following the same logic as Appendix E.1,

$BMgM - CMgM = \lambda_{E.2.4} - \lambda_{E.2.3}$

If $BMgM < CMgM$, then $\lambda_{E.2.3} > 0 \Rightarrow M = 0$, from Condition 4.

If $BMgM = CMgM$, then $\lambda_{E.2.4} = \lambda_{E.2.3} = 0 \Rightarrow 0 < M = M^{*} < \overline{M}$, from Conditions 1, 5, 6, 11.

If $BMgM > CMgM$, then $\lambda_{E.2.4} > 0$, $\lambda_{E.1.2} = 0 \Rightarrow M = \overline{M}$

In sum, the optimal sanction depends a cost-benefit analysis: if the gains obtained from the process of sanctioning does not compensate for its costs, then the fine is null; if there is an optimal value for which the Regulator can set a “price” for the externality to be internalized, then there is an internal optimal solution M^*; but if the benefits for the society of sanctioning always surpasses the associated costs, then the fine is maximal $\overline{M}$, generally exogenously imposed by legislation.

From Condition 2,

$(1-\psi)(-\beta+1-f)M = \lambda_{E.2.1} - \lambda_{E.2.2}$

$\Leftrightarrow f^* = 1-\beta-\frac{(\lambda_{E.2.1}-\lambda_{E.2.2})}{(1-\Phi)M}$

2.1 Considering $\lambda_{E.2.1} > 0 \Longrightarrow^3 f^* = \tilde{\tilde{f}} \Longrightarrow^4 \lambda_{E.2.2} = 0 \Longrightarrow^2 f^* = 1-\beta-\frac{\lambda_{E.2.1}}{(1-\Phi)M}$ and $\lambda_{E.2.3} = \lambda_{E.2.4} = 0$, that is, there is an internal optimal solution for M ($M = M^*$).

From Condition 1:

$\Phi\, \delta_{NS}(\sigma_c)\left(\frac{1}{2}\tilde{a}^2\right) + (1-\Phi)\left[\beta(1-f)-\frac{1}{2}(1-f)^2\right] - \frac{\Phi(1+\gamma_A+\gamma_J)\partial C_{NS}^{I}(M^*)}{\partial M} - (1-\Phi)\left[\frac{\partial CP(M^*)}{\partial M} + \mathrm{h}_{NS}(\sigma_c)\frac{\partial C_{NS}^{II}(M^*)}{\partial M}\right] = 0$

For simplicity on carrying on the calculations, consider:

$CMg_1 = \frac{(1+\gamma_A+\gamma_J)\partial C_{NS}^{I}(M^*)}{\partial M}$ and

$CMg_2 = \frac{\partial CP(M^*)}{\partial M} + \mathrm{h}_{NS}(\sigma_c)\frac{\partial C_{NS}^{II}(M^*)}{\partial M}$

Then the problem of an optimal factor reduction f converges to a quadratic equation of the form:

$f^2 - 2(1-\beta)f - (2\beta-1) - \frac{\Phi}{1-\Phi}\delta_{NS}(\sigma_c)(\tilde{a}^2) + 2\frac{\Phi}{1-\Phi}CMg_1 + 2CMg_2 = 0$

Which leads to an impossible first solution (given Condition 11) and the possible solution, given Δ:

$f = 1-\beta \mp \frac{\sqrt{\Delta}}{2}$

$f = 1-\beta+\frac{\sqrt{\Delta}}{2} \Longrightarrow \lambda_{C.2.1} = -(1-\psi)M\frac{\sqrt{\Delta}}{2} < 0$ Impossible solution

$f = 1-\beta-\frac{\sqrt{\Delta}}{2} \Longrightarrow \lambda_{C.2.1} = (1-\psi)M\frac{\sqrt{\Delta}}{2} > 0$ Possible solution

$\Delta = 4\left[\beta^2 + 2\left(\frac{\Phi}{1-\Phi}\left(CMg_1 - \delta_{NS}(\sigma_c)\left(\frac{1}{2}\tilde{a}^2\right)\right) + CMg_2\right)\right]$

Consider $l = \frac{\Phi}{1-\Phi}\left(CMg_1 - \delta_{NS}(\sigma_c)\left(\frac{1}{2}\tilde{a}^2\right)\right) + CMg_2 > 0$, then:

$\Delta = 4(\beta^2 + 2l) \Leftrightarrow \sqrt{\Delta} \approx 2\beta + j$

Substituting the possible solution for f, under optimality:

$f^* \approx 1 - 2\beta - \frac{j}{2}$

Considering this approximation (note that there is no equality in the previous equation), and substituting the value for $\bar{\bar{f}}$, then the corner solution exists when $\beta = \beta_1^*$:

$$\beta_{1NS}^* = \frac{1}{2}\left[\delta_{NS}(\sigma_c = 1)a + \frac{(\rho_A+\rho_J)\Pi_R}{M^*} - \frac{j}{2}\right]$$

2.2 Now consider $\lambda_{E.2.2} > 0 \implies^4 f^* = \tilde{f} \implies^3 \lambda_{E.2.1} = 0$ and $M = M^*$

From Condition 2:

$$f^* = 1 - \beta + \frac{\lambda_{E.2.2}}{(1-\Phi)M}$$

The calculations are analogous to the 2.1 section, in which:

$f = 1 - \beta + \frac{\sqrt{\Delta}}{2}$ and $\Delta = 4(\beta^2 + 2l)$

$\bar{f} = f^* \approx 1 - \beta + \beta + \frac{j}{2} \approx 1 + \frac{j}{2} > 1 > \bar{f}$ Contradiction

Therefore, there is no corner solution when $f^* = \tilde{f}$.

2.3 Consider now $\lambda_{E.2.1} = \lambda_{E.2.2} = 0 \implies^5 f > \tilde{f} \implies^6 f < \tilde{\tilde{f}}$ and $M = M^*$

From Condition 2:

$$f^* = 1 - \beta$$

From Condition 1:

$$\Delta = 0$$

$$\Leftrightarrow \ \beta_{2NS}^* = \sqrt{\frac{\Phi}{1-\Phi}(\delta_{NS}(\sigma_c)a^2 - 2CMg_1) - 2CMg_2}$$

Therefore, if $\beta = \beta_{2NS}^*$, there is an optimal internal solution with $\tilde{f} < f < \tilde{\tilde{f}}$.

6.5.3 APPENDIX E.3

This is the optimization problem of the Agency as to define M and f in the scenario of a low second instance incentive ($\theta_{NS}^* = \tilde{a}$) and high first instance incentives ($f \geq \tilde{\tilde{f}}$):

$\underset{M,f}{\text{Max}}\, \Phi\left[B\big((1-f)M\big) - C_{NS}^I(M) + \mathrm{g}_{NS}(\sigma_c)(\Sigma)\right] + (1-\Phi)[B((1-f)M) - CP(M) - \mathrm{h}_{NS}(\sigma_c)(C_{NS}^{II}(M) + k)]$ s.t. $f \geq \tilde{\tilde{f}}$, $M \geq 0$ and $M \leq \bar{M} < \Pi_R$

Its Lagrangian is:

$$\mathcal{L} = \underset{M,f}{\text{Max}}\, B\big((1-f)M\big) - \Phi[C_{NS}^I(M) - \mathrm{g}_{NS}(\sigma_c)(\Sigma - k)] - (1-\Phi)[CP(M) + \mathrm{h}_{NS}(\sigma_c)(C_{NS}^{II}(M) + k)] - \lambda_{E.3.1}\left(\tilde{\tilde{f}} - f\right) + \lambda_{E.3.2}M - \lambda_{E.3.3}(M - \bar{M})$$

The Kuhn-Tucker Conditions are:

25. $\frac{\partial \mathcal{L}}{\partial M} = 0$

25.1. $\Leftrightarrow \frac{\partial B((1-f)M)}{\partial((1-f)M)}\frac{\partial((1-f)M)}{\partial M} - \frac{\Phi \partial C_{NS}^{I}(M)}{\partial M} - (1-\Phi)\frac{\partial CP(M)}{\partial M} - (1-\Phi)\mathrm{h}_{NS}(\sigma_c)\frac{\partial C_{NS}^{II}(M)}{\partial M} + \lambda_{E.3.2} - \lambda_{E.3.3} = 0$

26. $\frac{\partial \mathcal{L}}{\partial f} = 0$

26.1. $\Leftrightarrow \frac{\partial B((1-f)M)}{\partial(f)} + \lambda_{E.3.1} = 0$

27. $-\lambda_{E.3.1}\left(\tilde{\tilde{f}} - f\right) = 0$
28. $\lambda_{C.3.2}M = 0$
29. $\lambda_{C.3.3}(M - \bar{M}) = 0$
30. $f \geq \tilde{\tilde{f}}$
31. $M \geq 0$
32. $M \leq \bar{M}$
33. $\lambda_{E.3.1}, \lambda_{E.3.2}, \lambda_{E.3.3} \geq 0$

From Condition 1:

$$\beta(1-f) - \frac{1}{2}(1-f)^2 - \frac{\Phi \partial C_{NS}^{I}(M)}{\partial M} - (1-\Phi)\left[\frac{\partial CP(M)}{\partial M} + \mathrm{h}_{NS}(\sigma_c)\frac{\partial C_{NS}^{II}(M)}{\partial M}\right] + \lambda_{E.3.2} - \lambda_{E.3.3} = 0$$

(Eq. 31)

Following the same logic as Appendix E.1 and E.2,

$BMgM - CMgM = \lambda_{E.3.3} - \lambda_{E.3.2}$

If $BMgM < CMgM$, then $\lambda_{E.3.2} > 0 \Rightarrow M = 0$, from Condition 4.

If $BMgM = CMgM$, then $\lambda_{E.3.3} = \lambda_{E.3.2} = 0 \Rightarrow 0 < M = M^* < \bar{M}$, from Conditions 1, 4, 5, 9.

If $BMgM > CMgM$, then $\lambda_{E.3.3} > 0$, $\lambda_{E.3.2} = 0 \Rightarrow M = \bar{M}$

In sum, the optimal sanction depends a cost-benefit analysis: if the gains obtained from the process of sanctioning does not compensate for its costs, then the fine is null; if there is an optimal value for which the Regulator can set a "price" for the externality to be internalized, then there is an internal optimal solution M^*; but if the benefits for the society of sanctioning always surpasses the associated costs, then the fine is maximal $\bar{M}$, generally exogenously imposed by legislation.

From Condition 2,

$(-\beta + 1 - f)M = -\lambda_{E.3.1}$

$\Leftrightarrow f^* = 1 - \beta + \frac{\lambda_{E.3.1}}{M}$

2.1 Considering $\lambda_{E.3.1} > 0 \Longrightarrow$[3] $f^* = \tilde{\tilde{f}}$ and $\lambda_{E.3.2} = \lambda_{E.3.3} = 0$, that is, there is an internal optimal solution for M ($M = M^*$).

From Condition 1:

$$\beta(1-f)-\frac{1}{2}(1-f)^2=\frac{\Phi\partial C_{NS}^{I}(M^*)}{\partial M}+(1-\Phi)\left[\frac{\partial CP(M)}{\partial M}+\mathrm{h}_{NS}(\sigma_c)\frac{\partial C_{NS}^{II}(M^*)}{\partial M}\right]$$

For simplicity on carrying on the calculations, consider:

$$\frac{\Phi\partial C_{NS}^{I}(M^*)}{\partial M}+(1-\Phi)\left[\frac{\partial CP(M)}{\partial M}+\mathrm{h}_{NS}(\sigma_c)\frac{\partial C_{NS}^{II}(M^*)}{\partial M}\right]=d$$

Then the problem of an optimal factor reduction f converges to a quadratic equation of the form:

$$f^2-2(1-\beta)f+(1-2\beta)-d=0$$

Which leads to an impossible first solution (given Condition 9) and the possible solution, given Δ:

$$f=1-\beta\mp\frac{\sqrt{\Delta}}{2}$$

$f=1-\beta+\frac{\sqrt{\Delta}}{2}\Longrightarrow\lambda_{E.3.1}=-M\frac{\sqrt{\Delta}}{2}<0$ Impossible solution

$f=1-\beta-\frac{\sqrt{\Delta}}{2}\Longrightarrow\lambda_{E.3.1}=M\frac{\sqrt{\Delta}}{2}>0$ Possible solution

$$\Delta=4[\beta^2-\psi CMg_1-(1-\psi)CMg_2]$$

Consider:

$$\Delta=4(\beta^2-v)\Leftrightarrow\sqrt{\Delta}\approx 2\beta-m$$

Substituting the possible solution for f, under optimality:

$$f^*\approx 1-\frac{m}{2}$$

Considering this approximation (note that there is no equality in the previous equation), and substituting the value for $\tilde{\tilde{f}}$, then the corner solution exists when:

$$f^*\approx 1-\frac{m}{2}=\tilde{\tilde{f}}=1-\frac{(\rho_A+\rho_J)\Pi_R}{M^*}-\delta_{NS}(\sigma_c=1)\tilde{a}$$

$$2\left[\frac{(\rho_A+\rho_J)\Pi_R}{M^*}+\delta_{NS}(\sigma_c=1)\tilde{a}\right]=m=s(CMg_1,CMg_2)$$

2.2 Now consider $\lambda_{E.3.1}=0\ \Longrightarrow^{3}\ f^*>\bar{f}$ and $M=M^*$

From Condition 2:

$$f^*=1-\beta_{4NS}^*\Longrightarrow^{1}\Delta=0\ \Leftrightarrow\beta^2=\Phi CMg_1+(1-\Phi)CMg_2$$

$$\Leftrightarrow\beta_{4NS}^*=\sqrt{\Phi\frac{\partial C_{NS}^{I}(M^*)}{\partial M}+(1-\Phi)\left(\frac{\partial CP(M)}{\partial M}+\mathrm{h}_{NS}(\sigma_c)\frac{\partial C_{NS}^{II}(M^*)}{\partial M}\right)}$$

Therefore, if $\beta=\beta_4^*$, there is an optimal internal solution with $f>\tilde{\tilde{f}}$.

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Pro-competition Regulation in Telecommunications: an Impact Evaluation

Regulação pró-competição em Telecomunicações: uma Avaliação de Impacto

ABSTRACT

This study examines the impact of asymmetric remedies imposed on service providers with Significant Market Power in Brazilian uncompetitive municipalities within the wholesale markets of dedicated lines, high-capacity data transport, and fixed access network infrastructure for data transmission. Employing a combination of Difference-in-Difference analysis and Propensity Score Matching techniques, this inquiry explores how wholesale price regulation influences market concentration, service density, small providers' market share, fiber investments, user grievances, and perceived quality, particularly with respect to pricing and service excellence. The findings unveil a spectrum of effects across diverse remedies and indicators, leading to a proposition advocating the complete deregulation of fixed access network infrastructure for data transmission, the partial deregulation of dedicated lines, and a heightened focus on the pivotal high-capacity data transport wholesale market. Notably, the study underscores the need for robust analysis and careful consideration of the implications of asymmetric remedies on market dynamics and regulatory policy.

Keywords: pro-competition standardization, asymmetric remedies, significant market power, wholesale market regulation, regulatory policy.

RESUMO

Este estudo analisa o impacto de remédios assimétricos impostos a provedores de serviços com Poder de Mercado Significativo em municípios brasileiros pouco competitivos nos mercados atacadistas de linhas dedicadas, transporte de dados de alta capacidade e infraestrutura de rede de acesso fixo para transmissão de dados. Utilizando uma combinação de análise de Diferenças em Diferenças e técnicas de Propensity Score Matching, este trabalho investiga como o controle

de preços no atacado influencia a concentração de mercado, densidade de serviço, participação de mercado de pequenos provedores, investimentos em fibra, queixas de usuários e qualidade percebida, especialmente em relação a preços e excelência do serviço. As descobertas revelam uma variedade de efeitos em relação a remédios diversos e indicadores, culminando em uma proposição que defende a completa desregulamentação do mercado de infraestrutura de rede de acesso fixo para transmissão de dados, a parcial desregulamentação de linhas dedicadas e um foco intensificado no mercado atacadista de transporte de dados de alta capacidade. Destaca-se que o estudo ressalta a necessidade de uma análise robusta e consideração cuidadosa das implicações dos remédios assimétricos na dinâmica do mercado e na política regulatória.

Palavras-chave: regulação pró-competição, remédios assimétricos, poder de mercado significativo, regulação de mercado atacadista, política regulatória.

1 INTRODUCTION

Telecommunications services were predominantly provided by state-owned enterprises until the 1980s when a wave of market liberalization began, leading to privatization of the sector in developed and developing countries. This shift brought about structural and institutional changes in the State-Market relations, which gave rise to the current concept of regulation[1] and the creation of regulatory agencies worldwide.

The Brazilian telecommunications regulatory agency, Agência Nacional de Telecomunicações (Anatel), was founded during that privatization context already at the end of the century, when political forces came together allowing those changes to take place. The transition to a Regulatory State demanded implementation of new government instruments to prevent monopolistic power among the newly privatized incumbents. As a result, this regulatory landmark in Brazil empowered Anatel to periodically reassess the regulation, aiming to promote competition and ensure compliance with technological and market developments.[2]

Since then, Anatel has pursued a pro-competition regulatory approach, with a particular focus on its competition goals regulation, known as the "Plano Geral de Metas

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[1] One exception to this linkage would be the United States of America, a country which, since the second half of the 19th century, regulate their natural monopoly utilities. In this country, other policies rather than privatization were adopted in the market liberalization period, such as contracting out, concession of vouchers, among others (PRIEST, 1993).
[2] Law nº 9.472, of July 16, 1997.

de Competição" (PGMC), which was approved in 2012.[3] The PGMC underwent its first revision in 2018[4]. In this revised version, Anatel made an effort to tailor various remedies in wholesale markets according to the level of competition observed in the retail sector.

The underlying principle of this regulation is that competition is assessed at the retail level and depending on the extent of problems there identified, regulatory remedies are imposed on providers with significant market power (SMP) at the wholesale level. Hence, the pro-competition regulatory policy in the Brazilian telecommunications sector since 2018 can be described as a fine-tuning process, addressing different competition issues and offering tailored solutions through its regulatory interventions.

Given the rapid technological advancements in the entire value-chain of Information and Communication Technologies (ICTs) and telecommunications, as well as profound market dynamics, continuous policy calibration is crucial. Also, it is important to assess the impact of previous regulatory instruments, following preferably established empirical procedures, adhering to best practice guidelines for performance evaluation.[5]

The regulatory framework was designed in such a way that municipalities experiencing more pronounced competition issues received wholesale price control measures, a more intensive regulatory remedy, while others with similar economic and social characteristics did not.

The regulation design then allows for the evaluation of the interventions by identifying treated and control groups, enabling the assessment of the impacts caused by specific regulations. By examining these differential treatments, the study seeks to gain insights into the effectiveness and outcomes of the regulatory measures implemented.

The empirical strategy employed in this study utilizes the difference-in-differences statistical technique to evaluate the regulatory impacts on fixed broadband in Brazilian municipalities. Specifically, the analysis focuses on municipalities that were subject to asymmetric regulation in wholesale price control of three different relevant markets, comparing them to municipalities that did not undergo such regulation. By employing this method, the study aims to measure and understand the effects of the regulatory interventions on fixed broadband services in different geographical areas.

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[3] Resolution nº 600, of November 8, 2012.
[4] Resolution nº 694, of July 17, 2018.
[5] OECD's The Governance of Regulators' Performance evaluation Chapter.

The study's findings, derived from four distinct models, offer insights into the effects of various regulatory remedies on market performance indicators and competition within the wholesale telecommunications sector.

Model 1 explores regulatory remedies for the industrial exploitation of dedicated lines market. Model 2 evaluates effects of high-capacity data transport price control measures, containing evidence of a notable impact on market performance and some consumer experience indicators, to be confirmed on Model 4. Model 3, on the other hand, analyses the regulatory instruments of fixed access network infrastructure for data transmission via copper pair at transmission rates equal to or less than 12 Mbps remedies, revealing absence of impactful results. Lastly, in Model 4, the integration of last two mentioned wholesale markets remedies reports results that imply improved competition in the retail market.

The findings reveal a range of effects across various remedies and indicators, culminating in a recommendation that supports complete deregulation of fixed access network infrastructure for data transmission, partial deregulation of dedicated lines, coupled with an emphasized focus on the critical high-capacity data transport wholesale market. This study brings light to the importance of evaluation of the pro-competition standardization, which calls for a heightened level of scrutiny and reflection when it comes to crafting policies and making necessary regulatory adjustments.

This study brings light to the importance of evaluation of the pro-competition standardization. This calls for a heightened level of scrutiny and reflection when it comes to crafting policies and making necessary regulatory adjustments.

In addition to this introduction, the study includes a section on regulatory details of the pro-competition policy in Brazilian telecommunications, a literature review, the methodology, presentation of results from the econometric models, conclusions, and supplementary information.

2 THE REGULATION IN BRAZIL

The rationale behind the pro-competition regulation in the telecommunications sector is based on a comprehensive qualitative and quantitative analysis of the retail markets, along with an investigation of the wholesale relevant markets. This analysis ultimately leads to the definition of asymmetric remedies and the assignment of significant market power (SMP), which determine the specific regulatory obligations to be followed. A detailed explanation is provided below.

The normative process begins with a market study as part of a Regulatory Impact Analysis (RIA) conducted by the technical staff of the regulatory agency. This study initially focuses on defining the relevant markets for telecommunications services provided to end consumers, considering both product and geographic dimensions. The formal paradigm for this definition draws from antitrust practices but also considers the specificities of network industries, as highlighted by Gual (2003).

Prior to delineating the retail relevant markets, a robust qualitative evaluation is conducted using Porter's analytical model, which synthesizes a set of techniques for defining business strategies and understanding the industry structure, competitive forces, and the behaviour of established market participants.

The Porter (1986) analysis considers competitive forces such as rivalry, contestability of potential entrants, substitute products and services, bargaining power of customers, and bargaining power of suppliers. These forces are examined for each retail market and provide insights for market definition.

Following the qualitative analysis, an effort is made to categorize the retail relevant markets into four distinct categories, aiming to quantify the level of competition across all Brazilian municipalities. The underlying premise of this quantitative approach is that social welfare can be optimized when regulation is tailored to address actual market failures and promote deregulation where reasonable competition exists.

To achieve this, Brazilian municipalities are categorized into four levels of competition: competitive (Category 1), potentially competitive (Category 2), uncompetitive (Category 3), and non-competitive (Category 4). This classification is based on four pillars: market structure, infrastructure, demand, and service diffusion. Each pillar is represented by a chosen variable that reflects the municipal condition in relation to the fixed broadband market. Their respective variables are: Herfindahl-Hirschman Index (HHI), the number of Fiber Supplier Providers, Local HDI, and service density.[6]

A formula is devised to determine the category to which each municipality belongs, based on the values of the variables used to measure the development of the afore mentioned pillars. If a municipality exhibits several players in the market, potential demand, available infrastructure, and high service diffusion, it is considered to have a

[6] Local Human Development Indicator (HDI) by UNPD. The variable considered only Income and Education subindices, applying a weighted average with factors of 80% and 20%, respectively.

competitive environment. Conversely, if there is a monopoly, low potential demand, supply bottlenecks, and an incipient market, it is denoted as non-competitive.

The thresholds for these variables were calibrated endogenously to ensure a relevant and statistically significant distinction between competition categories. Table 1 provides the specific thresholds utilized for the fixed broadband market, noting that Category 3 was defined residually.

Table 1 Categorization criteria for fixed broadband market

1 Competitive if	**2** Potentially Competitive if at least one block applies:		**4** Non-Competitive if
HHI ≤ 3800 and	HHI ≤ 5150 and	Service Diffusion > 0,88 and	HHI = 10000 and
Voice Infra Index > 3 and	Voice Infra Index > 1 and	Voice Infra Index > 1 and	Voice Infra Index = 2 and
Service Diffusion > 1,38	Service Diffusion > 0,88	Demand > 4,41	Demand ≤ 5,28
3 Uncompetitive residually - if not classified as 1, 2 or 4,			

Source: Anatel (2016).

This classification was utilized in conjunction with regulatory remedies to address market failures. The underlying principle was that only providers with market power could harm competition, leading to inefficiencies as a consequence of their behaviour. Therefore, the regulatory authority aimed to identify markets where competition was limited and unlikely to develop without public intervention.

In the telecommunications sector, vertical integration plays a crucial role due to network externalities. It is particularly important for regulators to apply precise regulatory remedies because competition bottlenecks in telecommunications are generally found in the wholesale segments (OECD, 2014).

Thereafter, the process involves identifying wholesale relevant markets where hypothetical monopolists might exert market power. The design of these markets in terms of product and geographic dimensions is established, followed by a triple test. The relevant market must meet the following cumulative conditions to be considered subject to *ex ante* asymmetric regulation within the scope of the PGMC: presence of high and non-transitory structural barriers to entry; maintenance of probability of exercising market power for a significant period; and insufficient competition legislation and available regulations to reduce the probability of exercising market power.

The decision to implement regulatory intervention depends on the results of the triple test. If structural barriers, the maintenance of market power, and insufficient competition legislation are identified, then regulatory measures are necessary to address competition issues. The next step is to list the providers to whom these remedies will be applied.

To determine providers with significant market power (SMP), a five-criteria test is conducted for each wholesale market. The criteria include (i) market share, the ability to exploit economies of (ii) scale and (iii) scope, (iv) control over infrastructure whose duplication is not economically viable, and (v) simultaneous operation in wholesale and retail markets.

The combination of these factors determines which companies possess SMP in each geographic unit of the relevant market. If no players with SMP are identified for a specific product and geography, it indicates that the market failures previously identified did not manifest in a significant manner to justify the imposition of asymmetric regulatory measures.

Asymmetric regulatory measures refer to the prescription of remedies that will be applied in the relevant wholesale markets with the aim of balancing competitive conditions in the associated retail markets. The dosimetry of measures depends on the severity of the competition limitation in the relevant markets. The range of measures available in the PGMC varies from milder forms of intervention, such as transparency measures, to more stringent ones that may involve accounting, functional, or structural separation.

In practical terms, the extent of the remedies is contingent upon the level of competition in the retail markets, which was quantified through the categorization process. This means that, for competitive municipalities, more complex measures like structural separation would not be suitable. Instead, simpler instruments such as transparency obligations would be appropriate. Table 2 provides a list of imposed regulatory remedies for the fixed broadband upstream wholesale markets.

Table 2 Regulatory Remedies for Fixed Broadband

Wholesale Market	**Category**	**Remedy**
Fixed access network infrastructure	1,2,3 and 4	Asymmetric regulatory measures of transparency, as well as equal and non-discriminatory treatment.
Fixed access network infrastructure	3	Wholesale product price control measures.
Industrial exploitation of dedicated lines	1,2,3 and 4	Asymmetric regulatory measures of transparency, as well as equal and non-discriminatory treatment.
Industrial exploitation of dedicated lines	3	Wholesale product price control measures.
Industrial exploitation of dedicated lines	4	Interconnection-focused price control measures.
High-capacity data transport	2 and 3	Asymmetric regulatory measures of transparency, as well as equal and non-discriminatory treatment.
High-capacity data transport	3	Wholesale product price control measures.
Passive Infrastructure	1,2,3 and 4	Asymmetric regulatory measures of transparency and equal and non-discriminatory treatment.
Passive Infrastructure	1,2,3 and 4	Wholesale product price control measures.

Source: Acts of the Board of Directors of Anatel.

The regulatory framework in Brazil encompasses seven wholesale markets, four of which are related to the upstream branch of the fixed broadband value chain. Out of the four mentioned wholesale markets, three of them are eligible for evaluation. These markets are as follows:

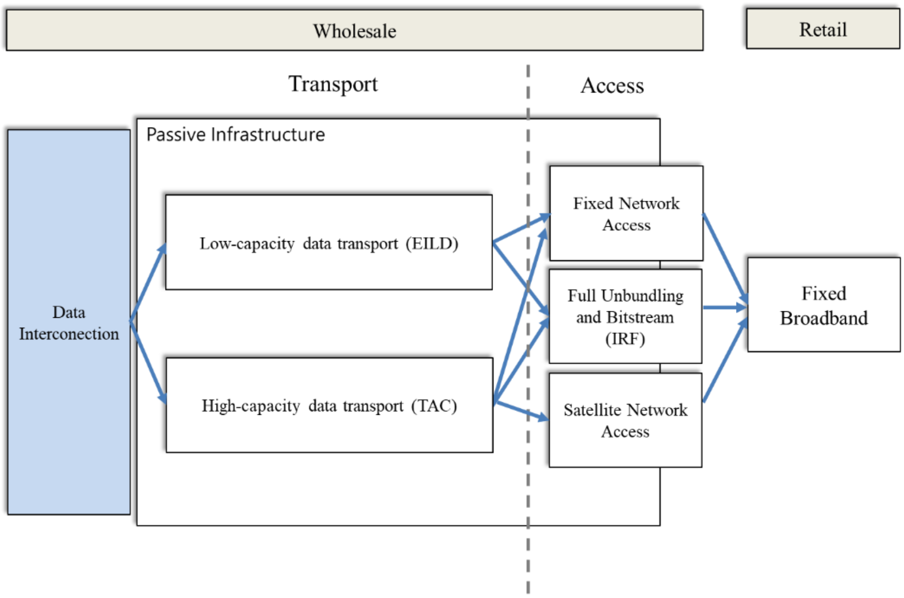
1. Fixed access network infrastructure for data transmission via copper pair at transmission rates equal to or less than 12 Mbps (henceforth IRF): its regulated products are *Bitstream*, which is the logical breakdown of the network connecting the subscriber or user termination to a concentration point designated by the requesting provider; and *Full Unbundling*, as the sharing of the entire frequency spectrum inherent to the metallic access pair.
2. High-capacity data transport (henceforth TAC): this market involves the transmission, reception, and delivery of IP (Internet Protocol) traffic at a capacity greater than 34 Mbps. It facilitates bidirectional traffic between pre-established addresses, adhering to standardized interfaces and quality and safety conditions.
3. Industrial exploitation of dedicated lines (henceforth EILD): in this market, a telecommunications service provider offers another provider a dedicated line with specific technical characteristics for the establishment of the latter's service network, whether for local or long-distance services.

The fourth wholesale market pertains to the passive infrastructure of pipelines and subducts. However, since price control regulation is universally imposed in this case, as indicated in on Table 2, there is no variation at the municipal level that allows for impact evaluation.

It is important to note that the assigned category for these three wholesale markets corresponds to the highest estimated category among all associated retail markets. For the first two wholesale markets mentioned, the associated retail markets are fixed broadband and mobile telephony. For the EILD market, the associated retail markets include fixed broadband, mobile telephony, and voice services.

The underlying wholesale markets act as network inputs to the provision of fixed broadband service to end-users, as illustrated in Figure 1, which represents the value chain for the studied markets.

Figure 1 Value Chain



Source: Author's own elaboration.

In summary, there are three markets that are suitable for analysis: IRF, TAC, and EILD. For IRF and TAC markets, price control measures are applied to Category 3 municipalities. As for the EILD market, it is further differentiated into Local and Long-Distance segments. Price control measures are then applied not only to Category 3 municipalities but also to Category 4 municipalities, restricting on interconnection goals.

A treated municipality is defined as one where price control regulatory instruments are imposed based on the previously explained categorization, and if there is at least one incumbent provider with significant market power (SMP). Table 3 provides an overview of the number of municipalities that are treated ($D = 1$) and not treated ($D = 0$) for each wholesale market, considering all possible combinations.

Table 3 Distribution of Regulatory Remedies

EILD	IRF	TAC	Category	total
0	0	0	1	6
0	0	0	2	503
0	0	0	3	28
0	0	0	4	1634
0	0	1	3	3
0	1	0	3	1
0	1	0	4	7
0	1	1	4	14
1	0	0	3	1357
1	0	1	3	250
1	1	0	3	41
1	1	1	3	1726

Source: Anatel.

In competitive retail markets, there is no need for the adoption of asymmetric regulatory measures as market forces already address efficiency and fairness. Conversely, in uncompetitive markets characterized by a lack of infrastructure, universalization public policies would be the appropriate remedy. This premise shaped the form of the regulation, where competitive and non-competitive markets are not subject to the heaviest regulatory instruments, but rather the uncompetitive ones.

Overall, Category 3 municipalities face the most substantial regulation, which includes price control measures in the wholesale markets, while non-discriminatory and isonomic treatment, along with transparency, are universally imposed.

The fact that regulatory remedies are applied selectively to certain municipalities, while others with similar socioeconomic and market factors are not subject to such interventions, creates an opportunity to study the impact of pro-competition telecommunications regulation in Brazil. Evaluating these impacts is crucial, not only to enhance existing market interventions but also due to the ongoing process of deregulation of *ex ante* wholesale market remedies in some countries.

For instance, the European Commission has recently updated the list of relevant markets in the electronic communications sector that are candidates for *ex ante* regulation, reducing it to only two wholesale markets: local access provided at a fixed location to deliver mass-market broadband services and bundles, and dedicated capacity mainly for business use requiring high-quality connectivity (EUROPEAN COMMISSION, 2020).

In 2023, the Spanish telecoms regulator, CNMC, following the recommendation of the European Commission, decided to roll back regulation in the wholesale market for mobile call termination due to the conclusion from the three-criteria test that the market tends towards effective competition.

Similarly, the Austrian regulator, Telekom Control Commission (TKK), has announced the full deregulation of fixed wholesale access markets in 2022, with the aim of accelerating fibre-optic expansion across the country. Following extensive market research, the regulator has determined to repeal obligations imposed on A1 Telekom Austria in the former wholesale markets for local access and central access.

3 THE ECONOMIC LITERATURE

Ex-post evaluations play a crucial role in determining whether a policy intervention has successfully achieved its objectives, and if not, understanding the reasons behind any shortcomings. This type of exercise provides valuable insights that can subsidise the design of future interventions for improved outcomes (OECD, 2016). Several studies have conducted such evaluations, and it is important to highlight some of the results obtained.

Prior to revising the empirical studies, it is worth noting that this study does not delve into the theoretical rationale of the regulation. Instead, it focuses on evaluating the impact of existing regulatory instruments. That is, the proposed analysis consists of verifying the effectiveness of the regulation, taking its theoretical underpinnings as given. Therefore, the literature revision directs attention to quantitative studies.

Faccio and Zingales (2022) conducted a study demonstrating that the way a government designs regulatory rules can have a significant impact on market concentration, competition levels, and prices. Their findings indicate that pro-competition regulation effectively reduces prices without adversely affecting the quality of services or investments.

Through an analysis of a panel of 148 countries, the study reveals that government's procompetitive policies, ranging from number portability to Voice over Internet Protocol (VoIP), have a substantial effect in reducing market concentration and prices. For instance, the introduction of number portability on average decreases the market share of the two largest operators in a country by 4 percentage points, reduces the price of a mobile-broadband internet plan with a 1GB data volume by US$10 per month, and decreases operators' EBIDTA margin by 4 percentage points.

Contrary to claims made by industry representatives, Faccio and Zingales (2022) state that markets with higher prices generally exhibit lower, not higher, quality of services. While the results do not establish a strong relationship between increased competition and higher quality, they effectively reject the opposing argument that reduced competition leads to improved quality.

The importance of broadband services has become increasingly evident as digitalization permeates all economic sectors. In fact, digital inclusion is recognized as a crosscutting theme across several Sustainable Development Goals (SDGs), with the United Nations acknowledging digital inclusion as a fundamental human right in its “declaration of Digital Interdependence”, as discussed at the UN Secretary General’s High-Level Panel on Digital Cooperation.[7]

However, many countries still face challenges in providing broadband access to all regions, and the digital divide has become even more pronounced during the global COVID-19 pandemic (OECD, 2021). The ongoing process of digitalization has been accelerated by the health crisis and the associated economic constraints, leading to rapid innovation in the digital sphere (World Bank, 2022).

Given these realities, and with a focus on modelling the factors that drive broadband access and affordability, Reddick et al. (2020) identified five key factors based on an extensive literature review. These factors include geographical disparities, competition, profit-based discrimination, technology deployment costs, and socio-economic factors. To the best extent possible, these factors are considered in the econometric modelling conducted in this study.

Briglauer et al. (2019) conducted an evaluation of a major European state aid program for broadband deployment in rural areas, specifically in the German state of Bavaria during 2010 and 2011. Using matched difference-in-differences estimation strategies, the authors found that municipalities receiving aid had significantly higher broadband coverage, ranging from 18.4 to 25.4 percentage points, compared to non-aided municipalities, when accounting for broadband quality.

However, the study also noted that while the increase in broadband coverage helped to close the digital divide, it did not lead to a significant increase in the number of employed or self-employed individuals or wages. The authors concluded that the increase in broadband coverage through state aid helped to prevent depopulation in rural areas but did not contribute to a further reduction of the economic divide in terms of job creation.

[7] Secretary-General’s High-level Panel on Digital Cooperation.

Briglauer et al.’s (2019) econometric approach serves as a reference point for this study, as the interventions evaluated in their research may share similarities with the interventions examined here.

Considering now policy implications rather than the technicalities of the evaluation, for Briglauer et al. (2019) not only do market conditions related to geography (e.g., specific topographical characteristics), historical decisions (e.g., legacy network deployments and regulations) and specific strategies at the firm level (e.g., network investments) shape the evolutionary trajectory of the Internet ecosystem, but they also significantly give rise to different dimensions of path dependencies. These factors give rise to various dimensions of path dependencies, such as legacy, regulatory and competitive, and strategic and complementary path dependencies.

Understanding these institutional elements and market-driven investment incentives can provide insights for the design of effective public policies, which is particularly relevant for subsequent analyses in this study.

4 THE METHODOLOGY

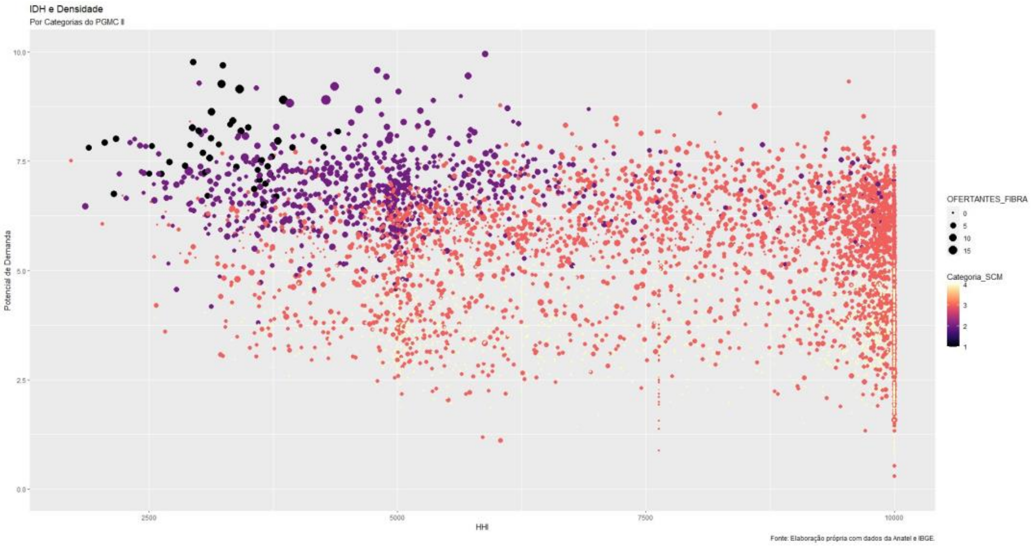
4.1 EMPIRICAL STRATEGY

The program under study is the wholesale markets price control regulation implemented by Anatel at the end of 2018, which primarily applies to a specific list of municipalities based on predetermined thresholds, that is, the assignment rule for these regulations is deterministic.

To evaluate the impact of this regulation, a comparative analysis can be conducted between municipalities considering several market performance indicators. This permits the examination of causality between the regulation and observed outcomes. Specifically, subsets of Category 3 municipalities, which received the treatment, can be paired with similar municipalities that did not receive the treatment.

It is important to note that the assignment rule adopted in the regulation is endogenous to the socioeconomic and market structure. Therefore, the ideal evaluation method of randomized assignment cannot be applied in this case. Additionally, since the assignment rule is based on multiple variables rather than a single threshold, it presents challenges in employing regression discontinuity approaches, as depicted in Figure 2. Identifying an appropriate instrumental variable in this context would also be a complex task.

Figure 2 Distribution of categorized municipalities by pillar variables



Source: Author's own elaboration. Data: Anatel.

The nature of the assignment and the data format make the difference-in-differences (DID) method a suitable approach for the analysis. DID is a quasi-experimental design that utilizes longitudinal data to estimate causal effects of a specific intervention or treatment (such as a passage of law, enactment of policy, or large-scale program implementation) by comparing changes in outcomes over time between a treatment group and a control group (BERTRAND et al., 2004).

Rather than comparing outcomes directly between the treatment and control groups after the intervention, the DID method focuses on comparing trends in outcomes between the two groups. The trend for an individual is calculated as the difference in outcome before and after the intervention. By subtracting the pre-intervention outcome from the post-intervention outcome, the method controls for the effect of all characteristics unique to that individual that do not change over time. This includes both observed and unobserved time-invariant characteristics (GERTLER et al., 2016).

In this study, natural candidates for the control group are municipalities in Category 2, 3 and 4. However, it is expected that group balance may be an issue since these categories encompass a wide range of economic realities. To address this, matching procedures such as propensity score matching can be employed, as done by Briglauer et al. (2019) in their difference-in-differences estimation.

Those authors affirm that, since broadband infrastructure is deployed by profit-maximizing telecommunication carriers that consider the local cost of deployment as well as the local market potential, common trends in pre-intervention stage would be reassuring since they would reflect a similar attractiveness of municipalities for broadband deployment. They argue that matching techniques ensure that the treatment and control groups have common pre-treatment trends in broadband availability and balanced economic characteristics.

This assumption is appropriate for this study, considering that broadband infrastructure deployment in Brazil is driven by more than seven thousand profit-maximizing telecommunication carriers, locally competing in an environment of punctual market interventions, being free enterprise the dominant principle. By taking these factors into account, the difference-in-differences analysis can provide valuable insights into the impact of the wholesale markets price control regulation on broadband market outcomes.

4.2 THE MODEL

To assess the impact of the pro-competition regulation, the following equation was estimated:

$$y_{it}^{m} = \beta_0^m + \beta_1^m T_i + \beta_2^m D_i + \beta_3^m D_i^m T_i \sum_{k=1} \beta_{k+3}^m X_{it}^k + \sum_{k=1} \gamma_k^m \theta_i^k + \varepsilon_{it}$$

The variable T_i is binary and equals one if an observation belongs to the post-intervention period, that is, for years after 2019 (this one included). The time window for the analysis ranges from 2015 to 2022.[8]

D_i^m is also a binary variable that indicates whether wholesale price control regulation took place in municipality i. For this variable to assume unitary value, two conditions must hold: to be assigned of Category 3; and to exist an incumbent provider with designated SMP in that market m.[9]

X_{it}^k encompass time-varying covariates including real GDP per capita, population, fibre suppliers, Local HDI, Gini coefficient, industry GDP share and broadband service penetration, while θ_i^k captures municipality-specific fixed effects and ε_{it} represents the error term of the static specification.

The dependent variable y_{it}^m encompasses three major blocks of indicators in terms of market performance, user complaints and perceived quality. The first one encompasses market concentration index (HHI), service density, market share of small providers in the retail fixed broadband market and fibre investments proxy.

[8] The regulated offers were first received in January 2019.
[9] The regulatory instrument also applies to Category 4 municipalities in the EILD market, but only for interconnection goals.

The second block relates to the complaints registered at Anatel by the service users. Here, two group of regressors were considered: price related and quality related complaints. Furthermore, an analysis was conducted differentiating responses of users consuming services of the treated and not treated providers, as well as an overall analysis considering all data. A user complaint index (UCI) was created, consisting of the sum of complaints received per 1000 service accesses. Therefore, there are three dependent variables for each type of user complaints: UCI non SMP providers; UCI for SMP providers; and UCI for all providers.

The third block relates to the service satisfaction and perceived quality survey the Agency conducted annually. The survey asks several questions to the interviewed, analogous the previous block, only price related and quality related answers were considered. Therefore, two types of dependent variables emerge: price related grades assumed by users; and service quality related grades assumed by users.[10] [11] For this block, the same differentiation of firms was done as above explained.

In sum, there are six dependent variables for the market performance block, and three dependent variables for the following blocks: price related user complaints, quality related user complaints, price related perceived quality and service quality related perceived quality. Therefore, there are 18 dependent variables, 2 types of models to estimated (all and matched sample) for 4 models of wholesale markets to be evaluated.

[10] Referring to the variables C1_1 and D1_1 from the survey.
[11] Referring to the variables C1_2 and C1_3 from the survey.

Table 4 List of dependent variables

Block	Dependent Variable
Market Performance	HHI
	Service Density
	Market Share of Small Providers
	Fiber Investment Proxy for non SMP providers
	Fiber Investment Proxy for SMP providers
	Fiber Investment Proxy for all providers
Price Related Complaints	Sum of price related complaints received per 1000 service accesses for non SMP providers
	Sum of price related complaints received per 1000 service accesses for SMP providers
	Sum of price related complaints received per 1000 service accesses for all providers
Quality Related Complaints	Sum of quality related complaints received per 1000 service accesses for non SMP providers
	Sum of quality related complaints received per 1000 service accesses for SMP providers
	Sum of quality related complaints received per 1000 service accesses for all providers
Price Related Perceived Quality	Price related grades assumed by users of non SMP providers
	Price related grades assumed by users of SMP providers
	Price related grades assumed by users of all providers
Service Quality Related Perceived Quality	Quality related grades assumed by users of non SMP providers
	Quality related grades assumed by users of SMP providers
	Quality related grades assumed by users of all providers

Source: Author's own elaboration.

Given that the assignment rule for the treatment is deterministic, based on the values of the four pillars detailed in Table 1, the most straightforward strategy of modelling the probability of being treated would be to construct a continuous propensity score based on the data of the moment the assignment was determined. Therefore, the propensity score is defined as the probit model of the following functional form:

$$P(D^m = 1|W) = \Phi(\gamma_0 + \sum_{k=1}^{4} \gamma_k W_i^k + \sum_{k=1}^{4} \gamma_{k+4} (W_i^k)^2)$$

Where W represents the four pillar variables used in the assignment and $\Phi(.)$ is the standard normal distribution function. Note that not only the level values were considered, but also its quadratic forms. This is because Category 4 municipalities are not treated even though its pillar variables values represent less market and economic development.

To differentiate the effects of the imposed price control interventions for each wholesale market, the formation of treated and control groups takes into consideration, as far as possible, municipalities that have received only the specific regulatory measure being evaluated. This means that for each wholesale market, the treated group consists of municipalities where its price control intervention has been the only one implemented, while the control group consists of municipalities that have not undergone the same intervention.

Model 1 evaluates the impacts of asymmetric remedies on $EILD$. The treated group consists of 1357 municipalities where the regulatory remedies were imposed only on $EILD$ wholesale markets ($D_i^{EILD} = 1$). The control group, on the other hand, includes 2165 municipalities where no regulatory remedies were imposed for $EILD$ and other wholesale markets and are not classified as competitive ($D_i^{TAC} = D_i^{IRF} = D_i^{EILD} = 0$ and $C^{EILD} \neq 1$). Accordingly, the underlying null hypothesis tests the significance of the coefficients β_3^{EILD}, which measure the impact of the price control instrument. If it is not rejected, municipalities with $D_i^{EILD} = 1$ may be used in the control groups of Models 2, 3 and 4.

Model 2 tests the impacts of price control instruments on Category 3 municipalities in the TAC wholesale market. In this case, the treated group consists of the 253 municipalities that are subject only to this wholesale price control. The control group comprises municipalities that have not received intervention at any market and are categorized as uncompetitive ($D_i^{TAC} = D_i^{IRF} = D_i^{EILD} = 0$ and $C^{TAC} = 3$).

Model 3 tests the impacts of price control instruments on Category 3 municipalities in the IRF wholesale market. In this case, the treated group consists of only 49 municipalities. The control group comprises uncompetitive municipalities that either have not received any intervention ($D_i^{TAC} = D_i^{IRF} = D_i^{EILD} = 0$ and $C^{IRF} = 3$).

Lastly, evaluation of the price controls in TAC and IRF jointly was conducted in Model 4, counting with 1740 treated municipalities ($D_i^{TAC} = D_i^{IRF} = 1$). The control group comprises all potentially competitive and uncompetitive municipalities that have not received price control remedies ($D_i^{TAC} = D_i^{IRF} = D_i^{EILD} = 0$ and $C^{IRF} = 2 \; or \; 3$). These results, hence, should be compared with the ones from previous Models for a comprehensive conclusion.

5 RESULTS

5.1 MODEL 1 – EILD

In this group of regressions, the asymmetric price control remedies imposed on providers with SMP on the wholesale market of industrial exploitation of dedicated lines on uncompetitive municipalities in Brazil is evaluated. There are 1357 treated municipalities and 2165 in the control group. The first block relates to the market performance regressions. The first column of each regression uses the entire sample, while the second reports the results utilizing the matched sample by propensity score.

The results of Model 1 indicate that there has been a statistically significant decrease in HHI of treated uncompetitive municipalities on EILD, but not significantly in economic terms (the HHI varies from 0 to 10000). On the other hand, it seems to have had a significant effect on service density, with an increase of 1 p.p. of the treated units comparing to the control group, it represents 4,3% of the mean of the indicator at the beginning of the validity of the norm. As to the small providers' market share in treated groups, in comparison to the control, it has decreased over time by 2,1 p.p., representing 3,7% of the variable mean in 2019.

Table 5 Results Model 1 – EILD – Market Performance

	Dependent variable:					
	HHI		Density		Small Providers' MS	
	(All)	(Match)	(All)	(Match)	(All)	(Match)
data_D	−1,077.548***	−1,070.215***	0.086***	0.110***	28.973***	28.298***
	(9.535)	(11.915)	(0.001)	(0.001)	(0.147)	(0.182)
D	276.194***	235.110***	−0.064***	−0.035***	10.108***	7.354***
	(11.550)	(12.352)	(0.001)	(0.001)	(0.178)	(0.188)
est_DiD	−36.404**	−44.544***	0.034***	0.010***	−2.878***	−2.172***
	(15.355)	(16.843)	(0.002)	(0.002)	(0.237)	(0.257)
pop	−0.0004***	−0.002***	0.00000***	0.00000***	−0.00001***	−0.0001***
	(0.00001)	(0.00004)	(0.000)	(0.000)	(0.00000)	(0.00000)
gdp	0.002***	0.002***	0.00000***	0.00000	−0.00002***	−0.00003***
	(0.0002)	(0.0002)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
share_Ind	−16.361***	−14.123***	0.001***	0.001***	0.101***	0.136***
	(0.342)	(0.366)	(0.00004)	(0.00004)	(0.005)	(0.006)
gini	−2,626.087***	−1,435.060***	−0.006	−0.119***	−53.774***	−32.762***
	(63.830)	(72.062)	(0.007)	(0.008)	(0.986)	(1.098)
hdi	−7,403.819***	−7,297.986***	1.982***	1.997***	−30.903***	−13.596***
	(67.587)	(84.817)	(0.007)	(0.010)	(1.044)	(1.293)
const	12,296.700***	11,704.300***	−1.116***	−1.102***	83.789***	65.676***
	(60.749)	(74.212)	(0.006)	(0.009)	(0.938)	(1.131)
Obs	267,635	206,227	267,635	206,227	267,635	206,227
Adj R^2	0.152	0.163	0.392	0.333	0.209	0.217
Note:	*p<0.1; **p<0.05; ***p<0.01					

Source: Author's own elaboration.

A question might raise if this fine-tuned regulation may distort investment decisions across treated and non-treated municipalities, or if the prerogative of contracting regulated inputs in the wholesale market may drive or inhibit investments in new technologies. To answer this, a proxy of fiber investments was constructed regarding non-SMP, SMP and all providers. The results indicate no evidence it has.

Consumers of non-established providers appear to express higher perceived experience with fixed broadband service prices in treated municipalities when compared to the control group. However, the regression has low explanatory adjustment, which calls for a cautious approach in drawing definitive conclusions. There are no significant effects regarding the quality related complaints regression series. There are also no significant results for the series of regressions concerning perceived quality.

In summary, there is limited evidence of competition enhancement or discernible improvements in price and quality perception among consumers, when it comes to induced effects stemming from price control measures in EILD. Nevertheless, a noteworthy effect on service coverage emerges, which merits careful consideration in the ultimate evaluation of this regulatory policy. Given this, it appears that there is room for a certain degree of deregulation of this wholesale market, even if it doesn't entail complete deregulation.

5.2 MODEL 2 – TAC

This group of regressions evaluates those municipalities that have received asymmetric remedies solely within the wholesale high-capacity data transport market. The number of observations is more limited in this case, with only 253 municipalities in the treated group and 1196 in the control group, being more severe for the user complaints and quality perceived block of regressions. It has been observed a small positive increase on HHI for the treated units, although a positive increase has also been spotted for the small providers market-share. As to the service density, no significant effect was found. It should be highlighted that the municipalities with $D_i^{EILD} = 1$ were considered here at the control group, given that the effects were not consistently robust.

Table 6 Results Model 2 – TAC – Market Performance

	Dependent variable:					
	HHI		Density		Small Providers' MS	
	(All)	(Match)	(All)	(Match)	(All)	(Match)
data_D	−1,172.486***	−1,129.361***	0.109***	0.089***	26.176***	25.062***
	(13.178)	(27.853)	(0.001)	(0.003)	(0.201)	(0.418)
D	−63.758***	−19.819	0.033***	0.001	−3.497***	−9.462***
	(23.067)	(28.708)	(0.002)	(0.003)	(0.352)	(0.431)
est_DiD	151.502***	119.677***	−0.016***	0.0001	1.983***	3.003***
	(31.512)	(39.394)	(0.003)	(0.004)	(0.481)	(0.591)
pop	−0.004***	−0.009***	0.00000***	0.00000***	−0.00004***	0.0001***
	(0.0002)	(0.0004)	(0.000)	(0.00000)	(0.00000)	(0.00001)
gdp	0.0003	−0.002***	0.00000	0.00000***	−0.00004***	−0.00002***
	(0.0002)	(0.0005)	(0.00000)	(0.00000)	(0.00000)	(0.00001)
share_Ind	−6.053***	−4.040***	0.001***	0.0003***	0.156***	0.126***
	(0.493)	(0.914)	(0.00005)	(0.0001)	(0.008)	(0.014)
gini	−779.132***	−323.253*	−0.267***	−0.295***	−20.811***	3.759
	(99.931)	(179.601)	(0.009)	(0.017)	(1.527)	(2.695)
hdi	−1,738.047***	−4,762.089***	1.346***	1.457***	13.935***	101.691***
	(116.899)	(180.226)	(0.011)	(0.017)	(1.786)	(2.705)
const	8,050.698***	9,849.839***	−0.629***	−0.662***	48.767***	−16.087***
	(108.744)	(179.708)	(0.010)	(0.017)	(1.662)	(2.697)
Obs	110,087	38,450	110,087	38,450	110,087	38,450
Adj R^2	0.088	0.117	0.270	0.387	0.170	0.234
Note:	*p<0.1; **p<0.05; ***p<0.01					

Source: Author's own elaboration.

An examination has also been carried out to determine whether the asymmetric remedies have influenced firms' incentives towards new technology investments, either in a negative or positive direction. The findings indicate that there is no substantiated evidence of such influence.

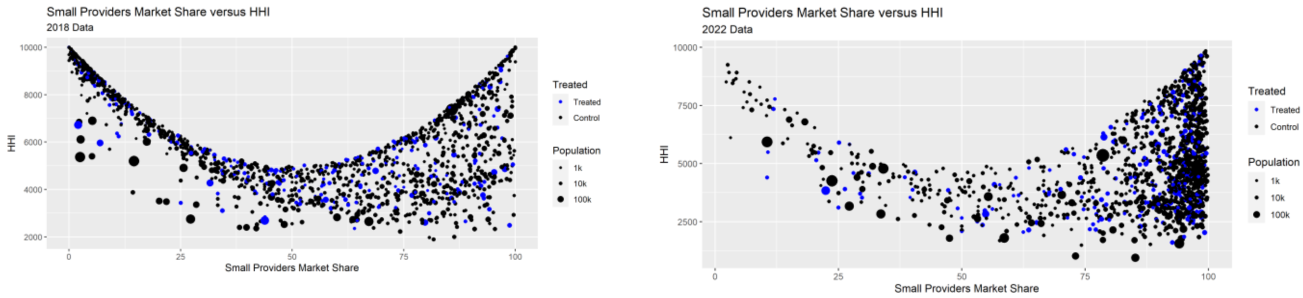
No significant impact is found neither on the price related complaints regressions nor on the perceived quality blocks. As to quality related complaints of SMP providers, a significant negative impact on the indicator is identified. This may indicate an induced quality standards effect driven by competition in the relevant market: established providers would have to be more vigilant over its service quality given the rise of rival companies.

Hence, even though there is a loss in terms of test power due to lack of data, the effects on market share of small providers and SMP providers' quality related complaints stand out, while the positive impacts of the treatment, even if economically negligible, on the most used indicator on competition, the HHI, is puzzling.

To further investigate on this, it's important to analyse the relation between small providers' market share and HHI throughout time. This is done in Figure 3 for 2018, where municipalities in Model 2 are plotted for the year prior to the validity of the studied regulatory policy, and 2022. There is a relevant shift of municipalities presenting high

HHI and low presence of small providers to a scenario in which it is observed high values for both indicators.

Figure 3 Small Providers' Market Share and HHI in 2018 and 2022



Source: Author's own elaboration. Data: Anatel.

That is, in a relevant part of Brazilian municipalities, the competition indicator has signalled a competition degradation, however, this result does not imply that incumbent providers with significant market power in the wholesale branch have become more dominant, but rather that emerging firms have indeed contested the market. Therefore, the analysis of the results of Table 6 should take into consideration these HHI details and perform a joint analysis considering this variable and the market share of emerging providers. The positive impact estimated on HHI may be due to the recent phenomenon in Brazil of successfully telecommunication services provision in small scale.

This conclusion may be used as input not only for the policies impact evaluation itself but also for future policies revision. This is particularly relevant once traditionally calculated indicators such as the HHI constitute a criterion for the retail markets competition categorization and, ultimately, for the treatment assignment of regulatory instruments.

5.3 MODEL 3 – IRF

In this session, the asymmetric remedies imposed on providers with SMP only on the wholesale market of fixed access network infrastructure for data transmission via copper are assessed. The number of observations is even more limited, with only 49 municipalities in the treated group and 1188 in the control group. The results of the model indicate a negative response in the estimations for HHI across the entire sample. However, for the matched sample, this response shifts to positive territory, although the economic impact remains inconsequential. For the service density regression, the results indicate a negative impact of the treatment. Lastly, for the small providers market-share, there is a significant negative effect in the entire sample and a non-significant for the matched sample. Furthermore, no statistically significant results are found for the user complaints and perceived quality blocks of regressions.

Table 7 Results Model 3 – IRF – Market Performance

	Dependent variable:					
	HHI		Density		Small Providers' MS	
	(All)	(Match)	(All)	(Match)	(All)	(Match)
data_D	−1,180.193***	−1,860.546***	0.110***	0.101***	26.311***	24.287***
	(13.246)	(59.316)	(0.001)	(0.006)	(0.204)	(1.018)
D	1,147.243***	286.513***	0.049***	−0.010*	1.646**	−6.147***
	(48.377)	(61.063)	(0.005)	(0.006)	(0.744)	(1.048)
est_DiD	−555.818***	144.822*	−0.038***	−0.027***	−3.322***	−1.125
	(66.484)	(83.881)	(0.006)	(0.008)	(1.022)	(1.440)
pop	−0.004***	−0.013***	0.00000***	−0.00000*	−0.00005***	0.00000
	(0.0002)	(0.001)	(0.00000)	(0.00000)	(0.00000)	(0.00001)
gdp	0.001***	−0.0003	−0.00000***	−0.00000***	−0.0001***	−0.0002***
	(0.0003)	(0.002)	(0.00000)	(0.00000)	(0.00000)	(0.00003)
share_Ind	−3.861***	17.515***	0.001***	0.004***	0.177***	0.554***
	(0.527)	(1.917)	(0.00005)	(0.0002)	(0.008)	(0.033)
gini	−1,229.133***	−502.978	−0.230***	−0.115***	−25.977***	−47.587***
	(108.743)	(425.949)	(0.010)	(0.042)	(1.672)	(7.310)
hdi	558.333***	1,710.999***	1.307***	1.843***	−5.062**	113.077***
	(130.775)	(429.096)	(0.012)	(0.042)	(2.011)	(7.364)
const	6,692.245***	6,402.199***	−0.617***	−0.945***	63.946***	3.390
	(118.531)	(407.608)	(0.011)	(0.040)	(1.822)	(6.996)
Obs	93,975	7,448	93,975	7,448	93,975	7,448
Adj R^2	0.099	0.261	0.241	0.393	0.163	0.236
Note:	*p<0.1; **p<0.05; ***p<0.01					

Source: Author's own elaboration.

Thus, the results indicate that there seems to be no impact of IRF remedies on market performance indicators and on the proxies of price and quality of fixed broadband service. These inputs can be used, with other rationale, to subsidise the deregulation of the wholesale market of fixed access network infrastructure.

5.4 Model 4 – TAC and IRF

There are 1740 municipalities that received price control remedies on TAC and IRF wholesale markets. The purpose of this model is to assess the effects of both instruments, counting with more degrees of freedom in relation to Models 2 and 3. The control group counts with 1873 municipalities. In this model, a noteworthy economic and statistically significant impact is observed on the competition index. This effect is consistent in magnitude across both samples, representing approximately 10% of the entire range of the indicator. Specifically, the treated units have exhibited an HHI that is over 1000 points lower than that of the control group.

Furthermore, it has been observed an outstanding effect on small providers market-share, with a 10 p.p. positive difference for the treated municipalities. On the other side, a negative effect was observed on density for this model.

Table 8 Results Model 4 – TAC and IRF – Market Performance

	Dependent variable:					
	HHI		Density		Small Providers' MS	
	(All)	(Match)	(All)	(Match)	(All)	(Match)
data_D	−986.670***	−1,015.997***	0.129***	0.132***	25.065***	25.842***
	(10.424)	(10.781)	(0.001)	(0.001)	(0.151)	(0.155)
D	1,417.163***	1,309.550***	−0.030***	−0.023***	−21.448***	−21.633***
	(10.944)	(11.094)	(0.001)	(0.001)	(0.158)	(0.160)
est_DiD	−1,109.211***	−1,076.936***	−0.013***	−0.015***	10.656***	9.905***
	(15.015)	(15.240)	(0.002)	(0.002)	(0.217)	(0.219)
pop	−0.0005***	−0.005***	0.00000***	0.00000***	−0.00001***	−0.0001***
	(0.00001)	(0.0001)	(0.000)	(0.000)	(0.00000)	(0.00000)
gdp	0.004***	0.002***	−0.00000***	−0.00000	−0.00001***	−0.00003***
	(0.0002)	(0.0002)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
share_Ind	−16.593***	−12.566***	0.001***	0.001***	0.110***	0.170***
	(0.304)	(0.313)	(0.00003)	(0.00003)	(0.004)	(0.005)
gini	−1,310.044***	−342.538***	0.059***	−0.055***	−56.381***	−36.901***
	(62.097)	(65.079)	(0.006)	(0.007)	(0.898)	(0.936)
hdi	−3,492.892***	−2,374.317***	1.940***	1.784***	−64.827***	−45.837***
	(72.660)	(75.308)	(0.007)	(0.008)	(1.050)	(1.083)
const	8,894.437***	7,849.933***	−1.156***	−1.008***	117.725***	96.273***
	(66.190)	(69.379)	(0.007)	(0.007)	(0.957)	(0.998)
Obs	274,551	264,443	274,551	264,443	274,551	264,443
Adj R^2	0.201	0.206	0.335	0.332	0.283	0.293

Note: *p<0.1; **p<0.05; ***p<0.01

Source: Author's own elaboration.

No evidence emerged indicating that these wholesale remedies impacted investment decisions among non-SMP, SMP and all providers. Additionally, reasonably well fitted models were estimated for SMP and all providers regarding the quality related complaints indicator, which indicate that the norm might have had an impact on firms' investments on consumer experience. Since the most prominent effect is the one from SMP providers, it might be suggested that this partially comes from the more intense competition it has been established in the retail relevant market. There is also a positive impact on consumer price related perceived quality, although its economic significance is quite small.

6 Conclusions

This study examined the impacts of wholesale price control asymmetric remedies in three wholesale markets, which were mandated by the Brazilian Telecommunications Regulator for providers with SMP in uncompetitive municipalities. The evaluation focused on their effects on fixed broadband market performance and consumer experience indicators.

Model 1 unveiled that EILD remedies generated a small reduction in HHI and on small providers' market share. Additionally, a rise in service density was observed when comparing the dynamics of variables between the treated and control groups. Based on this analysis, the results suggest that there exists space for a partial deregulation of this market.

In Model 2, TAC remedies yielded increases in both HHI and small providers' market share, while service density remained unaffected. Also, there seems to be an effect on the quality of the service provided by established providers. Since fewer observations were utilized and the results are mixed, further exploration on Model 4 should be conducted to evaluate the effects of this wholesale market regulatory remedies. Nonetheless, it's important to underscore that the analysis needs to be both discerning and cautious, not solely focused on the presented findings but also on the conventional indicators in use. For instance, the Herfindahl-Hirschman Index could indicate market concentration, as conventionally interpreted, but it could also be indicative of market contestation by emerging firms.

Model 3, dealing with IRF, indicated a complex response with positive HHI effects, service density exhibiting a negative response, and small providers' market share showing no significant impact. Hence, the results indicate that there seems to be no impact of IRF remedies onto market performance indicators. This input can be used, with other rationale, to subsidise the complete deregulation of this wholesale market.

Lastly, the integration of TAC and IRF remedies in Model 4 resulted in a substantial reduction in HHI, alongside a notable rise in small providers' market share, emphasizing a positive impact on competition. The treated municipalities, in comparison the ones that have not counted with the regulatory policy, showed an HHI over 1000 points (10% of the indicator range) lower and small providers' market share 10 p.p. higher. This suggests that there has been an important competition driver provided either by high-capacity data transport wholesale markets or by fixed access network infrastructure for data transmission via copper pair. Their respective models (Models 2 and 3), although with limited degrees of freedom in the analysis, indicate that there is no evidence of impact in the latter, while the former shows some notable effects.

Moreover, acknowledging not only the outcomes of the regressions but also qualitative market analysis, it is imperative to underscore that the significance of legacy networks in the domain of fixed broadband provision is waning. Concurrently, there is a pronounced surge in investments in innovative technologies, particularly fibre optics, which is gaining substantial traction even among long-established utility owner providers. This is response to a revolutionary new form of telecommunications consumption, either to communication, entertainment or teleworking, which encompasses high quality data transmission to which Full Unbundling seems to be no valuable input to this service provision.

Therefore, it seems reasonable for the regulator to focus on wholesale products that have demonstrated noteworthy impacts in the past and are expected to maintain their significance in the future. The high-capacity data transport appears to be this wholesale input.

Normative landmarks play a crucial role in driving societal change. Nonetheless, the static nature of norm content is continuously enriched over time through updates, as the regulator assimilates market and technological advancements. Following regulatory best practices, these updates can be augmented with the adoption of ex post policy evaluations. With the aid of quantitative methods, this analysis assesses the effectiveness of regulatory policies, comprehending their influence on the market and stakeholders, and providing insights to guide informed decision-making for future regulatory actions.

In this context, the PGMC can undoubtedly be regarded as the most substantial pro-competition norm within the telecommunications landscape. It was first approved back in 2012 and first revisited in 2018. This study conducted a regulatory impact analysis of some of its regulatory instruments, potentially constituting inputs to its subsequent revision.

Collectively, these findings accentuate the role of some pro-competition asymmetric remedies in influencing market dynamics, prompting further consideration for policy formulation and regulation adjustments.

7 APPENDIX

7.1 DATA

When working with econometric models, it is essential to provide comprehensive information about the variables used. Here are some additional details that can help improve the understanding and interpretation of the models.

7.1.1 Dependent Variables

HHI (hhi_{it}^{m}) – Herfindahl-Hirschman Index – is a commonly used measure of market concentration in industrial organization. It provides a quantitative measure of the degree of competition or concentration within a market or industry.

$$hhi_{it} = 10000 \sum_{j} \left(\frac{A_{jit}}{\sum_j A_{jit}} \right)^2$$

This index is calculated for the fixed broadband market considering providers access A_{jit} (Anatel's primary source data) to measure market share. The HHI is then the sum the squares of the market shares of all j providers operating in each i retail market geographic dimension throughout time t. It is considered the same measurement unit of the Brazilian regulator, with estimations closer to 0 indicating market decentralization while, on the other end, 10000 expressing monopoly situation.

Small providers market share ($spms_{it}$) measures the sum of market shares, also based on providers access A_{jit}, of all $\tilde{j}$ non-incumbents providers – providers not belonging to the Economic Groups of Oi, Telefonica, Claro, SKY and TIM. It could also be seen as a proxy for market concentration since it would capture the aggregate market share of providers that do not possess Significant Market Power in any wholesale market.

$$spms_{it} = 100 \ \frac{\sum_{j \in \tilde{J}} A_{jit}}{\sum_j A_{jit}}$$

The subset of small providers $\tilde{J}$ out of all the j providers is the same of the new definition of “small providers” revised on the same resolution that approved PGMC, the regulation in analysis. This definition states that small provider is the one holding a national market share of less than 5% (five percent) in each retail market it operates.[12] Anatel approved an Act stating that the providers mentioned above are not considered small providers. Additionally, the Act specifies that all other providers not mentioned are automatically designated as small providers.

Fiber Investments (fi_{jit}) is a proxy constructed for fiber investments over a year. It is defined as the variation of the fiber accesses (A^F_{jit}) proportional to all fixed broadband past accesses.

$$fi_{jit} = \frac{A^F_{jit} - A^F_{ji,t-1}}{A_{jit}}$$

User Complaints (uc_{jit}) represent the total number of complaints (c_{jit}) to the regulatory agency filed by users of fixed broadband services (Anatel’s primary source data) from j providers, relative to the base. It is worth noting that the subscript j remains in the formula, as differentiation can be employed in the analysis, such as examining the indicator for providers with and without Significant Market Power (SMP). The measurement unit is total complaint per one thousand users.

$$uc_{jit} = 1000 \frac{\sum_j c_{jit}}{\sum_j A_{jit}}$$

Quality of experience variables (qoe^s_{jit}) are summary of grades g^s_{ujit} assigned by users u on several service attributes s based on their experience consuming services from providers j. This data is collected annually by Anatel through a Survey on Consumer Satisfaction and Perceived

[12] The original term is “Prestadora de Pequeno Porte”, as stated in Anatel’s Resolution No. 694/2018.

Quality, which involves interviewing approximately fifty thousand fixed broadband users. The purpose of this survey is to assess the quality of services from the consumer's perspective.

$$qoe_{jit}^{s} = \frac{1}{n_{jit}^{u}} \sum\nolimits_{u} g_{ujit}^{s}$$

The utilized variable is the median response of users. The service attributes s are: overall satisfaction with the service ($s1$); price ($s2$); and connection quality ($s3$). qoe_{jit}^{s} varies on a scale from 0 to 10, the maximum grade.

7.1.2 Independent Variables

GDP *per capita* (gdp_{it}) – Real Gross Domestic Product *per capita* – municipal GDP deflated by Consumer Price Index IPCA and divided by population (IBGE as the data source). The unit measure is in thousands of Brazilian Reais (R$).[13]

Population (pop_{it}) stands for the population of the municipality. Source: IBGE.

Human Development Index (hdi_i) is UNPD's (United Nations Development Programme) composite statistical measure that assesses the overall development and well-being of a country's population. A joint task force was established between the UNDP, Ipea, and Fundação João Pinheiro to estimate the index at the municipal level, utilizing the 2010 IBGE census as support. There is no variation in time for this variable.

Gini Coefficient ($gini_i$) is a statistical measure that quantifies the level of income inequality within a population. The coefficient ranges from 0 to 1, with 0 representing perfect equality and 1 representing maximum inequality. Source: IBGE. There is no variation in time for this variable.

Industry GDP share (ind_{it}) is the share of gross value added in secondary sector, at municipal level. Source: IBGE.

Fiber Suppliers (fib_{it}) represents the count of distinct companies offering fixed broadband services utilizing fiber optic technology. Source: Anatel.

Broadband Service Penetration (pen_{it}) refers to the percentage or rate of adoption and usage of fixed broadband internet services by the population in the municipality. It was defined as the total sum of accesses (Anatel as data source) divided by the population (IBGE as data source).

[13] IBGE stands for "Instituto Brasileiro de Geografia e Estatística", which translates to the Brazilian Institute of Geography and Statistics. It is a government agency responsible for collecting and analysing data related to various socio-economic aspects of Brazil, including population, geography, and economic indicators. In addition to its role in producing demographic and geographic information, IBGE is also responsible for calculating the country's Gross Domestic Product (GDP).

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Technological Change and its Economic Features in the Digital Era: Literature Review and Guidelines

Mudança Tecnológica e suas Características Econômicas na Era Digital: Revisão da Literatura e Diretrizes

ABSTRACT

This study analyses how technological change brought by digitalization may impact economic life in several separate but correlated features. The rapid evolution of digital technology, accelerated by the impact of the COVID-19 pandemic, is fundamentally reshaping economies and altering growth patterns. Digital technologies, particularly those associated with Digital Transformation, have profound economic and social implications, transforming human relations and institutional frameworks. Despite its benefits, embracing Digital Transformation presents challenges, triggering societal upheavals with winners and losers. This study focuses on five main economic features of this digital technological transformation: the macroeconomic impacts of digitalization, including the i) productivity paradox; and the ii) reshaping of development strategies due to the augmenting economic relevance of the service sector; iii) widening social inequalities due to the digital divide; iv) increased market concentration led by the ascent of information technologies; and v) how new technologies such as artificial intelligence may impact the future of labour markets. The analysis is conducted through a literature review for each of those economic features, concluding with policy implications.

Keywords: Digital Transformation; Artificial Intelligence; Labour Market; Digital Policies; Big Techs.

RESUMO

Este estudo analisa como a mudança tecnológica trazida pela digitalização pode impactar a vida econômica em diversos aspectos. A rápida evolução de tecnologias digitais, acelerada pelo impacto da pandemia de COVID-19, está remodelando fundamentalmente as economias e alterando padrões de crescimento. As tecnologias digitais, especialmente aquelas associadas à Transformação Digital, têm implicações econômicas e sociais profundas, transformando relações humanas e estruturas institucionais. Apesar de seus benefícios, a adoção da transformação digital apresenta desafios, desencadeando disputas sociais com vencedores e perdedores. Este estudo se concentra em cinco características econômicas principais desta transformação tecnológica digital: os impactos macroeconômicos da digitalização, incluindo o i) paradoxo da produtividade; e ii) remodelação de estratégias de desenvolvimento devido à crescente relevância econômica do setor de serviços; iii) ampliação das desigualdades sociais devido à divisão digital; iv) aumento da concentração de mercado liderada pelo avanço das tecnologias da informação; e v) como novas tecnologias, como a inteligência artificial, podem impactar o futuro dos mercados de trabalho. A análise é conduzida por meio de uma revisão da literatura para cada uma dessas características econômicas, concluindo com implicações de política.

Palavras-chave: Transformação Digital; Inteligência Artificial; Mercado de Trabalho; Políticas Digitais; Gigantes da Tecnologia.

1 INTRODUCTION

The swift evolution of technology, expected to intensify due to the impact of the COVID-19 pandemic, is fundamentally altering economies and their patterns of growth. Digital technologies are transformative not only in economic terms but also socially, altering business models, human relations, and institutional frameworks.

Technological change nowadays cannot be disassociated with the degree Digital Transformation has reached into all sectors of the economies and with the revolutionary aspect it is taking. This is increasingly meaningful since the Information and Communication Technology (ICT) revolution back in the 70's. While the steam revolution took a century to influence globalization, the ICT revolution took a decade, and with the Digital Transformation, these phenomena are now occurring concurrently. While the technological progress in the Manufacture Industrial Revolutions was about doubling matter, in the Digital Economy, now it's about doubling electrons, which technical capacity has been experiencing consistent double-digit growth for almost four decades.

Referred to as the "2nd Unbundling" by Baldwin (2016), the ICT revolution marked a pivotal shift where telecommunications, once considered peripheral, now stands as a central and indispensable element, as it has radically lowered the cost of moving ideas. The likelihood that digital technologies may indeed assume General Purpose Technology (GPT) characteristics as they allow applications across different industries and sectors highlights the ground-breaking impact they may exert, paralleled with the industrial and ICT revolutions.

To better understand these phenomena, it is important to be specific in the utilized terminology. By Digital Transformation[1], it could be interpreted as usage of Information and Communication Technology, when not trivial automation is performed, but fundamentally new capabilities are created in business, public government, and in life of people and society (MARTIN, 2008). Furthermore, Digital Transformation can be seen as the changes that digital technology causes or influences in all aspects of human life (STOLTERMAN and FORS, 2004).

Digital Transformation embraces a large range of new technologies, including Internet of Things, blockchain, robotics, cognitive, cloud and quantum computing, artificial

[1] Reis et al. (2018) conducts a systematic review of 206 peer-reviewed articles on Digital Transformation. A summary was done based on this survey.

intelligence (AI), intelligence-assisting devices and algorithms (IA), augmented reality (AR), machine learning (ML), Big Data analytics, telecommuting, digital payments, digital platforms.[2]

These new technologies change the way firms produce goods and services, innovate, and interact with other firms, workers, consumers and governments. From the microeconomic point of view, embracing the digital revolution might be a necessary pathway to profit maximization. According to a survey by the Economist Intelligence Unit[3], nine out of 10 companies believe Digital Transformation is now a requirement for success, while almost four out of five (79%) say that without further Digital Transformation, they will be at a competitive disadvantage within three years.

Being more specific, Digital Transformation, on an individual level, improves the ability to meet customer demands; enhances resource-saving technological progress, raising the potential to produce more output with fewer resources (with a special focus on natural resources, relevant to commodity exporting developing countries); provides the capacity to engage in substantial cost reductions based on data intelligence; enables businesses to streamline operations, automate processes, leading to improved productivity; through digital platforms, opportunities are provided for businesses to reach a broader market and expand their customer base; fosters innovation and product development by gathering real-time feedback from market demands; offers data-driven insights to make informed decisions, optimizing strategies; among other features, such as in logistic management, workforce coordination.

However, this pace of technological evolution inevitably triggers societal upheavals, leading to both beneficiaries and those adversely affected[4]. This study delves into the hurdles presented by Digital Transformation under five main economic features and provides guidelines on regulatory and legislative branches to tackle some of its issues.

One macroeconomic issue that might regain prominence in the context of increasing digitalization[5] is the updated version of the modern productivity paradox, which states that the rapid development of information and communication technologies over the past two decades has coincided with a generalised slowdown in aggregate productivity growth.

Another relevant feature is how robotics and telemigration in a growing service-led development framework might reshape development strategies and balance of payments of developing countries in a positive or negative way. In sum, DigiTech may allow many emerging markets to directly export the source of their comparative advantage, depending on how their institutions allow it.

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[2] According to Silva et al. (2020), digital platforms are not a technological branch *per se*, but they configure as relevant new forms of industrial organization relying on big data, artificial intelligence, and cloud computing. They facilitate the interaction among economic agents and proactively overseeing network effects between them (BELLEFLAMME and PEITZ, 2021). Given its current importance, its effects shall also be examined. As stated by Kenney and Zysman (2016), if the industrial revolution revolved around the factory, today's transformations are orchestrated by these digital platforms.

[3] Why digital transformation means success in the long run.

[4] For a review on its cautions from economic theory perspective, see Korinek and Stiglitz (2021).

[5] Equivalent term for Digital Transformation.

Moreover, social inequalities may be amplified with the increasing digital divide, when those technologies are not adopted by all, remaining a gap between individuals, households, businesses, and geographic areas at different socioeconomic levels with regard both to their opportunity and capacity to access ICTs and to their use of the Internet for a wide variety of activities.

On the economic order front, the ascent of information technologies can potentially fuel concentration of market power on the hands of few dominant digital firms. Market dynamics influenced by these changes might lead the economy toward a less competitive equilibrium, in which these powerful entities could exploit their advantage, creating distortions that could counteract the positive impacts of innovation. These trade-offs of creating incentives for innovation versus the potential risk of decreased societal welfare should be considered on the responses to the technological and market evolutions.

Other relevant aspect of digital technological change relates to its impact on labour market. On the one side are the alarmist arguments that the oncoming advances in AI and robotics will spell the end of work by humans, while many economists on the other side claim that because technological breakthroughs in the past have eventually increased the demand for labour and wages, there is no reason to be concerned that this time it will be any different.

The aim of this study is to delve into the literature concerning these five primary economic aspects related to Digital Transformation: productivity, globotics, digital divide, market power, and the future of jobs. The subsequent section provides an in-depth analysis of each of these economic features.

2 CHANNELS OF DIGITALIZATION

2.1 PRODUCTIVITY

Digital technologies and data stand as the cornerstone of a transformative revolution. They have ushered in a new era where individuals, businesses, and governments operate, communicate, work, and create in markedly different ways compared to the past. Moreover, these changes are swiftly gaining momentum and accelerating at an unprecedented rate (OECD, 2019).

Nonetheless the swift advancement of information and communication technologies over the last two decades has coincided with a widespread deceleration in overall productivity growth, often referred to as the modern productivity paradox (ACEMOGLU et al., 2014).

This puzzle relates to the Solow (1987) paradox in the case of ICT (Information and Communication Technologies): transformative new technologies are seen everywhere but in the productivity statistics (BRYNJOLFSSON et al., 2018).

Indeed, on one side, there exist remarkable instances of potentially transformative new technologies capable of substantially enhancing productivity and economic well-being, for instance, as discussed in Brynjolfsson and McAfee (2014). Notably, early indications display the promising potential of these technologies, with recent advancements in AI performance serving as a prominent example.

However, simultaneously, the measured growth in productivity has notably decelerated over the past decade. This slowdown is not trivial; it has significantly halved the pace of productivity growth compared to the levels witnessed in the decade preceding this deceleration. Furthermore, this slowdown is not limited to specific regions; it is widespread, spanning across OECD countries and, more recently, observed among several significant emerging economies as well (SYVERSON, 2017).

Despite the ongoing process of digitalization, there has been a significant and sharp decline in labour productivity growth across OECD countries in recent decades. Importantly, this decline is not solely attributable to measurement issues. While uncertainties surrounding productivity measurement have escalated, primarily due to digitalization and the rising significance of intangible assets like algorithms and data, most researchers concur that mismeasurement is not the primary cause behind the observed slowdown in productivity (OECD, 2019).

Brynjolfsson et al. (2019) suggest three primary candidate explanations for the discrepancy between technological advancements and labour productivity growth: concentrated distribution of productivity gains; implementation delays; and mismeasurement.

Barriers hindering the widespread adoption of technology among firms, leading to the widening gap between slower-adopting firms and those at the forefront of best practices, have been cited as a potential explanation for this paradox (ANDREWS et al., 2016). The sluggish productivity growth observed in the “average” firm masks the notable and robust advancements achieved by a small group of firms: Andrews et al. (2016) shows that the rising labour productivity gap between global frontier and laggard firms largely reflects divergence in revenue based multi-factor productivity (MFPR). This leads to a compelling inference: the increasing gap in MFPR between leading global firms and the slower laggards may indeed signify a divergence in physical productivity or technology, encompassing a broad spectrum. Notably, contemporary innovation isn't solely about firms introducing technologically advanced goods and services; it also hinges on their adeptness in tacitly combining various intangibles elements — such as computerized information, innovative property, and economic competencies — within their production processes (CORRADO et al., 2009; ANDREWS et al., 2016).

The observed divergence in MFPR might seem surprising for several reasons. Firstly, neo-Schumpeterian growth theory (AGHION and HOWITT, 2006; ACEMOGLU, AGHION, and ZILIBOTTI, 2006) and models of competitive diffusion (Jovanovic and MacDonald, 1994) imply productivity convergence, suggesting that firms farther behind the global frontier should experience faster growth by capitalizing on a larger reservoir of untapped technologies and knowledge. Secondly, the extent of productivity divergence observed in the data poses a challenge to models of creative destruction and a competitive market environment where the process of market selection enhances productivity (AGHION and HOWITT, 1992; CABALLERO and HAMMOUR, 1994; CAMPBELL, 1998), thereby prompting questions regarding market competitiveness.

Andrews et. al (2016) also states that structural changes in the global economy – namely Digital Transformation, globalisation and the rising importance of tacit knowledge – could underpin multi-factor productivity divergence through two interrelated channels:

firstly, the escalating capacity of digital technologies to facilitate winner-takes-all dynamics in the global market – as discussed by Brynjolfsson and McAfee (2011) – has empowered technological frontrunners to widen their performance gap with slower-paced laggard firms; secondly, other potential explanation relates to a stalling technological diffusion channel, in which the escalating importance of tacit knowledge and the intricate nature of technologies have amplified the level of sophisticated supplementary investments needed for the effective adoption of new technologies. Additionally, it's plausible that a new technological phase is coming that expertise is tacitly possessed by a selected few, such as the early adopters who are in the process of learning, while the rest of the population is still trailing behind.

Despite the increased accessibility of broadband networks for many firms, the integration of more advanced digital tools and applications remains incomplete and showcases significant disparities among countries (MCKINSEY GLOBAL INSTITUTE, 2018). Consequently, delving into the factors driving technology adoption becomes imperative, as argued by Nicoletti et al. (2020).

Despite the above-mentioned evidence on the distribution of productivity gains partially explaining the paradox, Capello et al. (2022) states that it remains compelling that the concentration of labour productivity gains in few highly innovative and productive firms and sectors with a restricted share on the overall economy can hardly influence the dynamics of aggregate labour productivity, resulting in negligible effects on labour productivity growth.

This in fact ties to the second explanation of the paradox, which relies on the presence of implementation lags in the building and full-scale application of the new technologies. Key technologies such as general-purpose ones (artificial intelligence emerging as the most promising candidate in this regard) hold significant potential to impact the economy and overall welfare, but it might take considerable time, more than conventionally expected, to be able to grasp their tangible effects in terms of statistics (BRYNJOLFSSON et al., 2019).

These lags stem from two primary sources. Initially, there is a temporal factor wherein the new technologies require time to reach a critical mass capable of significantly impacting aggregate output. Should the adoption rate remain low, it could trigger adverse threshold effects, resulting in minimal or negligible effects on labour productivity growth. Additionally, in cases where technology adoption exhibits decreasing returns, the gains in labour productivity might remain limited or even absent. The presence of threshold effects in adoption can lead to misplaced optimism, for instance, when adoption levels are mistakenly perceived as enough to induce substantial effects on productivity growth, yet the attained level remains inadequate in comparison to the critical mass necessary to genuinely influence productivity growth (CAPELLO et al., 2022).

Secondly, emerging technologies, particularly general-purpose ones that are conceptual, original, and distant from immediate market applications, demand complementary investments, co-inventions, adjustments, and learning curves from adopting firms to surmount organizational inertia and bottlenecks (CAPELLO et al., 2022). The adoption trend of ICTs during the 1980s and 1990s exemplifies this delayed pattern, as highlighted by Brynjolfsson and Hitt (2003), Bresnahan et al. (2002), Brynjolfsson and Hitt (2000).

David (1991) draws similar conclusions regarding electrification, while recent findings on the impact of the Internet of Things on labour productivity growth appear to support this viewpoint (EDQUIST et al., 2021).

Lastly, mismeasurement in output can influence the observed level of labour productivity and subsequently affect its growth trajectory. This explanation has been advocated by various scholars, albeit with mixed evidence, e.g. Mokyr (2014), Alloway (2015), Hatzius and Dawsey (2015), and Smith (2015). However, recent studies contend that mismeasurement issues are unlikely to be the primary driver behind the observed modern productivity paradox (CARDARELLI and LUSINE, 2015; BYRNE et al., 2016; NAKAMURA and SOLOVEICHIK, 2015; SYVERSON, 2017).

However, the substitution of workers by new technologies might trigger the migration of workers from highly productive (i.e., innovative) sectors, where adoption occurs, to less productive ones, creating an intersectoral reallocation effect. This phenomenon could once more result in minimal or negligible aggregate productivity growth. In such scenarios, conventional indicators obscure and complicate the isolation of the impact of these intersectoral dependencies, which can significantly hinder productivity growth (DAUTH et al., 2021).

In the empirical realm, Nicoletti et al. (2020) explore a unique dataset covering digital technology usage across 25 industries in 25 European countries from 2010 to 2016, aiming to uncover the factors driving digital adoption in firms, particularly in cloud computing and back/front office integration. The focus is on structural and policy-related elements influencing firms' capabilities and motivations for adoption. Factors considered include infrastructural availability (e.g., high-speed broadband), managerial quality, workers' skills, and market settings (product, labour, and financial). The study initially establishes a significant positive link between the penetration of high-speed broadband and the adoption of these digital technologies. This validates the idea that improving high-quality broadband infrastructure complements the adoption of advanced digital applications, forming the foundation of a digital economy. Moreover, the research strongly supports the hypothesis that factors like low managerial quality, inadequate ICT skills, and mismatches between workers and job requirements hinder digital technology adoption and subsequent diffusion rates.

Gal et al. (2019) evaluate the impact of adopting various digital technologies on firm productivity. Using a blend of cross-country firm-level productivity data and industry-level data on digital technology adoption, the study employs an empirical framework that considers firm diversity. The findings strongly indicate that the adoption of digital technologies within an industry corresponds to improved productivity at the firm level. The effects are notably more pronounced in manufacturing and routine-based activities. Additionally, these effects tend to be more significant for already productive firms and less impactful in instances of skill shortages. This correlation suggests potential complementarities between digital technologies and other capital elements, such as skills, organizational structure, or intangible assets. Consequently, digital technologies might have contributed to the widening productivity disparities among firms. Consequently, policies aimed at promoting digital adoption should be accompanied by initiatives that facilitate the advancement of slower firms, particularly by enhancing access to skills.

Rivares et al. (2019) employ an innovative empirical strategy to evaluate the impact of online platform development on the productivity of service firms. Using Google Trends internet search data across ten OECD countries and four industries (hotels, restaurants, taxis, and retail trade), the authors establish a proxy measure of platform usage, linking it to firm-level productivity data in these sectors. Their findings indicate that platform development positively influences the productivity of established service firms and encourages the reallocation of labour toward more productive firms in these industries. This trend might be attributed to platform features like user reviews and ratings, which diminish information asymmetries between consumers and service providers, fostering heightened competition among providers.

The impact observed varies significantly depending on the specific type of platform involved. Platforms categorized as "aggregators", linking established service providers to consumers (such as Booking.com or TheFork), have consistently shown an inclination to enhance the productivity, profitability, and employment levels of existing service firms. Conversely, more disruptive platforms that introduce new provider types into competition with established ones (like Uber or Airbnb) have not shown a notable effect on the productivity of existing providers. However, these disruptive platforms tend to reduce the mark-ups, employment rates, and wages of established firms. Evaluating the productivity of these new providers presents challenges due to conceptual complexities and limitations in available data.

Platform markets frequently exhibit high concentration due to robust multi-sided network effects. Interestingly, the productivity gains driven by "aggregator" platforms seem to diminish when a platform maintains persistent dominance within its market. These findings from Rivares et al. (2019) underscore the importance of advocating for greater contestability in platform markets. Strategies to enhance this contestability include reducing switching costs between platforms and rigorously enforcing competition policy tools.

Furthermore, according to the study, stringent regulations in product and labour markets have been observed to impede the productivity of established service firms when platforms emerge. This hindrance might stem from the limitations these regulations impose on firms, restricting their ability to adapt to swiftly evolving economic landscapes. They conclude that this situation prompts a revaluation of regulations in the context of platform development: outdated rules or those disproportionately favouring established entities should be re-examined and potentially eliminated. Concurrently, new categories of service providers facilitated by platforms should gradually adhere to tax and regulatory standards akin to those governing competing industries. This step ensures fair competition and a level playing field.

The research on macroeconomic aspects of the productivity puzzle reveals two underlying microeconomic issues related with the lagging technology adoption and productivity dispersion: the access and skill-related digital divide, especially among firms, and market concentration. These topics will be further explored in the following two subsections.

2.2 DIGITAL DIVIDE

With the outbreak of the COVID-19 pandemic, nations worldwide relied on its telecommunications infrastructure as a source of economic resilience against the health and economic crisis – see Abrardi et al. (2023), Zhang (2021) and Katz et al. (2020) for impact measurements.

In fact, well before the pandemic, an extensive body of economic literature had already underscored the pivotal role of digital connectivity and services in propelling growth, amplifying productivity, nurturing employment prospects, advancing equity, and mitigating poverty – for a review, see Bertschek et al. (2016) and ITU (2020).

Digital technologies have also played a pivotal role in this battle, with e-learning, telecommuting, drones, robots, and electronic devices serving as effective tools against the outbreak. However, the inability of certain countries, individuals, and businesses to leverage these digital advancements results from the digital divide (AISSAOUI, 2022).

In other words, the onset of the pandemic highlighted the remarkable capacity of digital technologies to bolster economic and social resilience. Simultaneously, it laid bare the widening digital divide and its profound implications for social inclusion. Although the pandemic accelerated connectivity, stark inequalities in access and usage continue to impede full and equitable inclusion (WORLD BANK, 2022).

To fully harness the potential of Information and Communication Technology (ICT) and realize the developmental prospects of the digital era, addressing the digital divide stands as one of the most pressing challenges for the information society. The complexity of this issue stems from its vague terminology and multifaceted nature, encompassing diverse scenarios (RALLET and ROCHELANDET, 2004).

Extensive research has been conducted to analyse the digital divide phenomenon, as evidenced by studies conducted by Scheerder et al. (2017), Karar (2019), Hidalgo et al. (2020), Gladkova and Ragnedda (2020), and Unwin (2020), summarised by Aissaoui (2022).

Firstly, it is important to acknowledge for the several dimensions digital divide may encompass, its determinants, forms of measures and then proceed to an analysis on its relation to digitalization and economic performance.

OECD (2001) defines the digital divide as the disparity between individuals, households, businesses, and geographical regions across various socioeconomic levels in relation both to discrepancies in their access to ICTs and to their utilization of the Internet for diverse activities. Essentially, the digital divide may delineate differences among and within countries.

The capacity for individuals and businesses to harness the Internet varies notably within the OECD region and between OECD and non-member nations. The access to key telecommunications infrastructure is fundamental to any consideration of the issue, as it precedes and is more widely available than access to and use of the Internet. This comprehensive definition underscores the multi-dimensional nature of the digital divide, encompassing diverse aspects such as access, utilization, and performance, across global, regional, and social dimensions (AISSAOUI, 2022).

Geographically, the divisions within the digital landscape encompass several dimensions: disparities among developed nations; gaps between developed and developing countries; variations among different regions; and distinctions between rural and urban areas (RALLET and ROCHELANDET, 2004).

As society transitions into the digital era, the discourse surrounding digital disparities has taken on tangible significance. Academic research in this domain has primarily focused on issues related to access and technological resources. This "first-level digital divide" concerning access and equipment represents just one facet of numerous digital inequalities (AISSAOUI, 2022). As Bowie (2000) highlighted, even if universal access to free personal computers and reliable internet were achieved, technological empowerment would remain elusive for those lacking literacy and proficiency. Literacy emerges as a critical asset for individuals, regions, and nations in an information-driven society, Ben Youssef (2004) underscores that the true value of these technologies lies not merely in possession but in their utilization and the contributions made within the network.

Recent literature on these the social differences created by digitalisation has delineated three distinct types of disparities—access divide, use divide, and result/performance divide related to ICTs — termed as the first, second, and third-level divides, respectively (ALEXANDER et al., 2015; GLADKOVA and RAGNEDDA, 2020).

The concept of the first-level digital divide is rooted in the discrepancies of access to ICTs such as computers and the internet, delineating a division between those who can leverage ICT opportunities and those who cannot (MONTAGNIER et al., 2002). Initially emphasized by Long-Scott (1995) in the context of democratic participation, this division later became known as the first-level digital divide.

Measuring this divide entails various indicators, initially focusing on the rate of computer ownership and later expanding to include internet access (EASTIN et al., 2015). The initial metric selected was the number of main telephone lines per 100 inhabitants, introduced by the ITU in 1960. Subsequently, mobile telephony per 100 inhabitants emerged in 1982, initially employing analogous technology before transitioning to the digital one. As the debate evolved with the introduction of the Internet, communication services underwent significant development, leading to new indicators such as the number of computers and internet users per 100 inhabitants. Ultimately, the advent of high-speed broadband replaced the emphasis on the rate of internet connection, shifting the focus towards more qualitative indicators, such as bandwidth (AISSAOUI, 2022).

The second-level digital divide refers to the widening inequalities stemming from variations in digital skills and the ability to utilize ICTs effectively. Kling (1998) made the initial distinction between these divides, highlighting disparities in technical access and social access. This divide is further defined through three skill categories—instrumental, structural, and strategic, as outlined by Steyaert (2002). The first category encompasses fundamental skills, involving the handling of hardware and software, technical prowess, and critical reasoning abilities to navigate through technical hazards and other routine technical challenges. Contrastingly, the second category pertains to the evolving structure of information presentation and the novel methods of accessing online content. Finally, the third category of skills involves preparatory measures for proactive information retrieval, cultivating a decision-making approach based on available information, and a continual analysis of the environment to effectively utilize gathered information in both personal and professional spheres (STEYAERT, 2002).

However, measuring these skills encounters limitations due to a lack of comprehensive definitions, hindering scientific assessments (VAN DEURSEN and VAN DIJK, 2010). The primary challenge in measuring digital skills stems from the absence of comprehensive and universally accepted operational definitions. This limitation results in a scarcity of rigorous scientific assessments evaluating these skills (HARGITTAI, 2003). For instance, Scheerder et al. (2017) undertake a systematic literature review (PRISMA) to gauge medium-related, content-related, safety and security, and general skills. Conversely, Ben Youssef et al. (2022) utilize a questionnaire survey and a multinomial logit model to assess operational, formal, informational, and strategic skills. Meanwhile, Van Deursen and Van Dijk (2010) opt for performance tests instead of logit models. Given the intricacies involved in these statistical measurements, their utility in informing public policy becomes notably more complex.

The current crisis has magnified the importance of addressing digital skills disparities, particularly in the context of remote learning and work. Beyond access issues, the transition to online activities during lockdowns has underscored significant disparities in e-skills among various stakeholders. Consequently, reducing these inequalities has become a top priority worldwide. Moreover, the evolution of emerging technologies like AI and big data necessitates new competencies, highlighting knowledge inequalities that require further investigation (COTTER and REISDORF, 2020; Van DEURSEN et al., 2019, KOEHORST et al., 2021).

Unlike the first and second levels of the digital divide, which revolve around access to and usage of ICTs, the third level pertains to disparities in effectively mobilizing digital resources to achieve specific objectives (AISSAOUI, 2022). Even among users with similar equipment and adequate skills, their outcomes from internet use can vary significantly (STERN et al., 2009; VAN DEURSEN et al., 2014; HELSPER et al., 2015).

Notably, some internet users show quicker improvement in their performance (Ben Youssef, 2004). Conversely, Helsper et al. (2015) highlight that individuals consistently translating their internet use into higher offline returns often experience retroactive effects, as increased economic resources further enhance their digital skills. However, theoretical studies on tangible results of digital engagement at this third level remain limited (HELSPER et al., 2015).

To simplify, two levels of analysis within the third-level digital divide can be discerned, based on the consolidation conducted by Aissaoui (2022): the aggregate and the individual levels.

At the aggregate level, research addresses disparities in the performance of regions concerning information and communication technologies, exploring ICTs' contribution to productivity and economic growth (HWANG and SHIN, 2017; GORDON, 2002; BOYER, 1998; PRADHAN et al., 2018).

Czernich et al. (2009) introduced dummy variables to consider 10 percent and 20 percent broadband penetration in their models explaining broadband's contribution to OECD economies. They found a significant impact on GDP per capita, ranging between 0.9 and 1.5 percentage points, when broadband penetration reached 10 percent. However, the transition from 10 percent to 20 percent did not yield significant results. This led the authors to suggest that broadband saturation and diminishing returns occur at the 20 percent point. Gillett et al. (2006) also included saturation as an independent variable and observed a negative relationship with the increase in economic growth, possibly influenced by network effects.

In implicit confirmation of this suggestion, Qiang et al. (2009) found that the economic impact of a 1 percent increase in broadband is higher in low and middle-income economies and lower in high-income economies. Similarly, Shideler et al. (2007), in their study of broadband impact in Kentucky, discovered that economic impact is highest around the mean level of broadband saturation at the county level due to diminishing returns to scale. According to this study, a critical amount of broadband infrastructure may be needed to significantly increase employment, but once a community is fully built out, additional broadband infrastructure will not further contribute to employment growth. In the case of mobile telephony, Gruber and Koutroumpis (2011) also demonstrate that mobile telephony's effects on GDP growth correlate with wireless penetration growth until penetration rates reach 60 percent, beyond which the effects tend to subside.

At the individual level, two primary research avenues can be highlighted: the first delves into the impact of ICT on wage disparity between skilled and unskilled workers (MAROUANI and NILSSON, 2016; HE and LIU, 2008; CHU et al., 2015; ACEMOGLU, 2000, 2001, 2002, 2005). On the other side, the second avenue focuses on comprehending the advantageous aspects of Internet usage (WEI et al., 2011; STERN et al., 2009; HELSPER et al., 2015; SCHEERDER et al., 2017). Within this line of inquiry, Van Deursen and Helsper (2015) categorize offline outcomes into five areas: economic, social, institutional, political, and educational.

According to the first perspective, there is a long-established modelling that ICTs might demonstrate a differential impact, benefiting certain individuals more than others. Aghion and Williamson (2000) underscored the significance of technological advancements in elucidating internal inequality dynamics within nations. The integration of new technologies into production processes and overall corporate operations often prompts companies to favour hiring a more educated workforce over less-educated individuals. This preference may stem from the skilled workforce's adeptness at adopting and implementing new technologies within firms, thereby boosting their productivity while diminishing that of unskilled workers and subsequently fostering wage disparities.

Regarding the second research strand, empirical studies exploring the influence of Internet usage on economic outcomes suggest that individuals engaging more intensively with the Internet tend to enhance labour incomes and capitalize on employment prospects (DIMAGGIO and BONIKOWSKI, 2008; KUHN and MANSOUR, 2016).

Bhatnagar and Ghose (2004) focus on consumption and suggest that individuals benefiting from ICTs may enjoy a digital consumer dividend, availing themselves of goods and services at more favourable prices compared to those less favoured by technology. Additionally, Jensen (2007) demonstrates that the adoption of mobile phones by fishermen and wholesalers was associated with a dramatic reduction in price dispersion, the complete elimination of waste, and nearly perfect adherence to the Law of One Price, enhancing both consumer and producer welfare.

Social studies investigating ICT's impact reveal various advantages for technology-benefited individuals. ICT use, particularly the Internet, facilitates diversification and expansion of social connections, aiding in finding partners and expanding social circles (MUSCANELL and GUADAGNO, 2012). It also amplifies the volume and intensity of interactions within local communities (KAVANAUGH et al., 2005; KATZ and RICE, 2002).

At the political and institutional levels, individuals with enhanced ICT access benefit from direct interaction with technologically-advanced state institutions. Moreover, those with broader and diverse social networks exhibit higher participation in political and civic affairs. Numerous studies have examined the impact of ICT use on student education and performance (AISSAOUI, 2022). Moore and Kearsley (2011) emphasize the various formal and informal learning opportunities facilitated by the Internet. Nonetheless, according to Helsper et al. (2015), verifying whether these advancements directly correlate with improved educational outcomes remains a challenge.

Empirical studies addressing the first-level digital divide initially leaned toward technological determinism, emphasizing technology as the primary driver of societal evolution (SRINUAN and BOHLIN, 2011). However, this perspective has since evolved to incorporate socio-economic factors, recognizing its limitations in explaining the digital divide. The widespread access to ICTs worldwide has shifted focus towards disparities in their utilization and digital skillsets, in line with the second-level definition (VAN DEURSEN and VAN DIJK, 2018; HILBERT, 2016).

Based on the literature concerning the digital divide, particularly its second level, it can be concluded that there is a need for improved digital literacy within a human capital development framework. This perspective suggests that merely reducing the first-level digital divide may not be a sufficient solution in terms of development. In Brazil, sectoral regulation in telecommunications has primarily focused on addressing access issues, such as through auction service coverage obligations and investment commitments. While these regulations have a reasonable justification in driving social goals through efficient market forces, broader policy measures could complement these efforts to achieve more from information and communication technologies (ICTs), particularly in enhancing labour productivity through digital technologies.

Yet, the COVID-19 pandemic has underscored the persistent challenges of ICT access, bringing renewed urgency not only to address the access-based digital divide, but also, with the ongoing process of Digital Transformation, to further assess the new digital technologies various impacts on social and economic life.

Hence, a synergy between regulatory and public policy measures is crucial for tackling both the first and second levels of the digital divide. Additionally, it is imperative for regulatory and political entities to focus on the third-level aspect to subside future actions and shape evidence-based policies.

2.3 Market Power

The ascent of AI and other information technologies may lead not only to the deepening of economic concentration on developed industrial countries but also to greater market concentration within (and across, as digital markets go global) countries. Consequently, the economy might shift towards an equilibrium that is less competitive, marked by heightened market power by dominant firms. In thesis, entities possessing market power are well positioned to exploit it for their own benefits. These distortions could counteract some of the advantages of innovation, intensifying the negative distributive impacts of labour-saving or resource-saving advancements [6]. Considering any inequality-averse social welfare function, societal welfare could witness a decline if market power is used by the new dominant firms to the detriment of consumers.

Digital platforms have become increasingly relevant in today's business, as they promoted innovative ways of meeting supply with demand on various sectors, promoted new forms of communication and commerce, also implementing new consuming habits on urban transportation, entertainment and other branches of the social and economic life in the 21th century.

Because network effects are paramount to these business models, there is a natural tendency for concentration, which also raises concerns on the optimal regulatory and antitrust approach towards these digital firms. That is, alongside the valuable benefits provided by the new digital giant firms, denoted henceforth as "big techs", fear arises on how the market concentration might bring negative outcomes to the societies. For instance, to preserve the advantages gained, firms may resort to anticompetitive and rent-seeking behaviours, potentially offsetting the benefits derived from the creation and deployment of new digital technologies.

In fact, this trade-off of creating the rules of the game to foster innovation versus limiting economic harms derived from use of monopoly power of the dominant and most innovative firm draws back to the famous north American antitrust case against Microsoft, accused of maintaining its monopoly position in the personal computer (PC) market. Interestingly, Microsoft has acquired OpenAI, an AI research and deployment company that has had leading success on providing AI tools to the workplace. This trade-off then has had a vigorous resurgence under the eyes of legislator, regulators and antitrust authorities.

The economic analysis on how the increasing market power in digital markets should be viewed and dealt with is not a regular task. To name one feature, the deepening of two-sided markets might erroneously raise concerns about predation on the low-price side or even criticize excessive pricing on the high-price side, overlooking the fact that such price structures can also be adopted by small, entering platforms. As stated by Tirole (2015), regulators should exercise caution and avoid mechanically applying standard antitrust concepts where they may not be applicable.

Among these economic features, two main topics stand out: intellectual property rights, its derived institutions and the effects on the innovation dynamics in the digital sphere;

[6] This discussion is held in the Subsection Future of Jobs.

and the scope for competition regulation, paralleled with the utility regulation, already well established in various sectors of the economy.

Nevertheless, there is room for discussion whether antitrust and regulation should continue to rely on purely economic analysis, or whether it should also consider the reality in which firms can wield market power to acquire enormous political influence. Despite its importance, this subsection shall focus solely on economic features.

2.3.1 Intellectual Property Rights, Institutions, and Innovation

One of Schumpeter’s most famous concepts is “creative destruction”. He argued that innovation is a disruptive force that destroys existing economic structures and creates new ones. New innovations, technologies, and business models replace old ones, leading to dynamic and evolving economic systems.

Schumpeter suggested that innovation does not happen uniformly over time but occurs in cycles. Periods of intense innovation, often driven by technological breakthroughs, lead to economic growth and development. However, these cycles also involve periods of consolidation and reduced innovation. The Austrian economist acknowledged that successful innovators could gain temporary monopoly power as a reward for their entrepreneurial efforts. He saw this as a necessary incentive for entrepreneurs to take risks and invest in innovation (SCHUMPETER, 1943).

Schumpeter's fundamental argument revolves around the close relationship between monopolies and Research and Development (R&D). This connection is built on two distinct points: the idea that monopolies naturally foster R&D, and the proposition that inducing firms to undertake R&D requires accepting the creation of monopolies as a necessary, albeit undesirable, outcome. Tirole (2003) chooses to overlook the first argument, which is contentious and not central to Schumpeter's thesis. Instead, he focuses on the second argument, which assigns innovation the status of a public good that can be promoted through a patent system.

Any innovation developed by a single firm offers valuable information to other firms at minimal or no expense. Although all firms are ready to utilize this information, none of them is willing to invest the substantial sums of money required for its production without compensation. In practice, this compensation often takes the form of a patent, granting the innovating firm a temporary monopoly and enabling it to recover its R&D costs. The challenge of the patent system lies in the fact that, while it encourages R&D, it hinders the widespread adoption of innovation, resulting in a non-competitive situation (TIROLE, 2003).

In his ground-breaking 1962 article, Arrow (1962) posed the question: what is the gain from innovation to a firm that is the only one to undertake R&D, given that its innovation is protected by a patent of unlimited duration? Tirole (2003) analyses it under a monopoly scenario[7], and considering competition, in which the monopoly can be threatened by entry, taking place the race for patents. These analyses can shed some light into the incentive structures in the digital landscape.

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[7] Where, socially, a monopolist has too low incentive to introduce a new product, once he cannot fully appropriate the social surplus, unless he can price-discriminate perfectly.

In a scenario for drastic innovations, there is a tendency towards "entry" into the output market (not necessarily competition, it could be a replacement of the prior monopolist). Therefore, innovation is achieved early, with the monopolist concerned with the possibility of the innovation by the entrant (FUDENBERG and TIROLE, 1986). Alternatively, for non-drastic innovations, there is a tendency for the monopoly to persist, because the monopolist has higher probability of obtaining the patent (TIROLE, 2003).

If the task would be to classify the new DigiTech revolution into either of these categories, it would be reasonable to put it into the first one, as it is expected that, if the artificial general intelligence is achieved, computers would be able to outperform humans on most of intellectual activities. Even without the confirmation of this scenario, the general-purpose nature of AI may indeed reshape economic relations in a considerable manner – for studies exemplifying the revolutionary aspects of new digital technologies, see Trajtenberg (2019) and Mateu and Plutchart (2019).

Other studies confirm that, when the lagging firm draws even, both entrant and monopolist intensify their research effort, even though the latter tending to invest more (GROSSMAN and SHAPIRO, 1987; JUDD, 1985). This contrasts with the statement of Holmstrom (1989) that small firms are responsible for a disproportionate share of innovative research, arguing that larger firms are at a comparative disadvantage in conducting highly innovative research.

This is particularly relevant as laws and regulations deal with the traditional trade-offs of spillovers versus patent protection, efficiency effects versus replacement effects, as well as with the dynamic incentives to innovation.

One additional element is that the innovation process of a particular service may be engendered by upstream firms, not vertically integrated, with outsourcing R&D assuming significant market efficiency. Take for instance the technological mobile telecommunication generations (5G being the latest implemented one), the development of the new technologies are predominantly done by specialized firms (e.g. Nokia, Erikson, Huawei) while the provision of the service to the final consumer is done by telecommunication operators. That is, the output firm is not the one necessarily engaging in R&D that shall reshape the nature of the product, and, consequently, the market. This form of organization is seen as an efficient way of the market dealing with transaction costs, as pioneeringly advocated by Coase (1937), leaving to the State its recognition into norms on patents and industrial property in general.

As to this institutional framework on intellectual property, it should be highlighted the significance of the Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS), which played a significant role in shaping intellectual property legislation across the globe. The agreement established a set of minimum standards for the protection of intellectual property rights[8], encouraging innovation, fostering trade and technology transfer[9].

[8] Copyright and related rights (i.e. the rights of performers, producers of sound recordings and broadcasting organizations); trademarks including service marks; geographical indications including appellations of origin; industrial designs; patents including the protection of new varieties of plants; the layout-designs of integrated circuits; and undisclosed information including trade secrets and test data.

[9] Overview: the TRIPS Agreement.

Despite critics, counterarguments to these reinstate that the prevailing intellectual property system ought to be praised. Its main argument is that the speed with which the covid-19 vaccines were developed, viewed as one of science's greatest achievements, stems from the stability and incentive structure fostered by this very own system. Even though the legislation on patents and its implementation may not enter details on competition fostering, sectoral regulation may, as next section explores.[10]

2.3.2 Governmental responses to Market Concentration

Governments can view market concentration on two traditional perspectives: through the Chicago school of thought, in which concentration might be a by-product of efficiency; or by the Harvard school lenses, in which market structure is determinant for behaviour.

In a pre-pandemic phase, there was already a growing concern on the market concentration taken by Big Techs – for that, see Teachout and Khan (2014) and Rahman (2018) – which was accompanied by criticism that their ante-Chicago approach was tantamount to an abandonment of potential efficiency gains (PETIT, 2020).

In a post-pandemic world, this criticism seems to have vanished, while the arguments on competition concerns have evolved to more complex arguments than the ones based on structure versus behaviour debate (MARTY, 2021).

To begin with, it is convenient to highlight some characteristics of digital ecosystems. To Rahman (2018), these activities are characterized by high barriers to entry, large potential sunk costs, increasing returns, and advantages based on diversification. Therefore, it is an infrastructure whose value depends on the downstream activity it enables. Symmetrically, denial of access or discriminatory conditions of access has a significant effect on complementors and users. Finally, it is a socially necessary infrastructure, and unilateral control of it places complementors in a vulnerable position. At the same time, the firms in question can act as gatekeepers for the circulation of goods and services and the diffusion of ideas.

On one side, it is argued that the competitive advantages of dominant operators in the Tech sector, including their technological, financial, and informational strengths, along with the network effects, economies of scale, and scope they enjoy, make it nearly impossible for an equally efficient competitor to emerge and potentially replace them in the future. This dominance, coupled with a lack of competitive pressure, could lead to the abuse of market power. In this situation, their market position becomes less contestable, and barriers to entry become insurmountable (MARTY, 2021).

Two barriers to entry seem to be increasingly relevant, under the competition scope: i) data and algorithms, as the possession of massive, updated and diversified data flow is paramount to these new business models, presented as key advantages of dominant companies (RUBINFEL and GAL, 2017); and ii) network effects, as the evolution of digital platforms in multi-sided markets positions them as the new network industries, primarily driven by the significant impact of direct, indirect, and algorithmic network effects (MONTERO and FINGER, 2021).

[10] https://www.economist.com/by-invitation/2021/04/20/michelle-mcmurry-heath-on-maintaining-intellectual-property-amid-covid-19

On the other, there is evidence that there are some firms that have been able to enter markets efficiently without having an initial stock of data or algorithms, such as Airbnb, Uber and, more recently, Zoom (LAMBRECHT and TUCKER, 2017). Additionally, network effects may not constitute a deterministic barrier to entry or exit, given examples of network effects reversion, e.g. MySpace, and based on the fact that new entrants may thwart them by adopting a differentiated strategy based on exclusivities negotiated with content publishers, as demonstrated in the video game console industry by the entry of Microsoft with its Xbox, to the detriment of Sony and its PlayStation 2 (LEE, 2013).

If the former argument is valid, regulation in the digital ecosystem may take place to foster competition, paralleled with the one conducted, for instance, in the telecommunications sector, given its network industry nature similarities. For instance, two imperfect solutions could be imported from the latter to the former: infrastructure sharing and structural separation.

The first solution would involve extending the theory of essential infrastructures to assets that are not truly in a natural monopoly situation and might even appear as convenient facilities (RIDYARD, 2004). This could include access to data, computational capacities, or even algorithms, considering that a new entrant might not be able to replicate them objectively, either in technical or financial terms.

One potential avenue is ensuring access to data. With the anticipated surge in the volume of data collected, especially due to the growth of the Internet of Things, firms' ability to access these data streams for training their AI will become increasingly crucial. This challenge is particularly significant for new entrants or firms not integrated into the ecosystems of tech giants, thus being excluded from these data flows. Ensuring universal and unimpeded access to data could mitigate entry barriers for non-dominant companies and, most importantly, enable them to offer algorithms that are at least potentially as efficient as those of the dominant operators (MARTY, 2021).

The proposed solutions, while addressing certain challenges, also face counterarguments. For instance, granting new entrants access to essential assets like data may raise concerns about the exploitation of extensive and risky investments made by dominant operators. Asymmetric regulatory tools, if not carefully implemented, could impact the incentives for investment across the market, leading to issues such as the expropriation of competitive advantage and free-rider behavior. These concerns not only jeopardize consumer welfare but also pose a threat to overall innovation (MARTY, 2021). If even more radical regulatory remedies are envisaged, such as algorithm sharing, as suggested by Gal and Petit (2021), it might reduce firms' incentives to invest and innovate by diminishing the potential advantages of collecting and processing data. Additionally, if AI algorithms were trained on similar data, the risks of algorithmic collusion could be heightened (MALAURIE-VIGNAL, 2021).

The second proposed solution involves limiting the capacity of dominant operators to diversify their activities. This approach extends the principle of the special responsibility of dominant operators and introduces a principle of speciality, similar to the regulations imposed on French public companies holding exclusive rights before the liberalization of network industries. The concept is to recognize that any diversification by dominant operators may give them an advantage over competitors in downstream or related markets. Since competitors lack the same resources generated by the dominance in the initial market, they cannot compete effectively with the dominant firm diversifying on its own merits (MARTY, 2021).

The risks associated with this regulatory tool in terms of consumer welfare and innovation may be substantial. Firstly, such a remedy may hinder the dominant operator from enhancing its offerings to the advantage of consumers by integrating various services. Secondly, it might perpetuate inefficient offerings in related markets. Preventing distortion of competition could potentially impede the competitive process by prohibiting dominant companies from implementing practices whose net effect could be (even if conditionally) beneficial to consumers. This uncertainty suggests, in principle, a case-by-case approach to practices that may be self-preferential, rather than a blanket *per se* prohibition – rule or regulation that prohibits certain conduct or practices without requiring proof of anti-competitive effects (MARTY, 2021). Nonetheless, this approach raises concerns about potential under-enforcement, as challenges arise in measuring algorithmic manipulation practices without exposing oneself to symmetrical risks of false positives (wrongly sanctioning) or false negatives – erroneously considering the practices as compliant with competition rules (CABRAL et al., 2021).

Despite the current uncertainties, the potential for a more significant disruption with the advancements of specific digital technologies, such as AI and quantum computing, adds considerable complexity to the debate. The technical capacity of competition authorities may be put to the test, given the necessity of correctly understanding its regulatory role and mapping market dynamics, measuring economic impacts, identifying relevant counterfactuals, and pricing competitive risks.

These innovations should, to use a somewhat overused Schumpeterian expression, lead to creative destruction that could dismantle the structural advantages enjoyed by dominant operators and lower barriers to entry.[11] This dynamic phenomenon adds uncertainties onto whether ex-ante regulation should indeed be introduced or if market forces and antitrust will be sufficient in the future for the benefits from technology to be obtained without causing economic harms to society. Therefore, one valuable evaluation would be to acknowledge if future technological change made possible by AI and its counterparts would act as tools for more creative destruction or consolidation of dominant positions.

Marty (2020) argues that these innovations stand little chance of escaping the control of the current dominant operators, who are likely to be their primary developers, whether through external growth (acquiring startups) or internal growth (research and development, learning by doing, etc.). Furthermore, the effectiveness of AI experiences significant growth when supported by massive, constantly updated, diversified, and high-quality data flows[12]. Consequently, a competitor without access to comparable data flows could be inherently less efficient than the dominant operator. Lastly, the availability of tools for early-stage detection of competitive threats and future opportunities

[11] Marty (2021), for instance, states that Microsoft's position of strength was eroded much less during the first decade of this century by the competitive disputes the company faced than by the development of the Internet and, especially, the growth of mobile technology.
[12] EU Commission, DG Competition Press Release IP/19/4291, 17 July 2019.

(nowcasting, sentiment analysis) may empower incumbent dominant operators to either self-disrupt and perpetuate their position or block potential competitors (Marty, 2021).

In response to the two main behaviours exhibited by tech giants as gatekeepers and engaging in self-preferencing (to which technical solutions were outlined), various jurisdictions are deliberating and implementing regulatory frameworks, each at different stages of legislative progress. The European Union has enacted the Digital Markets Act, the United States has introduced the Platform Competition and Opportunity Act and the Ending Platform Monopolies Act, Japan has passed the Act on Improving Transparency and Fairness of Digital Platforms, and the United Kingdom has a proposal, the Digital Markets, Competition, and Consumer Bill, currently under review. In Brazil, Bill 2768/2022 currently under review in Congress aims to establish rules for the organization, functioning, and operation of markets mediated by digital platforms.

In sum, there are some aspects to be considered prior to the institutional structuring. The first and obvious one is to internalize the discussions that have already taken place in the aforementioned countries, understanding the extent to which the imported definition of problems and construction of solutions could be valuable inputs to the analysis of the Brazilian digital economy.

Furthermore, potential future landmarks can yield long-term benefits if the economic trade-offs are technically well-motivated, ideally with theoretical and empirical research as impact assessment accessories, focusing on how regulation may tackle market imperfections without curbing the innovative process and economic growth.

Lastly, the arising regulatory activities should take advantage of established network industries regulation but may also be flexible enough to envision alternative solutions, which was already championed by Tirole (2015) for “ages”, considering the outstanding dynamism of the digital ecosystem.

2.4 GLOBOTICS

Globalization and robotics, often termed “globotics” are rapidly reshaping the global economy, propelled by the exponential advancements in digital technology. These technological strides, doubling in magnitude approximately every few years, are driving this transformative shift at an explosive pace. The repercussions of this evolution are expected to significantly impact developed and developing nations, as highlighted in studies by Brynjolfsson and McAfee (2014) and Baldwin (2019).

Additionally, Baldwin (2019), prior to the pandemic, theorized that the convergence of “teleworking” and the rise of artificial intelligence would prompt a substantial realignment with profound implications: a fresh surge of globalization, particularly within the service sector. Baldwin coined the term “telemigration” to illustrate individuals residing in one country while employed by companies situated in another. The universal health crisis has accelerated these phenomenon.

In sum, the combination of globotics and telemigration alongside with other digital technologies enhancements is quite likely to fundamentally alter the landscape of development in substantial and noteworthy manners. The conventional manufacturing-centric development theories may give room to the service-led ones. In this regard, it’s important to analyse development theory and how this technology phenomenon may demand its reassessment.

Baldwin and Forslid (2020) posit that the globotics transformation is poised to disrupt the conventional manufacturing-driven development trajectory akin to China's, while simultaneously fostering the evolution towards a service-oriented development journey, similar to the one India is pursuing.

The Globotics Transformation will be the third great economic transformation to shape societies over the past three hundred years, as proposed by Baldwin (2019). In this new era, digitalization will assume a pivotal role on economic and social development. Before exploring the potential avenues through which these transformations might unfold, it is essential to provide a brief historical contextualization of the first two related revolutions.

The first great transformation initiated in the early 1700s, denoted by the author as the Great Transformation, transformed societies from rural to urban and from agriculture to industrial. Launched by the Steam Revolution and all the mechanization that followed, the Industrial Revolution is far more than just the tale of steam power; it was the spark that ignited it all. Above all else, steam enabled people to surpass the constraints of muscle power, whether human or animal, and produce vast quantities of valuable energy at their command. This paved the way for factories and large-scale production, for the development of railways and extensive transportation networks. In essence, it laid the groundwork for modern life (BRYNJOLFSSON and McAFEE, 2014). This pivotal era marked, in the terms of these authors, humanity's First Machine Age — the first time progress was driven primarily by technological innovation—and it stands, they argue, as the most profound period of transformation the world has ever witnessed.

The second great economic transformation, called by the Baldwin (2019) as the "Services Transformation", shifted from 1973 onwards the focus from industry to services, being launched by the evolution of computing power and the continuous development of Information and Communication Technologies. Brynjolfsson and McAfee (2014) call this transition as the Second Machine Age, as computers and other digital advances did for mental power what the steam engine and its extensions did for muscle power. These authors argue that when goods are digitized — transformed into bits that are storable on computers and transmissible over networks — they take on fascinating and unique characteristics: they become subject to a different economic framework, where abundance is the standard rather than scarcity.

The Globotics Transformation has been launched by a third technological impulse: digital technology and, or the Digitech impulse, is radically different from the horsepower and ICT revolutions. Allowed by machine learning, computing is achieving new grounds from conscious thought to an intuitive and unconscious one and becoming to surpass humans in instinctual mental tasks (BALDWIN, 2019).

The Laws of Digitech encompass four principles that shed light on the unconventional characteristics of digital technologies. Moore's Law, for instance, asserts that computer processing speeds double roughly every 18 months, despite the increasing research costs associated with this advancement[13]. Gilder's Law, also known as the law of telecoms, indicates that the overall telecommunications system capacity (measured in b/s) triples every three years, with bandwidth expanding at least three times faster than computing power. Metcalfe's Law highlights that the value of a network amplifies at a rate surpassing the mere increase in connected individuals. Finally, Varian's Law postulates that while digital components tend to approach a cost of nearly zero, digital products hold exceptionally high value.

Baldwin (2019) argues that the Globotics Transformation will differ from the other two in two important ways. Firstly, Digitech's impact is going to be more heavily felt in the service sector, given its size and large employment capacity, its impacts tend to be greater than the ICT revolution's. Secondly, unlike the transformations that occurred in the nineteenth and twentieth centuries, which took 100 and 10 years, respectively, and globalization to arise after the automation, now globalization and automation occur *pari passu*.

This feature of the current transformation may have implications for labour market dynamics in all countries, since the new globalization wave provided by digital technologies such as telecommuting, deep learning translation, augmented reality, holoportation, telepresence robots, collaborative platforms tend to bring people closer to each other, despite their physical or cultural distance.

Indeed, the Great Transformation reduced the costs of moving goods and separating production from consumption. The Service Transformation reduced the costs of moving ideas, separating production stages. Now the Globotics Transformation is reducing the costs of "moving people", separating physical locations of production factors and output.

Needless to say, from 2020 onwards, the world was compelled to embrace remote work due to external factors, making remote workers less remote, in the words of Kilic and Marin (2020), unequivocally accelerating the pace of these societal transformations reliant on digital technologies.

In G7 countries, businesses are increasingly relying on remote workers for various tasks. Primarily, these remote workers reside within the same nation as the companies. This trend may be the result of wage variations and talent scarcities rather than globalization. However, a growing number of companies are inclining towards hiring foreign-based online service workers, often referred to as "telemigrants" (BALDWIN, 2019).

To acknowledge the extent of these changes ultimately implies on gauging the rate at which telemigration will achieve. The answer will differ across the various types of service since some are able to integrate remote workers with greater ease or have more widely accepted standards (BALDWIN and FORSLID, 2020).

There are three compelling factors that indicate telemigration is set to accelerate its growth across nearly all sectors faster than commonly anticipated: i) The rate at which Digitech is diminishing the language barrier, with machine translation fundamentally reshaping the global supply of service workers; ii) the trend toward remote work, with a

[13] It now takes seventeen times more research hours to double processing speeds than it did in 1971 (BALDWIN, 2019).

restructuring undertaken by companies and governments, specifically designed to accommodate and integrate telecommuting employees seamlessly, heavily dependent on collaborative platforms, especially after the pandemics, as defended by Brynjolfsson et al. (2020); and iii) the evolution of virtual and augmented realities, which allows for non-verbal communication in a way traditional technologies cannot.

Indeed, within the landscape shaped by globotics and telemigration, there is a clear trend of services gaining heightened value in this emerging Digital Era—an observable phenomenon already reflected in macroeconomic statistics. Nonetheless, it is worth noting that the emphasis of economic literature on services is relatively recent, considering that manufacturing has been regarded as an important engine of growth.

The evolving landscape of manufacturing is not a recent revelation. Loungani et al. (2017) highlight a burgeoning body of evidence that contests the longstanding beliefs in economic development that industrialization serves as the primary driver of growth. Hallward-Driemeier and Nayyar (2017) echo this challenge, emphasizing how globalization and emerging technologies are reshaping the attractiveness and viability of manufacture-based development strategy.

The conceptual frameworks supporting manufacturing-centric development have a noteworthy lineage, originating from what Krugman referred to as “high development theory”.

In fact, development economics has historically marginalized or even disregarded the significance of the service sector, a bias underscored by Loungani and Mishra (2014) as a deeply rooted prejudice. This bias against services finds historical roots; for instance, Adam Smith, in his renowned work, questioned the societal value of services rendered by various professions, citing “churchmen, lawyers, physicians, men of letters of all kinds, players, buffoons, musicians, opera-singers, opera-dancers, etc.” Karl Marx also deemed several service activities as “faux frais” of production — expenses incurred in the capital’s productive utilization but not contributing intrinsic value. This perspective resonated in Soviet planning, which de-emphasized services compared to heavy industry, a model that initially inspired early post-war development thinkers. Similarly, Baumol (1967) propagated the belief that services constitute a sector resistant to productivity enhancements.

It is a fact that many of today's prosperous nations attained wealth through industrialization. However, since approximately 1970, these same nations have experienced deindustrialization due to the impact of globotics. In this regard, there is growing evidence of the service sector importance.

Braga (2019) asserts that while industrial development has historically driven export-led growth, the distinctive potential of ICT-enabled services challenges traditional paradigms. By exclusively relying on electronic cross-border delivery, these services offer an avenue for export diversification that transcends conventional limitations, thus becoming accessible even to countries lacking developed physical trade infrastructure, as long as the telecommunications infrastructure is reasonably deployed.

Digitech may enable numerous emerging markets to directly export their comparative advantages — such as low-cost labour given its productivity — without the necessity of manufacturing and exporting those goods, in the first place. In a Ricardian model of comparative advantage trade, trade in goods is essentially a veil for trade in labour services. Digital technology is simply revealing this reality by removing that veil. Consequently, the subsequent surge in service trade is anticipated to yield an overall net export gain for emerging markets while representing an overall net import gain for developed economies (BALDWIN and FORSLID, 2020).

Utilizing a disaggregated annual panel data on global trade in services for 192 countries, Loungani et al. (2017) demonstrated that the trading of services is gaining substantial traction within global trade, progressively emerging as a pivotal element in global production. The analysis presented delineates global patterns in service trade and presents empirical observations highlighting the variations among countries across multiple dimensions of service exports. This study posits that not only are trading services catching up with the exportation of goods in numerous countries, but they also have the potential to perpetuate the robust globalization initiated by exported goods. It contends that such a development could significantly impact shifts in structural transformation, labour allocation, and income distribution.

Mattoo (2018) discussed what the new emphasis on services means for international cooperation efforts, and Heuser et al. (2017) examined the role of services in global value chains.

This is no new topic and it already counts with some policy recommendations. The 2018 report by the Pathways for Prosperity Commission titled “Charting Pathways for Inclusive Growth” identifies 'Global trade in services' as Pathway Three. Within the report, various strategies are outlined to unlock these pathways, with particular emphasis placed on how governments and businesses can foster the development of a digitally prepared nation. Several recommendations align closely with those proposed by UNCTAD across numerous publications on e-readiness. These recommendations emphasize five pivotal pillars: facilitating digital infrastructure, establishing supportive legal and regulatory frameworks, nurturing human capital, enabling financial support, and fostering effective coordination.

One policy aspect that can act as an inhibitor of this process would be the lack of regulatory cooperation pro service globalization. In some cases, regulatory cooperation could be far reaching and lead to harmonization or mutual recognition, which would eliminate the costs of regulatory heterogeneity for firms and liberate them from the uncertainty of discretionary licensing (TRACHTMAN, 2014). In other cases, regulatory cooperation could be valuable even if it only involves greater mutual understanding of how regulatory discretion in each jurisdiction will be exercised because that too would lend predictability to commitments.

For instance, rigid labour market regulations may prevent developing countries to enter the Globotics Transformation age as participants of an increasing international trade on services. Additionally, commercial and industrial policies focusing solely on physical products may not capture the potential of international trade.

2.5 Future of Jobs

Advancements in artificial intelligence (AI) and related automation technologies have sparked apprehensions regarding job displacement and heightened inequality. This apprehension is pervasive in high-income countries, and its ramifications may be more profound for developing nations and emerging market economies once these regions heavily depend on abundant labour and natural resources as their comparative advantage in the global economy. The potential decline in returns to both labour and natural resources, coupled with the winner-takes-all dynamics introduced by new information technologies, raises concerns about further impoverishment in the developing world. Such a scenario could jeopardize the swift advancements that have characterized development success over the past five decades, posing a threat to the strides made in poverty reduction and inequality alleviation (STIGLITZ et al., 2021).

The past decade has witnessed rapid advancements in AI driven by novel machine learning techniques and the accessibility of vast datasets. This momentum of change is anticipated to escalate in the upcoming years, as pointed out by Neapolitan and Jiang (2018) and Russell (2019), with AI applications already impacting businesses – see Agarwal, Gans, and Goldfarb (2019). Some analysts perceive this as a precursor to a jobless future (FORD, 2015; WEST, 2018; SUSSKIND, 2020), while others view the impending AI revolution as a mean to enhance human productivity and redefine work experiences (MCKINSEY GLOBAL INSTITUTE, 2018).

The predictions in studies examining which jobs may be replaced by automation and AI in the coming decades vary widely. Estimates range from a relatively small percentage of 14% of all jobs (OECD, 2019) to approximately 20–25% (HARRIS et al., 2018), and even higher figures of almost 50% according to studies by Frey and Osborne (2017) and McKinsey Global Institute (2018). Even the lower estimates suggest a significant impact, particularly as the effects may be concentrated in specific industries and among certain groups of workers, notably unskilled and routine jobs.

On one side, there are alarmist claims suggesting that the imminent progress in AI and robotics will lead to the extinction of human employment. On the other, numerous economists argue that historical technological advancements have consistently brought creative destruction but never with disastrous consequences to workers, in the broad perspective. Through these lenses, there would be no compelling reason to anticipate that the Digital Transformation Era will diverge from the trend since the Industrial Revolution.

Furthermore, AI technologies present instances where they either substitute or supplement human labour. This duality arises from AI's expansive technological capacity, capable of both functions. Consequently, the extent of job displacement attributable to AI largely hinges on societal and business decisions, as outlined by Acemoglu et. al (2022).

In essence, robotics, present AI practices and other digital technologies are perpetuating a trend seen in prior automation technologies: leveraging machines and computers to replace human labour across an expanding spectrum of tasks and industrial operations.

Over time, automation has involved substituting machines for human labour, resulting in the displacement of workers from the tasks being automated. However, this displacement effect is not integral, or only incidentally considered, in most macroeconomic and labour economic production function models. The typical approach represents production, either in aggregate or within sectors, as a function denoted by $F(AL, BK)$, where L signifies labour and K represents capital. In this context, technology is often seen as "factor-augmenting", multiplying these production factors akin to parameters A and B in the production function. Although it might seem natural to model automation as an increase in B, signifying capital-augmenting technological change, this form of technological advancement does not inherently cause displacement and typically increases labour demand and wages (ACEMOGLU and RESTREPO, 2016). Additionally, automation is not primarily about evolving more productive versions of existing machines but entails introducing new machinery to perform tasks previously undertaken by human labour (ACEMOGLU et. al, 2022).

Labour-augmenting technological changes, denoted by an increase in A, do result in a particular type of displacement if the elasticity of substitution between capital and labour is low. However, in general, this kind of technological change also broadens labour demand, particularly if capital adjusts over the long term (ACEMOGLU and RESTREPO, 2016). Furthermore, the examples provided by Acemoglu and Restrepo (2018) underscore that automation doesn't directly enhance labour; rather, it transforms the production process, enabling more tasks to be executed by machines.

The existence of a displacement effect doesn't imply that automation will invariably diminish labour demand. In fact, historical records show various periods when automation is correlated with expanded labour demand and even elevated wages (ACEMOGLU and RESTREPO, 2018). According to these authors, there are several reasons why automation, led by digitalization, for instance, may engage in positive impact on labour demand:

1. The Productivity Effect: by reducing the cost of certain tasks, automation boosts labour demand in tasks not automated (AUTOR, 2015). Particularly, automation leads to capital substituting for labour because capital, at the margin, performs certain tasks more cost-effectively than labour used to, reducing prices of goods and services undergoing automated production processes, thereby augmenting household wealth and increasing demand for goods and services.
2. Capital Accumulation: automation increases production's capital intensity, prompting further capital accumulation (e.g., by elevating the capital rental rate). This accumulation escalates labour demand. This may have been a pivotal adjustment channel for the British and American economies during the Industrial Revolution and the mechanization of agriculture in the first half of the 20th century, marked by rapid capital accumulation (ALLEN, 2009; OLMSTEAD and RHODE, 2001). According to some neoclassical models of economic growth (though based on restrictive assumptions), capital accumulation could be potent enough to consistently raise wages in the long term due to automation – see Acemoglu and Restrepo (2016) –, but a more conservative prediction is that it will act as a countervailing effect.

3. Deepening of Automation: automation's displacement effect occurs at the extensive margin — expanding the set of tasks producible by capital. But if technological advancements increase capital's productivity in already automated tasks, this wouldn't create additional displacement, as labour was already replaced by capital in those tasks. However, it generates the same productivity effects noted earlier, boosting labour demand.
4. The Reinstatement Effect: similar to how automation displaces, creating new tasks reinstates labour demand, notably increasing labour's share in national income. Consequently, technological progress may balance automation's impacts by generating new tasks, contributing to a more equitable growth path. Periods characterized by intensive automation have frequently aligned with the rise of new job opportunities, activities, industries, and tasks. In the study conducted by Acemoglu and Restrepo (2016), the period spanning from 1980 to 2010 showcases that approximately half of the growth in employment can be attributed to the introduction and expansion of novel tasks and job designations.

The task-based framework provided by Acemoglu and Restrepo (2018) outlines that the generation of new labour-intensive tasks (tasks in which labour holds a comparative advantage over capital) might serve as the most powerful factor balancing the growth process amidst rapid automation.

If, as suggested by Acemoglu and Restrepo (2018), automation has endured for centuries, despite the notable countervailing factors mentioned, an anticipated unbalanced growth trajectory might have entailed a continual decline in the labour share of national income since the onset of the Industrial Revolution. However, historical evidence refutes this hypothesis (see, for instance, Kuznets (1966); Acemoglu (2009)), indicating the existence of other potent forces propelling a shift towards more labour-intensive production to counteract the effects of automation.

Considerable uncertainty surrounds the impact of artificial intelligence, even among experts in the field. Some contend that AI holds less significance compared to the major innovations of the 20th century, predicting its impact on the economy to be rather limited. For instance, Gordon (2016) argues that indoor toilets and electricity exerted far more profound consequences on people's standards of living than more contemporary innovations.

Conversely, others go to the extent of forecasting that AI will usher in a level of technological progress unparalleled in human history. In this regard, Harari (2023) states that AI will change the course of human history, in fact, its medium-term development will lead to the end of human-dominated history. The author argues that there will be a shift of authority from humans to algorithms and the remarkable efficiency of AI in executing tasks, including those traditionally considered within the realm of human creativity, is predicted to give rise to a unique societal challenge — a segment of individuals not merely unemployed but deemed unemployable. This "useless" class may emerge as a consequence of lacking the skills required by the evolving economy, rendering them obsolete in the face of AI's capabilities.

The divergence in these perspectives reflects the ongoing debate and uncertainty surrounding the true potential and implications of AI. Therefore, it is convenient to summarize the existing research on both views.

Several observers express concerns that artificial intelligence and other digital technologies may lead to labour-saving effects, resulting in a decline in the demand for labour at existing factor prices. In such a scenario, equilibrium wages could decrease, negatively impacting workers.

Over the last half-century, the United States and many other countries appear to have undergone technological progress biased against workers with lower education levels engaged in routine tasks. This bias may have even been labour-saving, contributing to a reduction in the real income of these workers. Indeed, it is conceivable that closer integration of wealthier nations with poorer countries, characterized by a more abundant supply of unskilled labour, could exert downward pressure on the wages in the affluent countries (BHAGWATI, 2004).

In this scenario, Berg et al. (2018) center their attention on the varying impacts of technological progress across different worker groups, indicating that technological advancements could be detrimental to unskilled labour as it can be easily substituted by robots. In contrast, high-skilled labour is likely to complement robots, resulting in benefits from technological progress. Consequently, there is a risk that technological advances may lead to significant increases in inequality.

Automation could also exacerbate inequality along other dimensions, such as in sectors where women predominantly hold routine jobs. Brussevich et al. (2018) estimate that 26 million female jobs across 30 countries (28 OECD member countries, Cyprus, and Singapore) are at a high risk (likelihood greater than 70 percent) of displacement by technology within two decades.

Acemoglu and Restrepo (2019) develop a specific model suggesting that the capital-biased displacement of workers by robots has the potential to diminish the labour share of income and might be labour-saving, particularly if the productivity gains from robots are moderate.

The effects on inequality hinge also on the presence of other scarce limiting factors in the economy, such as natural resources or land. These factors may derive benefits from technological progress and subsequently become scarcer as the factors “capital” and “machine-replacing labour” become more abundant and affordable. Korinek and Stiglitz (2021) demonstrate that in such a scenario, without government intervention, labour may face adverse outcomes from technological progress even in the long run.

Globally, similar dynamics may unfold. While labour-saving technological progress would make the world as a whole richer, it would disproportionately impact developing countries that have a comparative advantage in cheap labour. If worldwide demand for labour, especially unskilled labour, declines, these countries would experience a significant deterioration in their terms of trade and lose a substantial fraction of their export income. Labour-saving progress might not only create winners and losers within the affected developing countries but also render entire countries on a net basis worse off (STIGLITZ et al., 2021). Alonso et al. (2020) find that improvements in the productivity of "robots" could drive divergence, as advanced countries benefit more from computerization given their higher initial capital stock.

Stiglitz et al. (2021) argue that, if it turns out that artificial intelligence is indeed labour-saving, particularly for unskilled labour, the repercussions for developing countries could be profound. As unskilled labour constitutes their comparative advantage and a relative richness, the ongoing convergence in living standards between developing and developed countries, a hallmark of the past half-century, could come to a halt or even reverse. Additionally, they state that this scenario would pose significant challenges for domestic policy within developing countries. In many regions, inequalities within developing countries surpass those in developed ones. AI has the potential to exacerbate these inequalities, and developing countries often lack the institutional capacities to effectively counteract them.

In the empirical stance, Autor et al. (2003) highlight that during the 1970s to the 1990s, computerization substituted for an increasing number of routine tasks, while simultaneously enhancing the productivity of workers involved in non-routine jobs requiring problem-solving and complex communication tasks. These technological shifts might have accounted for nearly two-thirds of the relative demand shift towards college-educated labour during that period.

More recently, Acemoglu and Restrepo (2022) estimated substantial adverse effects on employment and wages resulting from the introduction of industrial robots in the US, particularly concentrated in manufacturing and among routine manual, blue-collar, assembly, and related occupations. This contributes to an understanding of the pronounced increase in wage dispersion across skill groups over the past five decades.

An even more radical viewpoint, dating back to John von Neumann, suggests that AI could eventually reach a stage where AI systems attain human levels of general intelligence. This implies that they could engage in research, design improved versions of themselves, and recursively self-improve, leading to an accelerating pace of technological progress. In the words of von Neumann, this could result in "the appearance of approaching some essential singularity in the history of the race beyond which human affairs, as we know them, could not continue" – see Ulam (1958).

It has become rather commonplace to encounter negative impact assessments of AI and other digital technologies. However, despite the undoubtedly risks associated with these technologies, the valuable benefits should not be overlooked, and a certain degree of optimism can also be nurtured.

Even if technological progress initially leads to labour-saving outcomes, it may also spur additional accumulation of capital that complements labour, ultimately benefiting labour in the long run. For instance, Caselli and Manning (2019) demonstrate that in an economy with only capital and labour, where long-run capital accumulation is influenced by an exogenous interest rate, labour will consistently experience gains.

It is also conceivable that other forms of technological advances could benefit workers. Intelligence-assisting devices and algorithms (IA) may be complementary to labour rather than substituting for it, thereby enhancing the prospects of labour. Innovations falling into this category may include augmented reality (AR), machine learning (ML) algorithms assisting in analysing complex data, and other forms of integrating AI with humans. IA innovations may help workers become more productive in their jobs by taking over or improving specific tasks (STIGLITZ et al., 2021). Additionally, automation technologies often impact specific tasks but not entire jobs, as jobs typically consist of multiple tasks, see, e.g., Acemoglu and Autor (2011).

The fear of a drastic upheaval is not recent, neither are the predictions of the development of AI. Armstrong et al. (2014) observe that for the past six decades, analysts, from the initial Dartmouth conference, passing through Dreyfus's criticism of AI, Searle's Chinese Room paper, Kurzweil's predictions in the "Age of Spiritual Machines", to Omohundro's "AI Drives" paper, have consistently anticipated the development of general AI to occur within 15-25 years from whenever the prediction was made.

Another argument worth noting is that the technological singularity, a point at which the economic impacts of digital technologies are expected to be drastic (positively, for optimists, or negatively, for pessimists), may never actually occur. This possible turning point, whether termed the Globotics Transformation, the Third Age Machine, or by any other variant, is discussed by Walsh (2016), who suggests that if this new digital phenomenon does not exhibit revolutionary characteristics, the influence of AI on the economy and society might be less substantial than what both pessimists and optimists have anticipated.The concept that the swift advancement in computation and artificial intelligence will reach a critical point or singularity, beyond which economic growth will rapidly accelerate due to an ever-increasing pace of improvements permeating the economy is tested by Nordhaus (2015). The author formulates a growth model incorporating the Singularity and conducts various tests to assess whether societies are swiftly approaching it, being the pivotal question the substitutability between information and traditional inputs. The tests conducted by Nordhaus (2015) indicate that the Singularity is not imminent.

Based on the first type of previsions presented, arguments on an institutional response against the backlashes of AI arise. In general, they advocate for AI regulation, but do not delve into how this new regulation should be configured. For these new forms of public responses to be reasonably moulded, it is firstly necessary to delineate the social problem, convince society that it exists and to acknowledge if the existing institutions cannot resolve it or do not possess technical capabilities to do so.

Additionally, it is paramount to better comprehend the channels thought which possible policy tools might exert its effects, considering the underlying trade-offs. For instance, regulating technology aiming to counteract effects on labour market may limit innovation from economic growth perspective. The potential gains from minimizing the displacement effect in the short-term might not compensate for the opportunity cost of the reinstatement effect in the medium or long term.

Therefore, it is crucial to foster an international debate on these issues within a multi-stakeholder environment. This debate not only puts to test the divergent theories but also allows for the discussion of global concerns that require international coordination and thus enables the formulation of guidelines that can be adapted by countries at varying stages of development.

3 CONCLUSION

The evolving economic and technological landscape, particularly with the profound impacts of new digital technologies raises questions if the existing institutions are sufficient to tackle market failures, prevent social disruption and foster international cooperation. Furthermore, doubts arise as how policies should be configurated not only in the future but already in the present considering potential consequences of digitalization.

In this context, it is important to analyse these institutional issues under the light of the three transformative turning points discussed throughout the previous sections: the industrial, ICTs and digital revolutions.

Experience shows the importance of international cooperation in institutionalized terms given the outbreak of physical goods globalization and stability of great nations' interests, as stated by Polanyi (2001).

That is, the globalization of goods that occurred a century after the Industrial Revolution and their derived impacts were only balanced after the creation of the United Nations (including its specialized agencies), the International Monetary Fund, the World Bank, the General Agreement on Tariffs and Trade (which laid ground to the posterior creation of the World Trade Organization – WTO) and other multilateral institutions. They allowed actual international coordination, necessary macroeconomic adjustments and microeconomic corrections.

The ICTs revolution brought the globalization of ideas only a decade after its surge, however, no revolutionary international institutional reordering was necessary to balance its effects. The existing institutions continued to operate with stability, which outcomes were sufficient to address the economic impacts of the then new technological advancements. For instance, intellectual property issues were addressed through the Trade-Related Aspects of Intellectual Property Rights (TRIPS) Agreement under the direction of WTO. Furthermore, a more pronounced role was assumed by the International Telecommunication Union (ITU), since the development of the telecommunication demanded international standardization.

At the national level, significant changes were observed, particularly with the privatization of state-owned telecom companies and the opening up of markets to new providers alongside with creation of independent regulatory agencies.

The suggested current digital revolution is endorsed to generate globalization of people in parallel with its occurrence. As a consequence of its effects, doubts arise as how the state responses should be shaped and organized.

The conclusion of this study goes in the direction of taking advantage of existing institutions and reshaping its objectives. Extensive body of work was accumulated by the international and national institutions that regulate telecommunications and ICTs, given the similarity of the network industry characteristics and the structural interdependence inside the internet value chain, it would be a loss not to benefit from this acquired knowledge.

Regardless of which type of regulation may take place, from labour market to digital platforms, it is paramount for these new policies to be founded on a reasonable amount of evidence – or at least theoretical reasoning. As it has been stated throughout the technical subsections, the economic impacts need to be reasonably assessed prior to any intervention, without which costly economic impacts may emerge as consequence.

Furthermore, the new regulator should have a critic view of what can be imported from traditional network industry regulation to the digital ecosystem once new issues arise. Economic pro-competition regulation of tech firms such as making data sharing compulsory is different from technology regulation on its use and applications on human relations, such as robots impersonating real people. There is a widespread argument among experts (and non-experts) regarding the necessity of regulating AI, in particular; however, there is a lack of precise understanding on how to effectively implement such regulations. Entities with this new task should take its time to assess prospective regulatory impacts, considering that excessively restrictive regulations may curb benefits of the technology and eventually economic growth.

Complementarily, in the policy field, some public responses may take the form of technical regulation, while others may be naturally fall under public policy, guided by political forces. Considering the five topics this study has explored and its respective literature review, some policy implications are summarised below.

Policies aimed at revitalizing technological diffusion have become increasingly important, as evidence suggests that the slowdown in aggregate productivity growth is partly due to a widening productivity gap within sectors. This gap reflects the challenges faced by less advanced firms in catching up to industry leaders by adopting the latest technologies and business practices. The literature supports the notion that the adoption of digital technologies in firms is influenced by two main factors that can be shaped by policy interventions.

Firstly, improving the deployment of high-quality broadband infrastructure is crucial for the adoption of more advanced digital applications. Secondly, enhancing digital skills contributes to the human capital formation, being these policies from basic digital literacy promotion to higher education in ICT. These policies relate to the topics outlined in the digital divide section.

Based on the literature review on the digital divide, particularly its second level, it can be concluded that there is indeed a need for improved digital literacy within a human capital development framework. This perspective suggests that merely reducing the first-level digital divide may not be a sufficient solution for development. In Brazil, sectoral regulations in telecommunications have primarily focused on addressing access issues, such as through auction service coverage obligations and investment commitments. While these regulations have a reasonable justification in driving social goals through efficient market forces, broader policy measures could complement these efforts to achieve more from information and communication technologies (ICTs), particularly in enhancing labour productivity through digital technologies.

However, the COVID-19 pandemic has highlighted the persistent challenges of ICT access, emphasizing the need not only to address the access-based digital divide but also, with the ongoing process of Digital Transformation, to further assess the various impacts of new digital technologies on social and economic life.

Therefore, a synergy between regulatory and public policy measures is crucial for tackling both the first and second levels of the digital divide. Additionally, it is imperative for regulatory and political entities to focus on the third-level aspect to inform future actions and shape evidence-based policies.

Regarding competition tackling in the digital sphere, before structuring the institutional framework, several aspects need to be considered. Firstly, it's important to internalize the discussions and experiences from other countries, understanding how their approaches to defining and solving problems could provide valuable insights for analysing the Brazilian digital economy.

Secondly, future regulatory landmarks should aim for long-term benefits while carefully considering economic trade-offs. This should ideally be supported by theoretical and empirical research to assess the impacts of regulation on market imperfections, ensuring it does not hinder the innovation process or economic growth.

Lastly, regulatory activities should be informed by established practices in regulating network industries but should also be flexible enough to accommodate alternative solutions. This flexibility is crucial given the dynamic nature of the digital ecosystem.

As discussed in the fourth subsection, the Digital Transformation is altering possible development paths, considering the rapid growth of value creation within the digital sphere. In this regard, rigid labour market regulations may prevent developing countries to enter the Globotics Transformation age as participants of an increasing international trade on services. Moreover, commercial and industrial policies that exclusively target physical products may overlook the potential of international trade.

Finally, considering possible impacts of digital technologies on labour makets, there are arguments advocating for institutional responses to address the challenges posed by AI. While these arguments generally support AI regulation, they often lack specific details on how such regulation should be structured. To effectively shape these new forms of public responses, several key steps must be taken.

Firstly, it is important to clearly define the social problem at hand, convince society of its existence, and determine whether existing institutions lack the capacity to address it. Additionally, understanding the mechanisms through which policy tools may have an impact is crucial, as it involves navigating various trade-offs. For example, regulating technology to mitigate its impact on the labour market may stifle innovation and economic growth. The potential gains from minimizing the displacement effect in the short-term might not compensate for the opportunity cost of the reinstatement effect in the medium or long term.

Therefore, fostering an international debate within a multi-stakeholder environment is essential. This debate serves to test divergent theories and allows for the discussion of global concerns that require international coordination. It also enables the formulation of guidelines that can be adapted by countries at different stages of development.

This conclusion reiterates the initial statements regarding institutional foundations. The optimal public responses to address regulatory and policy issues arising from digitalization may find their place within the frameworks of international and national institutions that have played pivotal roles in paving the way for balanced development, particularly with the advent of the ICT revolution.

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