ESTIMATING RAIN ATTENUATION AT THZ FREQUENCIES FROM HISTORICAL DATA COLLECTED IN BRASILIA, DF

LUCAS V. MORAIS

DISSERTAÇÃO DE MESTRADO EM ENGENHARIA ELÉTRICA

DEPARTAMENTO DE ENGENHARIA ELÉTRICA

FACULDADE DE TECNOLOGIA

UNIVERSIDADE DE BRASÍLIA

UNIVERSIDADE DE BRASÍLIA
FACULDADE DE TECNOLOGIA
DEPARTAMENTO DE ENGENHARIA ELÉTRICA

ESTIMATING RAIN ATTENUATION AT THZ FREQUENCIES FROM HISTORICAL DATA COLLECTED IN BRASILIA, DF

UMA ESTIMATIVA DA ATENUAÇÃO DE ONDAS ELETROMAGNETICAS PROPAGANDO-SE EM THZ DEVIDO AO REGIME DE CHUVAS CONSIDERANDO DADOS HISTÓRICOS COLETADOS EM BRASILIA,DF

LUCAS V. MORAIS

ORIENTADOR: LEONARDO MENEZES, DR.

DISSERTAÇÃO DE MESTRADO EM
ENGENHARIA ELÉTRICA

PUBLICAÇÃO: PPGENE.DM-780/22

BRASÍLIA/DF: JUNHO - 2022

UNIVERSIDADE DE BRASÍLIA
FACULDADE DE TECNOLOGIA
DEPARTAMENTO DE ENGENHARIA ELÉTRICA

ESTIMATING RAIN ATTENUATION AT THZ FREQUENCIES FROM HISTORICAL DATA COLLECTED IN BRASILIA, DF

LUCAS V. MORAIS

DISSERTAÇÃO DE MESTRADO SUBMETIDA AO DEPARTAMENTO DE ENGENHARIA ELÉTRICA DA FACULDADE DE TECNOLOGIA DA UNIVERSIDADE DE BRASÍLIA COMO PARTE DOS REQUISITOS NECESSÁRIOS PARA A OBTENÇÃO DO GRAU DE MESTRE.

APROVADA POR:

Prof. Dr. Leonardo Menezes — ENE/Universidade de Brasília
Orientador

Prof. Dr. Ugo Dias — Dep.Engenharia Elétrica/Universidade de Brasília
Membro Interno

Prof. Dr.Leonardo Brito – Dep.Engenharia Elétrica/UFG
Membro Externo

BRASÍLIA, 09 DE JUNHO DE 2022.

FICHA CATALOGRÁFICA

LUCAS V. MORAIS	
Estimating Rain Attenuation at THz Frequencies from Historical Data Collected in Brasilia, DF [Distrito Federal] 2022.	
xi, 53p., 210 x 297 mm (ENE/FT/UnB, Mestre, Engenharia Elétrica, 2022).	
Dissertação de mestrado – Universidade de Brasília, Faculdade de Tecnologia.	
Departamento de Engenharia Elétrica	
1. Distribuição das Gotas de Chuva	2. Terahertz
3. Teoria de Mie	4. Atenuação por Chuvas
I. ENE/FT/UnB	II. Título (série)

REFERÊNCIA BIBLIOGRÁFICA

LUCAS V. MORAIS. (2022). Estimating Rain Attenuation at THz Frequencies from Historical Data Collected in Brasilia, DF . Dissertação de mestrado em Engenharia Elétrica, Publicação PPGENE.DM-780/22, Departamento de Engenharia Elétrica, Universidade de Brasília, Brasília, DF, 53p.

CESSÃO DE DIREITOS

AUTOR: Lucas V. Morais
TÍTULO: Estimating Rain Attenuation at THz Frequencies from Historical Data Collected in Brasilia, DF .
GRAU: Mestre ANO: 2022

____

Lucas V. Morais

Departamento de Engenharia Elétrica (ENE) - FT
Universidade de Brasília (UnB)
Campus Darcy Ribeiro
CEP 70919-970 - Brasília - DF - Brasil

Dedicated to my beloved parents and wife who have provided me encouragement and support throughout my studies.

ACKNOWLEDGMENTS

I would like to thank my advisor Dr. Leonardo Menezes for giving me all the support and guidance during this project.

RESUMO

Título: Uma estimativa da Atenuação de Ondas Eletromagneticas propagando-se em THz Devido ao Regime de Chuvas Considerando Dados Históricos Coletados em Brasilia,DF
Autor: Lucas V. Morais
Orientador: Leonardo Menezes, Dr.
Programa de Pós-Graduação em Engenharia Elétrica
Brasília, 09 de junho de 2022

A seguinte dissertação de mestrado propõe um caso de estudo que visa estimar a atenuação causada pela chuva no espectro em Terahertz. Usando dados da chuva coletados durante 20 anos em Brasília, DF, Brasil pelo Instituto Nacional de Meteorologia (INMET) foi possível realizar uma estimativa da atenuação média causada pela chuva. Norteado por um processo estatístico que usa como base a teoria de Monte Carlo e também a Transformada da Incerteza foi possível obter uma razoável estimativa teórica do impacto das chuvas em Brasília no contexto da comunicação em THz. Para calcular valores médios de atenuação usamos a Teoria de Mie em conjunto com a distribuição de Weibull e em seguida comparamos com resultados clássicos obtidos pela norma da ITU-R. A estimativa proposta por esse trabalho mostra que a atenuação média varia entre 1,91 dB/km e 2,66 dB/km. Para eventos de chuvas com taxas acima de 10 mm/hr, nossos resultados demonstraram que a atenuação média varia entre 8,56 e 15,96 dB/km.

Palavras-chave: Distribuição das Gotas de Chuva, Terahertz, Teoria de Mie, Atenuação por Chuvas.

ABSTRACT

Title: Estimating Rain Attenuation at THz Frequencies from Historical Data Collected in Brasilia, DF
Author: Lucas V. Morais
Supervisor: Leonardo Menezes, Dr.
Graduate Program in Telecommunication and Network Systems
Brasília, DF, June 09th, 2022

This work proposes a case study for estimating rain attenuation with rainfall data collected in Brasilia, Brazil at THz frequencies using Mie Theory and Drop Size Distribution. To address this goal, we used measured rainfall rate data for the past 20 years collected at the National Institute of Meteorology (INMET). A statistical approach that uses Monte Carlo simulation was applied to obtain a reasonable estimation of rainfall attenuation in the Terahertz spectrum. To evaluate the method's accuracy, we performed a comparison between the rain attenuation calculated by Mie Theory and ITU-R model. The estimation proposed in this work showed that the mean attenuation varies between 1.91 to 2.66 dB/km. For rain events greater than 10 mm/h, results showed that the mean attenuation varies between 8.56 to 15.96 dB/km.

Keywords: Drop Size Distribution, Terahertz, Mie Theory, Rain Attenuation.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1 INTRODUCTION

The data traffic through wireless systems has been increasing exponentially in the last decades. The demand for high-speed data systems has been pushing researchers all over the world to find new solutions for wireless systems. The next generation of mobile communication 6G has been promised to be a great step for future applications based on the internet of things (IoT). Among the applications that can emerge through IoT are connected appliances, smart home security systems, autonomous equipment, ultra-high-speed wireless internet, smart factory equipment beyond several's new applications [4]. Efforts to advance physical devices and the adoption of unused spectrum bands are at core of this development. The most recent research has been showing the Terahertz spectrum as one of the main technologies to be used in this path to high-speed wireless communication. Terahertz is based on frequencies from 100 GHz to 3 THz - that's the range currently authorized for use.

Those frequencies are very important when talking about the future of communication systems because it is still an unused spectrum. They have the potential for an enormous range of new applications. The emergence of new technologies in communication systems has demanded studies toward greater data transmission capacity. Shorter wavelengths provide a high ability to improve the rate of transmission and recently have been used in many indoor applications. A major limitation of outdoor propagation at THz is related to the high absorption and scattering from atmospheric gases and particles. Also, the rain might be a large source of scattering and absorption of EM at THz.

Rain has significant impacts on terahertz communications due to attenuation that is caused by absorption and scattering of electromagnetic (EM) radiation. These effects are essentially produced by water droplets. Given the effect of rain on terahertz communication, it is fundamental to estimate the effects of water drops for outside applications on THz. This work seeks answers about the effect of rain on THz propagation in the city of Brasilia, DF, Brazil. Based on rainfall data collected in Brasilia, it is possible to estimate its effects on wave propagation and then project real applications that can be done in the THz spectrum.

1.1 INITIAL CONSIDERATIONS

There are two main approaches to calculating rain attenuation: i) empirically using well-known models such as ITU-R; and ii) theoretically using Mie theory, which considers a drop of water a perfect sphere and a Drop Size Distribution (DSD). The DSD provides the distribution of raindrops according to their diameter [5, 6, 7]. There are many published studies based on rain attenuation that covers a specific frequency [8, 9, 10, 2, 1]. Ishii [9] uses experimental values of 96, 140, 225, 313, and 355 GHz to estimate rain attenuation and concluded that the best DSD fit to the frequency of 313-355 GHz was using Mie theory combined with the Weibull distribution. Norouzian et al. [2] compared the rain attenuation at 77 GHz and 300 GHz by using ITU-R and Mie Theory with DSD distributions. They concluded that the model that had the best fit to the data collected in Birmingham, UK was with Weibull distribution. Therefore, the approach using Mie Theory and Weibull distribution for THz attenuation shows the best fit for DSD theoretical model.

This study provides an estimation for the effects of rainfall attenuation in THz frequency that uses historical rainfall data collected over 20 years in Brasilia, DF, Brazil. The theoretical model is based on Mie Theory and Weibull distribution for droplets of water. A comparison between the theoretical and empirical ITU-R was also done to predict the divergence present in these models.

We used the rainfall fall rate measure per hour collected in Brazil between 2001 and 2020 at the weather station localized in Brasilia, Brazil (latitude: -15:789343, Longitude: -47:925756, Height: 1160.9 m) to estimate how the regime of rain would impact the transmission in different seasons of the year. When estimating rainfall mean rate based on collected data, it is possible to generalize attenuation throughout the whole terahertz spectrum, which can be especially useful for a large range of future applications [11, 12, 13, 14]. Due to the lack of studies related to the theoretical calculation of rain attenuation in Brazil, the main goal of this work is to clarify and study the methods already described in the literature to find the influence of rainfall using different approaches and real data collected over 20 years in Brasilia, DF.

1.2 OBJECTIVES

Given the difficulties of propagating THz waves through space, the following work has the objective of answering: **"How is the quantitative theoretical estimation of attenuation due rain in THz given a specific regime of rain in Brasilia, Brazil"**. To achieve this main goal, secondary objectives have been defined:

- Find the regime of rain in Brasilia.
- Describe the distribution of rainfall rate in Brasilia throughout the year.
- Describe and calculated attenuation using empirical model based in ITU-R.
- Describe and calculated attenuation using Mie scattering and absorption theory.
- Compare the theoretical and empirical model of calculating attenuation due rain.

1.3 ORGANIZATION

This paper claims to give a perspective on how the rain in the Midwest of Brazil, more specifically in Brasilia, can affect the propagation in the THz spectrum. This work was divided into four main parts. The first part gives an introduction with motivations to study this topic and also gives all the information about how the work was organized. The second part is the theoretical background that it is necessary to understand all the techniques that were used to reach the results. The third part is the analysis of all data related to rainfall in the city of Brasilia.

In the third part, we use quantitative data from the institute of meteorology to have a statistical analysis of the behavior of rain in Brasilia. All the data that was used in this work are available in INMET website. By using the data, it was possible to have a an idea of how is the behavior of the rainfall regime in the city of Brasilia and also an idea in the whole Midwest of Brazil. We used a statistical method based on finding the probability distribution function to describe how is the occurrence of a specific rainfall rate. Using the PDF of rainfall rates, it is possible to calculate the mean attenuation caused by rain in THz propagation.

The fourth part was dedicated to calculating the mean attenuation for a wave propagating in THz through space in a presence of rain. To calculate this attenuation it was used two approaches:

i) Using the Mie theory of scattering together with a drop size distribution of rainfall droplets.

ii) Using the empirical model from ITU-R [15].

The first model is very useful because can give an approximation for rainfall attenuation based on the theory that can be used for a very large range of frequencies. To make this approximation, we have to consider the scattering and absorption caused by rain considering the rain a perfect sphere. The second model is an empirical method based in observations by ITU-R. This model has a limit in the range of frequencies that were tested. To obtain an estimation for attenuation, it is necessary to statistically treat the data collected at INMET. We used two statistical methods to simulate and estimate rainfall attenuation:

i) Monte Carlo [16].

ii) Unscented Transform [17].

Monte Carlo simulation is a technique that performs an estimation based on a quantitative analysis by using a probability distribution. On other hand, the UT is a mathematical procedure used to estimate the results based on a probability distribution that is characterized by a finite set of statistics values.

Finally, the fifth part is the conclusion and some insights for future works that can be done to validate empirically the estimations that we have done here.

1.4 PUBLICATIONS

The following list contains the publication we made from this research:

Morais, L., Menezes, L., and Moraes, P. (2021). Rain Attenuation at THz Frequencies from Historical Data Collected in Brasilia, Brazil. In 2021 USNC-URSI Radio Science Meeting (USCN-URSI RSM). 2021 USNC-URSI Radio Science Meeting (USNC-URSI RSM). IEEE. https://doi.org/10.23919/usnc-ursirsm52661.2021.9552345

Morais, L. V. de, Menezes, L., and Moraes, P. (2021). Estimating Brasilia Rain Attenuation at THz Frequencies from Historical Data Based in Monte Carlo Simulation and Unscented Transform. In Anais do XXXIX Simpósio Brasileiro de Telecomunicações e Processamento de Sinais. XXXIX Simpósio Brasileiro de Telecomunicações e Processamento de Sinais. Sociedade Brasileira de Telecomunicações. https://doi.org/10.14209/sbrt.2021.1570724130

2 THEORETICAL BACKGROUND

2.1 MIE THEORY

Analyzing attenuation from scattering might be a very challenging task. Mie Theory is based on a solution of Maxwell's Equations assuming that the drop of water is a perfect sphere and solving the equations using spherical Bessel functions.As a matter of clarification, it will be disposed a brief resume of Mie theory that was described in a elegant form by Craig F.Bohren and Donald R. Huffman [18]. For a time-harmonic, linear, isotropic, and a homogeneous medium the electric and magnetic field must satisfy the wave equation given by:

$$\nabla^2\mathbf{E} + k^2\mathbf{E} = 0 \tag{2.1}$$

$$\nabla^2\mathbf{H} + k^2\mathbf{H} = 0 \tag{2.2}$$

Where $k^2 = \omega^2\epsilon\mu$ and be divergence-free

$$\nabla \cdot \mathbf{E} = 0$$
$$\nabla \cdot \mathbf{H} = 0$$

Also, **E** and **H** are not independent:

$$\nabla \times \mathbf{E} = j\omega\mu\mathbf{H}$$

$$\nabla \times \mathbf{H} = -j\omega\epsilon\mathbf{E}$$

For a given scalar function ψ and a constant vector **c** it is possible to suppose a function M:

$$\mathbf{M} = \nabla \times (\mathbf{c}\psi) \tag{2.3}$$

$$\nabla \cdot \mathbf{M} = 0$$

Applying Equation 2.3 into wave equation 2.1:

$$\nabla^2 \left[\nabla \times (\mathbf{c}\psi)\right] + k^2 \left[\nabla \times (\mathbf{c}\psi)\right] = 0$$

Using some vectors identities,

$$\nabla \times \left[\mathbf{c}(\nabla^2\psi + k^2\psi)\right] = 0$$

Which gives a straightforward consequence,

$$\nabla^2\psi + k^2\psi = 0 \tag{2.4}$$

Constructing from M another vector function that also satisfy the wave equation might be done by using N as:

$$\mathbf{N} = \frac{\nabla \times \mathbf{M}}{k} \tag{2.5}$$

$$\nabla^2\mathbf{N} + k^2\mathbf{N} = 0 \tag{2.6}$$

$$\nabla \times \mathbf{N} = \nabla \times \left(\frac{\nabla \times \mathbf{M}}{k}\right)$$

$$\nabla \times \mathbf{N} = \nabla \left(\nabla \cdot \frac{\mathbf{M}}{\mathbf{k}}\right) - \nabla^2 \left(\frac{\mathbf{M}}{k}\right)$$

$$\nabla \times \mathbf{N} = -\frac{1}{\mathbf{k}}\nabla^2\mathbf{M} = -\frac{1}{\mathbf{k}}\left(-k^2\mathbf{M}\right)$$

$$\nabla \times \mathbf{N} = k\mathbf{N} \tag{2.7}$$

The problem of finding solutions in a vector domain can now be treated as a scalar problem. Equation 2.4 can be written in spherical polar coordinates as it follows:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2 sin\theta}\frac{\partial^2\psi}{\partial\phi^2} + k^2\psi = 0 \tag{2.8}$$

$$\psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi) \tag{2.9}$$

Seeking for particular solutions to Equation 2.8 in the form of Equation 2.9 by replacing equation 2.9 into Equation 2.8 yield the three separated equations:

$$\frac{d^2\Phi}{d\phi} + m^2\Phi = 0 \tag{2.10}$$

$$\frac{1}{sin\theta}\frac{d}{d\theta}\left(sin\theta\frac{d\Theta}{d\theta}\right) + \left[n(n+1) - \frac{m^2}{sin^2\theta}\right]\Theta = 0 \tag{2.11}$$

$$\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \left[k^2r^2 - n(n+1)\right]R = 0 \tag{2.12}$$

Now, let is find the solution for Equations 2.10, 2.11, and 2.12. The linearly independent solution for Equation 2.10 are

$$\Phi_e = cos(m\phi)$$

$$\Phi_o = sin(m\phi)$$

Being the subscripts *e* and *o* denoting even and odd. Also, m is an integer or zero. The solution of Equation 2.11 is associated with Legendre functions of the first kind $P_n^m(cos\theta)$ of degree n and order m, theses functions are orthogonal:

$$\int_{-1}^{1} P_n^m cos(\theta) P_{n'}^m cos(\theta) d(cos\theta) = \delta_{n'n}\frac{2}{2n+1}\frac{(n+m)!}{(n-m)!}$$

Where the $\delta_{n'n}$ is the Kronecker delta, if $n = n'$ is 1 and zero otherwise.

By introducing the dimensionless variable $\rho = kr$ and $Z = R\sqrt{\rho}$, Equation 2.12 becomes,

$$\rho\frac{d}{d\rho}\left(\rho\frac{dZ}{d\rho}\right) + \left[\rho^2 - (n+\frac{1}{2})^2\right]Z = 0 \tag{2.13}$$

Therefor, the linearly independent solutions to 2.12 and 2.13 are the spherical Bessel functions:

$$j_n(\rho) = \sqrt{\frac{\pi}{2\rho}}J_{n+1/2}(\rho) \tag{2.14}$$

$$y_n(\rho) = \sqrt{\frac{\pi}{2\rho}}Y_{n+1/2}(\rho) \tag{2.15}$$

Any linear combination of j_n and y_n is also a solution to 2.12. Therefor, the Hankel functions given by Equations 2.16 and 2.17 are also possible solutions.

$$h_n^{(1)}(\rho) = j_n(\rho) + jy_n(\rho) \tag{2.16}$$

$$h_n^{(2)}(\rho) = j_n(\rho) - jy_n(\rho) \tag{2.17}$$

Finally, we can rewrite equation 2.9 as combination of previous results:

$$\psi_{emn} = cos(m\phi)P_n^m(cos\theta)z_n(kr) \tag{2.18}$$

$$\psi_{omn} = sin(m\phi)P_n^m(cos\theta)z_n(kr) \tag{2.19}$$

Where z_n is any of the four spherical Bessel functions j_n, y_n, $h_n^{(1)}$, or $h_n^{(2)}$. By choosing for convenience the constant **c** in equation 2.3 as **r**, we can rewrite equation 2.3 and 2.5 as it follows:

$$\boxed{\mathbf{M}_{emn} = \nabla \times (\mathbf{r}\psi_{emn})} \tag{2.20}$$

$$\boxed{\mathbf{N}_{emn} = \frac{\nabla \times \mathbf{M}_{emn}}{\mathbf{k}}} \tag{2.21}$$

$$\boxed{\mathbf{M}_{omn} = \nabla \times (\mathbf{r}\psi_{omn})} \tag{2.22}$$

$$\boxed{\mathbf{N}_{omn} = \frac{\nabla \times \mathbf{M}_{omn}}{\mathbf{k}}} \tag{2.23}$$

By solving Equations 2.20 and 2.22 in terms of spherical components:

$$\mathbf{M}_{emn} = \frac{-m}{sin\theta} sin(m\phi)P_n^m(cos\theta)z_n(\rho)\hat{\mathbf{e}}_\theta - cos(m\phi)\frac{dP_n^m(cos\theta)}{d\theta}z_n(\rho)\hat{\mathbf{e}}_\phi \tag{2.24}$$

$$\mathbf{M}_{omn} = \frac{m}{sin\theta} cos(m\phi)P_n^m(cos\theta)z_n(\rho)\hat{\mathbf{e}}_\theta - sin(m\phi)\frac{dP_n^m(cos\theta)}{d\theta}z_n(\rho)\hat{\mathbf{e}}_\phi \tag{2.25}$$

$$\begin{aligned}\mathbf{N}_{emn} = {} & \frac{z_n(\rho)}{\rho}cos(m\phi)n(n+1)P_n^m(cos\theta)\hat{\mathbf{e}}_r + cos(m\phi)\frac{dP_n^m(cos\theta)}{d\theta}\frac{1}{\rho}\frac{d}{d\rho}\left[\rho z_n(\rho)\right]\hat{\mathbf{e}}_\theta \\ & - msin(m\phi)\frac{P_n^m(cos\theta)}{sin\theta}\frac{1}{\rho}\frac{d}{d\rho}\left[\rho z_n(\rho)\right]\hat{\mathbf{e}}_\phi\end{aligned} \tag{2.26}$$

$$\mathbf{N}_{omn} = \frac{z_n(\rho)}{\rho} sin(m\phi) n(n+1) P_n^m(cos\theta)\hat{\mathbf{e}}_r + sin(m\phi)\frac{dP_n^m(cos\theta)}{d\theta}\frac{1}{\rho}\frac{d}{d\rho}\left[\rho z_n(\rho)\right]\hat{\mathbf{e}}_\theta \\ + m\ cos(m\phi)\frac{P_n^m(cos\theta)}{sin\theta}\frac{1}{\rho}\frac{d}{d\rho}\left[\rho z_n(\rho)\right]\hat{\mathbf{e}}_\phi \tag{2.27}$$

The next step is working with the expansion of a plane wave x-polarized in spherical coordinates.

$$\mathbf{E}_i = E_0 e^{j\mathbf{k}\cdot\mathbf{r}cos\theta}\hat{\mathbf{e}}_x \tag{2.28}$$

$$\hat{\mathbf{e}}_x = sin\theta\ cos\phi\ \hat{\mathbf{e}}_r + cos\theta \cos\phi \hat{\mathbf{e}}_\theta - sin\phi\hat{\mathbf{e}}_\phi \tag{2.29}$$

Expanding the equation 2.28 in vector spherical harmonics:

$$E_i = \sum_{m=0}^{\infty}\sum_{n=m}^{\infty}\left(\mathbf{B}_{emn}\mathbf{M}_{emn} + \mathbf{B}_{omn}\mathbf{M}_{omn} + \mathbf{A}_{emn}\mathbf{N}_{emn} + \mathbf{A}_{omn}\mathbf{N}_{omn}\right) \tag{2.30}$$

In which, the **A** and **B** are the coefficients of spherical expansion. Because $sin(m\phi)$ and $cos(m'\phi)$ are mutual orthogonal, the orthogonality imply that all vector harmonics of different order m are mutually orthogonal. The associated legendre function P_n^m is related to the *m*th of Legendre polynomial P_n,

$$P_n^m(\mu) = (1-\mu^2)^{m/2}\frac{d^m P_n(\mu)}{d\mu^m} \tag{2.31}$$

where,

$$\mu = cos\theta$$

The orthogonality implies that the coefficients are in the form of,

$$B_{emn} = \frac{\int_0^{2\pi}\int_0^{\pi}\mathbf{E}_i\cdot\mathbf{M}_{emn} sin\theta\ d\theta\ d\phi}{\int_0^{2\pi}\int_0^{\pi}|\mathbf{M}_{emn}|^2 sin\theta\ d\theta\ d\phi} \tag{2.32}$$

Using orthogonality and the Equations 2.22-2.27,

$$B_{emn} = A_{omn} = 0$$

Moreove, the others coefficients vanish unless m=1. The incident field is finite at the origin, so $j_n(kr)$ is the spherical Bessel function in ψ_{oln} and ψ_{eln}. The expansion for $\mathbf{E}_i$ has the form of,

$$\mathbf{E}_i = \sum_{n=1}^{\infty}(B_{oln}\mathbf{M}^{(1)}_{oln} + A_{eln}\mathbf{N}^{(1)}_{eln}) \tag{2.33}$$

Where the subscript (1) is indicating that $z_n(\rho)$ used in ψ is the $j_n(k_i r)$ and the coefficients are given by

$$B_{oln} = j^n E_0 \frac{2n+1}{n(n+1)} \tag{2.34}$$

$$A_{eln} = -jE_0 j^n \frac{2n+1}{n(n+1)} \tag{2.35}$$

The expansion of a plane wave in spherical harmonics,

$$\boxed{\mathbf{E}_i = E_0 \sum_{n=1}^{\infty} j^n \frac{2n+1}{n(n+1)}(\mathbf{M}^{(1)}_{oln} - j\mathbf{N}^{(1)}_{eln})} \tag{2.36}$$

The curl of 2.36 gives the incident magnetic field,

$$\boxed{\mathbf{H}_i = \frac{-k}{\omega\mu} E_0 \sum_{n=1}^{\infty} j^n \frac{2n+1}{n(n+1)}(\mathbf{M}^{(1)}_{eln} + j\mathbf{N}^{(1)}_{oln})} \tag{2.37}$$

The boundary conditions between the sphere and surrounding medium,

$$\begin{aligned}(\mathbf{E}_i + \mathbf{E}_s - \mathbf{E}_l) \times \hat{\mathbf{e}}_r &= (\mathbf{H}_i + \mathbf{H}_s - \mathbf{H}_l) \times \hat{\mathbf{e}}_r \\ &= 0\end{aligned} \tag{2.38}$$

Where the indices i, s, and l indicate incident, scattered, and inside fields. Using the boundary conditions and the orthogonality of vector harmonics it is possible to find the expressions for inside and scattered fields.

$$\boxed{\mathbf{E}_l = E_0 \sum_{n=1}^{\infty} j^n \frac{2n+1}{n(n+1)}(c_n\mathbf{M}^{(1)}_{oln} - jd_n\mathbf{N}^{(1)}_{eln})} \tag{2.39}$$

$$\boxed{\mathbf{H}_l = \frac{-k}{\omega\mu} E_0 \sum_{n=1}^{\infty} j^n \frac{2n+1}{n(n+1)}(d_n\mathbf{M}^{(1)}_{eln} + jc_n\mathbf{N}^{(1)}_{pln})} \tag{2.40}$$

For the scattered field is convenient switch our $z_n(\rho)$ to the Spherical Hankel[1] as it follows:

[1] see page 93 in reference [18] for more information

$$\boxed{\mathbf{E}_s = E_0 \sum_{n=1}^{\infty} j^n \frac{2n+1}{n(n+1)} (j a_n \mathbf{N}_{eln}^{(3)} - b_n \mathbf{M}_{eln}^{(3)})} \tag{2.41}$$

$$\boxed{\mathbf{H}_s = \frac{k}{\omega\mu} E_0 \sum_{n=1}^{\infty} j^n \frac{2n+1}{n(n+1)} (j b_n \mathbf{N}_{oln}^{(3)} + a_n \mathbf{M}_{eln}^{(3)})} \tag{2.42}$$

The superscript (3) indicate the spherical harmonics by Henkel functions $h_n^{(1)}$. The unknown coefficients a_n,b_n,c_n and d_n work as a weight for the equations. The four coefficients are given by[2]:

$$a_n = \frac{\mu m^2 j_n(mx)[x j_n(x)]' - \mu_1 j_n(x)[mx j_n(mx)]'}{\mu m^2 j_n(mx)[x h_n^{(1)}(x)]' - \mu_1 h_n^{(1)}(x)[mx j_n(mx)]'}$$

$$b_n = \frac{\mu_1 j_n(mx)[x j_n(x)]' - \mu j_n(x)[mx j_n(mx)]'}{\mu_1 j_n(mx)[x h_n^{(1)}(x)]' - \mu h_n^{(1)}(x)[mx j_n(mx)]'}$$

$$c_n = \frac{\mu_1 j_n(x)[x h_n^{(1)}(x)]' - \mu_1 h_n^{(1)}(x)[x j_n(x)]'}{\mu_1 j_n(mx)[x h_n^{(1)}(x)]' - \mu h_n^{(1)}(x)[mx j_n(mx)]'}$$

$$d_n = \frac{\mu_1 m j_n(x)[x h_n^{(1)}(x)]' - \mu_1 m h_n^{(1)}(x)[x j_n(x)]'}{\mu m^2 j_n(mx)[x h_n^{(1)}(x)]' - \mu_1 h_n^{(1)}(x)[mx j_n(mx)]'}$$

$$x = ka = \frac{2\pi N \frac{a}{2}}{\lambda}$$

$$m = \frac{k_1}{k} = \frac{N_1}{N}$$

k- Wave number medium.

k_1 - Wave number of particle

N - Refractive indices medium

N_1 - Refractive indices particle

a - Diameter of a Sphere [m]

By using the relation for rate of scattering energy as it follows and defining the extinction energy as the sum of internal and scattering energy we obtain,

[2]see pag.100 in reference [18] for more information

$$W_s = \frac{1}{2} Re \left\{ \int_0^{2\pi} \int_0^{\pi} (E_{s\theta} H_{s\phi}^* - E_{s\phi} H_{s\theta}^*) r^2 sin\theta d\theta d\phi \right\} \tag{2.43}$$

$$W_{ext} = \frac{1}{2} Re \left\{ \int_0^{2\pi} \int_0^{\pi} (E_{i\phi} H_{s\theta}^* - E_{i\theta} H_{s\phi}^* - E_{s\theta} H_{i\phi}^* + E_{s\phi} H_{i\theta}^*) r^2 sin\theta d\theta d\phi \right\} \tag{2.44}$$

The cross sections coefficients can now be defined as being a rate between power $[W]$ and intensity of irradiance$[W/m^2]$:

$$\boxed{C_{sca} = \frac{W_s}{I} = \frac{2\pi}{k^2} \sum_{n=1}^{\infty} (2n+1)(|a_n|^2 + |b_n|^2)} \tag{2.45}$$

$$\boxed{C_{ext} = \frac{W_{ext}}{I} = \frac{2\pi}{k^2} \sum_{n=1}^{\infty} (2n+1) Re(a_n + b_n)} \tag{2.46}$$

Where,

$$a_n = \frac{\mu m^2 J_n(mx)[xJ_n(x)]' - \mu_1 J_n(x)[mxJ_n(mx)]'}{\mu m^2 J_n(mx)[xh_n^{(1)}(x)]' - \mu_1 h_n^{(1)}(x)[mxJ_n(mx)]'}$$

$$b_n = \frac{\mu_1 J_n(mx)[xJ_n(x)]' - \mu J_n(x)[mxJ_n(mx)]'}{\mu_1 J_n(mx)[xh_n^{(1)}(x)]' - \mu h_n^{(1)}(x)[mxJ_n(mx)]'}$$

$$x = ka = \frac{2\pi N a}{\lambda}$$

$$m = \frac{k_1}{k} = \frac{N_1}{N}$$

k- Wave number.

I - Irradiance $[W/m^2]$

N_1 - Refractive indices particle

N - Refractive indices medium

R - Rain rate intensity [mm/h]

a - Diameter Sphere [m]

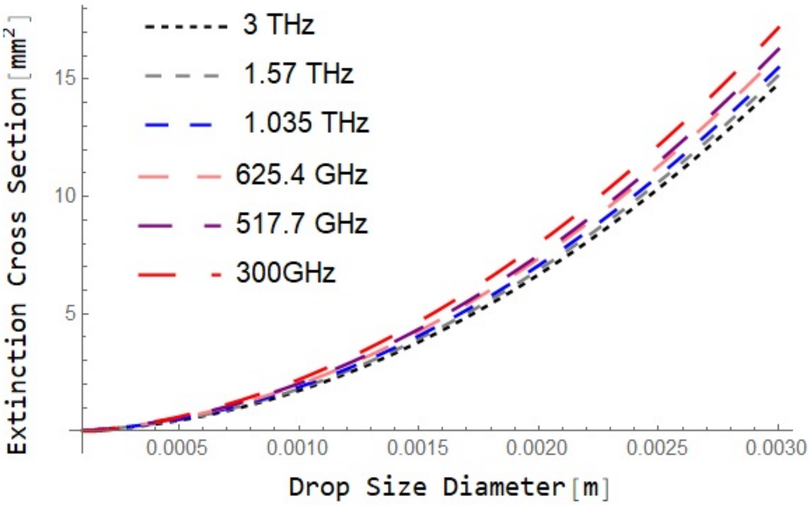


Figure 2.1 – Extinction Cross Section Vs Diameter of Rain Drop for 300GHz to 3 THz

Figure 2.1 shows the Extinction Cross Section in mm^2 versus the drop size distribution in m for a selected range of frequencies in the THz spectrum. It is easy to note that as the frequency increases, the extinction cross section decreases for a specific value of rain drop diameter. The Mie theory is the key to have a theoretical model for estimate the scattering and the absorption of THz waves by water drops. Therefor, we have to consider the water drops perfect spheres. In this work, we have the assumption that the wave is a perfect sphere for all the calculations that uses Mie Theory. Therefor, equation 2.46 will be used in the theoretical calculation for attenuation using Mie Theory and DSD.

2.2 DROP SIZE DISTRIBUTION

The Drop Size Distribution (DSD) is a basic property of rainfall precipitation that has been investigated through decades for a large number of researchers all over the world. The DSD is the frequency distribution of a drop sizes that is characteristic of a given fall of rain. The DSD is the distribution of rain's drops as a function of its diameter. Basically, there are three process responsible for the formation of rainfall drops: condensation, acummulation between water drops, and the collision of drops. The DSD has been an important topic of research because can give a good estimation about how it is the physical behavior of rain in a given region [21, 25, 24, 23]. The main ways to calculated the DSD in the last few years was by using Marshall/Palmer, Weibull and Gamma distributions. This chapter claims to give a better understanding of this tools to estimate DSD.

2.2.1 Marshall and Palmer

Marshall and Palmer [26] were responsible for finding an empirical model based on an exponential function. They were mainly inspired to research this subjective based in the sensitive of operational radar (predominantly C- or S- band) to rain drops. Also the topic is required in ice accretion on aircraft, which occurs when supercooled water droplets freeze instantaneously on impact and then form rime ice or return to the previous state of water along the wing, soil erosion, and purely meteorological matters. They collected empirical data in Ottawa, Canada in 1948 and by using the filter-paper method, which is based on the treatment of a paper with a suitable power dye and then pass the water drops through it. The stains made by the drops get permanent because of the previous treatment of paper. Using the data collected by the filter-paper method and using some mathematical fitting on the data, it was possible to find the following model:

$$\begin{aligned} N(D) &= N_0 \, e^{-\Lambda D} \\ N_0 &= 8000 \, m^{-3} mm^{-1} \\ \Lambda &= 4.1 R^{-0.21} mm^{-1} \end{aligned} \tag{2.47}$$

Where N_0 is a constant parameter for all rainfall rate (R) in mm/hr and Λ is the distribution slope parameter given in mm^{-1} and D is the diameter of rainfall drops. This model was for a long time the main way to calculate the drop size distribution. Figure 2.2 shows the drop size distribution based on MP for some values of rainfall rate. It is possible to see in fig. 2.2 that as the drop diameter size increases, the DSD given by $N(D)$ decreases. Drop size distribution is also a function of rainfall rate (R), which is a parameter of Λ, and we can see that as the rainfall rate increases, the drop rate decreases.

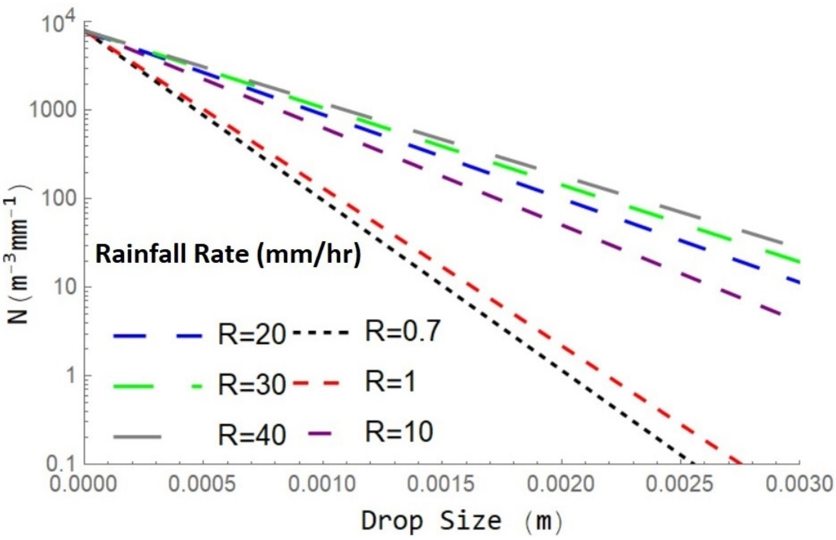


Figure 2.2 – Drop Size Distribution based on Marshall and Palmer

Historically, Marshall and Palmer (MP) distribution was the first tool used in the estimation of rainfall attenuation. There are a plenty of studies in this field that uses the MP as a way to calculate attenuation [8, 9, 10, 2, 1]. In this work, we did not use the MP as a way to calculate the drop size distribution and consequently attenuation because there are recent works in literature that shows empirically that the Weibull Distribuion as the most accurate DSD to theoretically calculate rainfall attenuation based in Mie Theory [2, 9].

2.2.2 Weibull Distribution

The classical Weibull probability density function is given by:

$$f(x_i, \eta, \sigma) = \frac{\eta}{\sigma}\left(\frac{x_i}{\sigma}\right)^{\eta-1} Exp\left[-\left(\frac{x_i}{\sigma}\right)^{\eta}\right] \tag{2.48}$$

for $\eta > 0$, $\sigma > 0$ and $x_i \geq 0$

The value of $\eta < 1$ indicates that the failure rate decreases over "time". This happens if there is significant "infant mortality". For $\eta = 1$ indicates that the failure rate is constant over time. The value of $\eta > 1$ indicates that the failure rate increases with time. In eq. 2.48 has x_i as input the data series variable of a known process either random or deterministic and σ is a type of scale parameter. The Weibull distribution was first used to explain rainfall attenuation in 1982 by Sekine and Lind [1]. Where the parameters N_0, η, and ρ were fitting by real data. When used to explain DSD, Sekine and Lind [1] claims that there are a minimum of two input parameters to correctly explain the phenomena of rainfall distribution - the diameter of drop size and the number of drops per unit of volume. Therefor, it follows that the DSD/Weibull should be represented as it follows:

$$N(D) = N_0 \times f(x_i = D_i, \sigma, \eta) \tag{2.49}$$

and thus,

$$N(D) = N_0 \frac{\eta}{\sigma}\left(\frac{D}{\sigma}\right)^{\eta-1} Exp\left[-\left(\frac{D}{\sigma}\right)^{\eta}\right] \tag{2.50}$$

Sekine and Lind [1] used the weibull distribution to calculate rain fall attenuation in 1982. The authors used Weibull DSD model together with the data collected by Babkin et al. [33] to fit the parameters (N_0, η, σ). Equations 2.51 shows the calculated parameters.

$$\begin{aligned} N_0 &= 1000 \;\; m^{-3} \\ \eta &= 0.95 R^{0.14} \\ \sigma &= 0.26 R^{0.44} \;\; mm \end{aligned} \tag{2.51}$$

In 2019 Norouzian et al. [2] used some DSD models together with Mie theory to calculated theoretical values for rain attenuation at the frequencies of 77GHz and 300GHz in order to compare with the experimental values that were obtained by using a transmission and receptor antenna located in an in-obstructed two-way path with length of 320 m. They concluded that for this range of THz frequencies the Weibull distribution together with Mie theory were the best theoretical model in comparison with the real results coming from the experiment. In [2], the authors also suggests new values for the parameters used in Weibull distribution. Norouzian et al. [2] used the data collected to fit the parameters given in eq. 2.50 and eq. 2.52.

$$\begin{aligned} N_0 &= 1100R^{0.53} \;\; m^{-3} \\ \eta &= 0.9R^{0.05} \\ \sigma &= 0.24R^{0.2} \;\; mm \end{aligned} \tag{2.52}$$

In order to illustrate the behavior of Weibull distribution based on the parameters calculated in eq. 2.51 proposed by Sekine and Lind [1], we plot the Fig. 2.3 that shows the DSD in terms of drop size diameter given in maters. We also plot different curves based in the values of rainfall rate given in mm/hr that goes between 0.7 to 40 mm/h . It is possible to notice that for low values of rainfall rate, the behavior of Weibull Distribution can change considerably when we compare to rates above 10 mm/h.

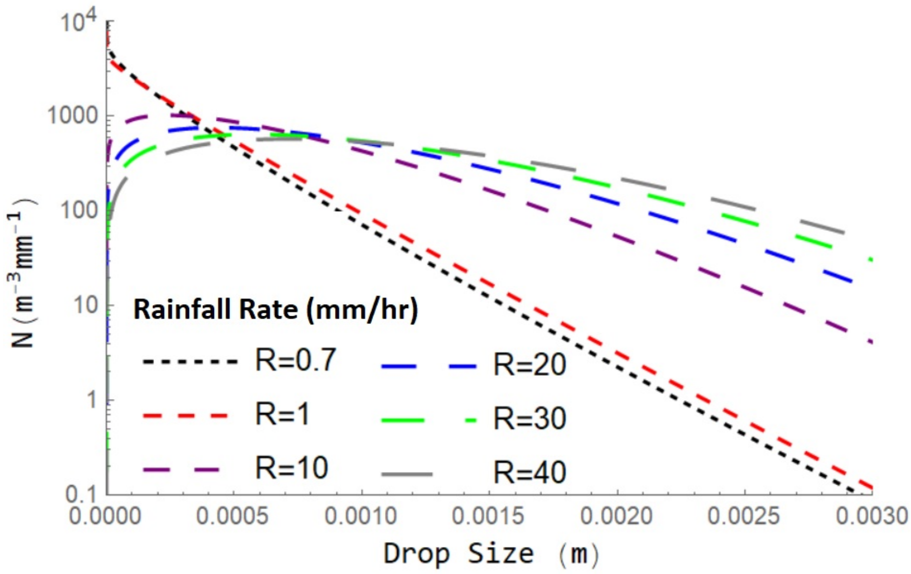


Figure 2.3 – Drop Size Distribution Using the Classical Weibull Distribution Proposed by Seking and Lind [1].

By using the modified parameter given in eq. 2.52 that was proposed by Nourian et al. [2] in 2019, we plot fig. 2.4 for the same rain fall rate as before. It is easy to notice that the new parameters proposed in [2] change the behavior of the curves especially for small drop size diameters. Based in the most recent studies related to this field, we used Weibull distribution with parameters calculated in [2] together with Mie theory to estimate a theoretical rainfall attenuation based in the data collected in Brasilia, Brazil.

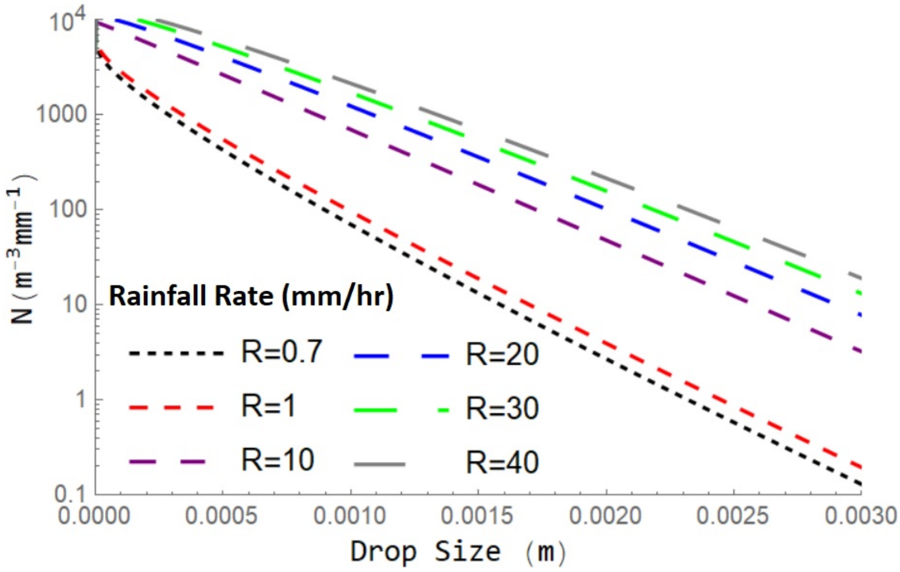


Figure 2.4 – Drop Size Distribution Using the Weibull Distribution proposed by Nourian et al. [2].

2.2.3 Gamma Distribution

Usually, the gamma distribution is used to describing averaged precipitation statistics [27]. The gamma distribution with three parameters (N_0, μ and Λ) where N_0 is a parameter given in ($m^{-3}cm^{-1-\mu}$) being μ a dimensionless and Λ is a slope parameter (mm^{-1}) was first proposed by Ulbrich in 1983 [3]. Figure 2.5 shows the behavior of Gamma DSD for different rainfall rates.

$$N(D) = N_0 D^{\mu} Exp(-\Lambda D) \tag{2.53}$$

$$\begin{aligned} N_0 &= 1.41 \times 10^6 R^{-0.52} \\ \Lambda &= 9.48 R^{-0.2} \\ \mu &= 3.62 \end{aligned} \tag{2.54}$$

The Gamma Distribution is a well known way to calculate rainfall attenuation based in Mie Theory and eq. 2.55.

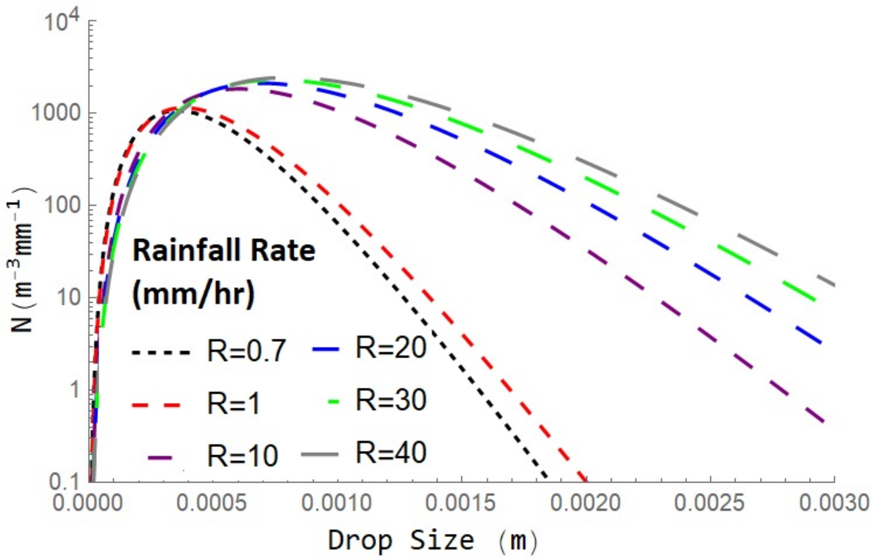


Figure 2.5 – Drop Size Distribution Using the Gamma Distribution proposed by Ulbrich.[3].

2.3 RAINFALL ATTENUATION

Using Mie theory and the concepts presented by Olsen [20], it is well-known that the rain attenuation can be calculated by integrating (2.55) over all raindrop diameters.

$$A\left(\frac{db}{km}\right) = 4.343\int_0^{a_{max}} C_{ext}(a,\lambda,m)N(a,R)da \tag{2.55}$$

Where N(a,R) is the DSD which is the number of drops per unit of volume and diameter of drops.

As we said before, many studies done using THz and Mie Theory have shown that the Weibull distribution combined with Mie theory have the best fit with empirical experiments[8, 9, 10, 2, 1]. Ishii [9] uses some experimental values in the range of THz and concluded that the best DSD fit to the frequency of 313-355 GHz was using Mie with Weibull distribution. Norouzian et al. [2] also compared some DSD distributions with fits collected empirically and for the range of frequencies in THz the Mie Theory combined with Weibull Distribution has shown the best fit to reality. Therefor, It is possible apply equations (2.46), (2.50), (2.52), and (2.55) to calculate a theoretical value for rainfall attenuation in a range of THz frequency.

We can also apply a more classical approach to find attenuation by using the empirical model from ITU-R [15] that describes rainfall attenuation throughout 1 to 1000 GHz. Equation 2.56 shows the ITU-R model to calculate the attenuation due to rain.

$$\boxed{A_{ITU} = kR^{\alpha}} \tag{2.56}$$

The parameters k and α are determined as functions of frequency in the range from 1 to 1000 GHz [15].

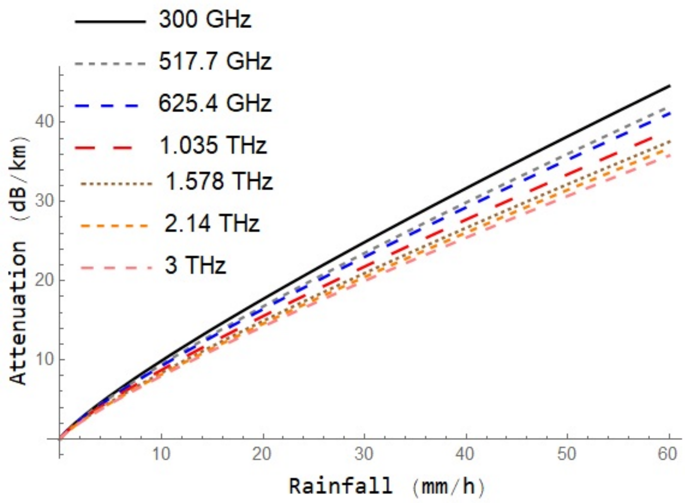
$$log_{10}(k) = \sum_{j=1}^{4} \left(a_j Exp \left[- \left(\frac{log_{10}(f) - b_j}{c_j} \right)^2 \right] \right) + m_k log_{10} f + c_k \tag{2.57}$$

$$\alpha = \sum_{j=1}^{5} \left(a_j Exp \left[- \left(\frac{log_{10}(f) - b_j}{c_j} \right)^2 \right] \right) + m_\alpha log_{10} f + c_\alpha \tag{2.58}$$

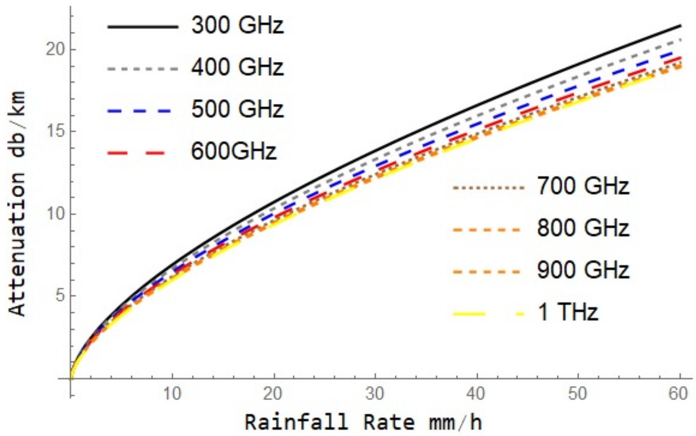
Where:
f: frequency (GHz)
k: either k_H or k_v
α: either α_H or α_v

The coefficients (k_H , α_H) for horizontal polarization and (k_v, α_v) for vertical polarization, are given in the recommendation ITU-R [15] with specific values for the parameters (a_j, b_j, c_j, m_k, and c_k).

Based in these two models, we can have an estimation of how rain can affect the propagation in THz. Figure 2.6.(a) shows the rainfall attenuation versus rainfall rate for some frequencies in THz by using Mie/Weibull theory. Figure 2.6.(a) shows the ITU-R model. By comparing both models, it is possible to verify that as the rainfall rate increases, the estimation in these two models for attenuation is diverging.



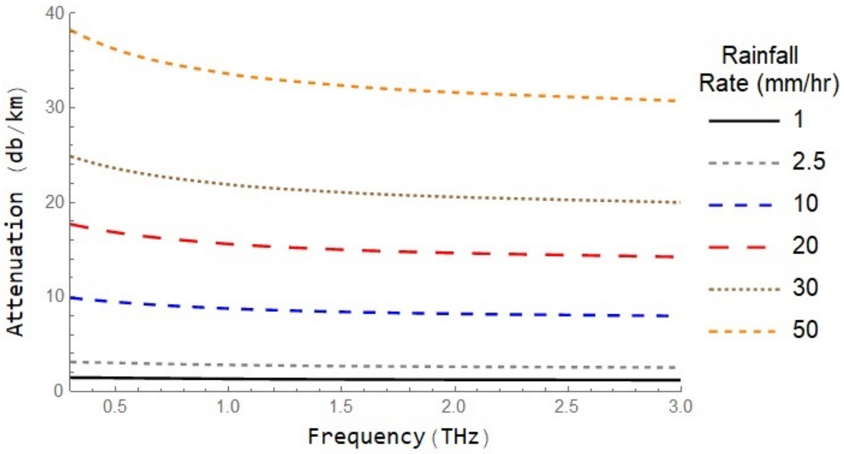
a)



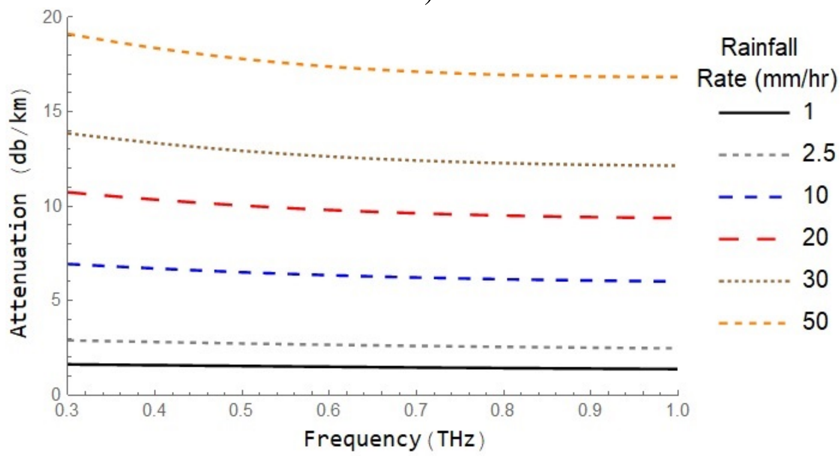
b)

Figure 2.6 – (a) Attenuation versus rainfall rate for frequencies between 300GHZ-3THz by using Mie/Weibull theory. (b) Attenuation versus rainfall rate for frequencies between 300GHz-1THz by using ITU-R model.

To have a better picture about the behavior of attenuation, Fig. 2.7 shows the attenuation as a function of frequency between 0.3 to 3 THz for some values of rainfall rate. Figure. 2.7.(a) shows the frequency vs attenuatuion by using Mie/Weibull theory and Fig. 2.7.(b) uses ITU-R parameters. It is easy to note that the divergence between ITU-R and Mie/Weibull is growing as the rainfall intensity rate rises.



a)



b)

Figure 2.7 – (a) Attenuation versus frequency in THz for some different rates of rainfall that uses Mie/Weibull theory. (b) Attenuation versus frequency in THz for some different rates of rainfall that uses ITU-R parameters.

Given these points, we are going to use equation 2.55 to calculated the attenuation in dB/km. Therefor, it will be necessary to find the cross section coefficient C_{ext} together with the drop size distribution $N(a, R)$. In order to compare models and their estimations, the next step will be to calculate the same attenuation, but now using the ITU empirical method given by equation 2.56.

2.4 STATISTICAL TECHNIQUES

The process of finding an estimation for rain attenuation at THz in Brasilia, DF uses the rainfall rate data collected over 20 years at INMET as input for all the calculations. As we could see in eq. 2.55 and eq. 2.56 the attenuation needs the rainfall rate (R) as input to give A(dB/Km). The rainfall rate is an non-deterministic parameter that needs to be treat statistically, consequently the Attenuation also needs a statistical approach to be correctly estimated. In this chapter we will be exploring the two methods that we used to estimate attenuation.

In order to achieve an estimation of rain attenuation with data collected at INMET, we used two simulation approaches:

i) Monte Carlo based on the distribution of rainfall rates throughout the wet season [16].

ii) Unscented Transform, which can give an estimation of rain attenuation based on few simulations with calculated weights [17].

2.4.1 Monte Carlo

The Monte Carlo method was invented by John Von Neumann and Stanislaw Ulan during the second world war. He was seeking for a way to improve decisions in uncertain conditions. The method received his name because its procedure can be compered to a roulette game. Monte Carlo simulation is based on repeated random sampling and statistical analysis to compute results [28]. It calculates results by iterating a set of random values from probability functions and outcome a set of values that can give a perspective of how some event can occur.

In this work, we used some collected data from INMET that measured the rain fall rate along two decades. By using the data, we can have a probability distribution function to use as base for applying random values that Monte Carlo needs in order to calculated the output function. Figure 2.8 shows the basic strategy to estimate rainfall attenuation. First, we used the collected data about rainfall rate in Brasilia, Brazil. Second step, We use the data to obtain a distribution function of rainfall. Third phase was to obtain random points based on the distribution. Fourth, we numerically use those points as input to a process that calculates attenuation in dB/km. Finally, we can estimate based in Monte Carlo the behavior of attenuation due rainfall.

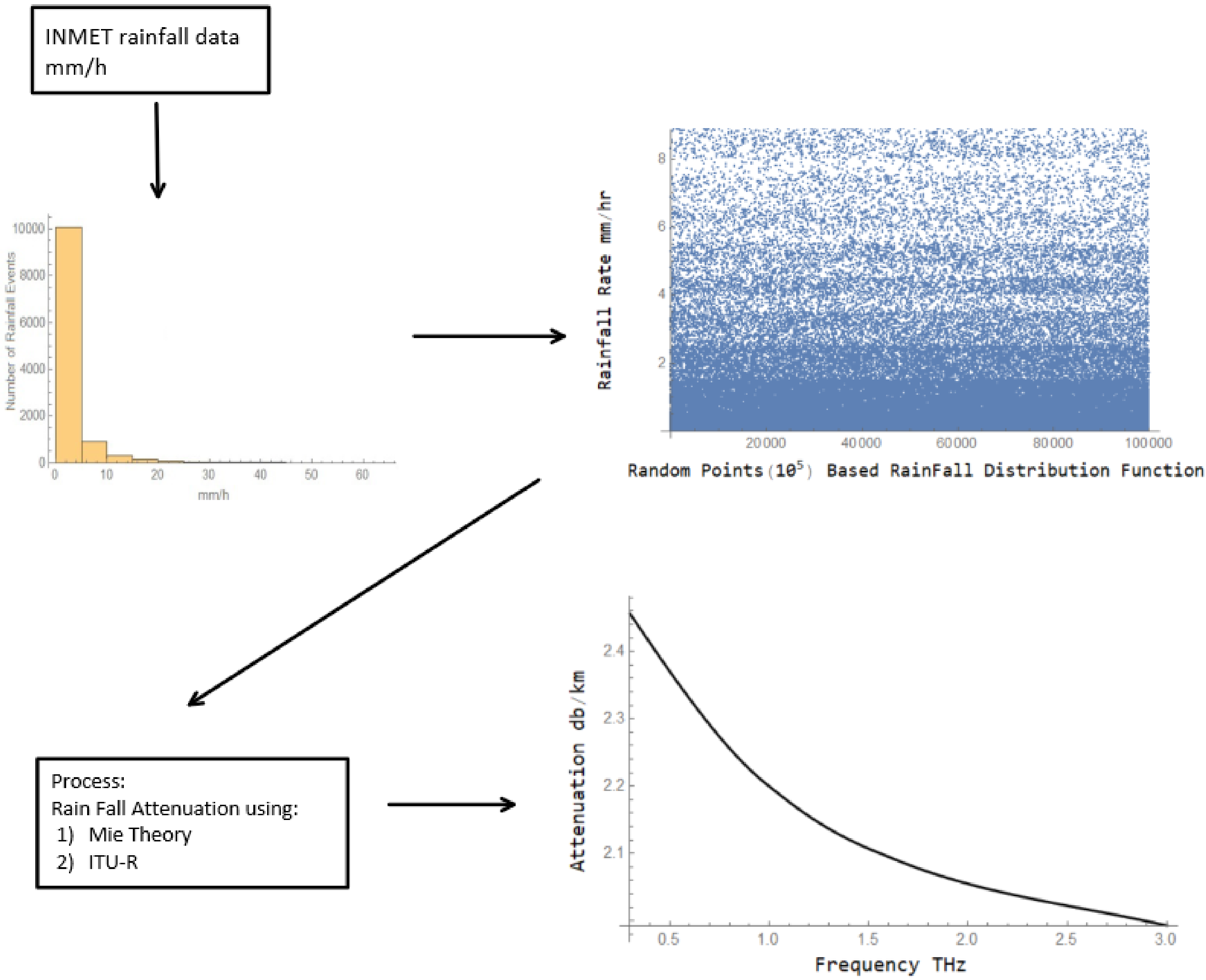


Figure 2.8 – The basic structure used to estimate attenuation by using the data collected at INMET.

2.4.2 Unscented Transform

The unscented transform was created by Julier and Uhlmann in 1997 as he presented his thesis as a form of "distribution approximation filter" [30, 29, 17]. The main idea it is to approximate a non linear mapping function by some specific points (sigma points). In our case, the UT does a discrete approximation of rainfall rate PDF and then uses the calculated sigma points as discrete input to calculate rain fall attenuation.

The UT uses the discrete moment function to calculate the sigma poins.

$$E\{[f(x)]^k\} = \sum_{i=1}[f(x_i)]^k p(x_i) = \int f(x)p(x)dx \tag{2.59}$$

If we define that the function $f(x)$ has continuous derivative, we can expand its mapping to the Taylor series as it follows:

$$f(x) = f(0) + \frac{df(x)}{dx}|_{x=0}\, x + \frac{1}{2!}\frac{d^2f(x)}{dx^2}|_{x=0}\, x^2 + ... \tag{2.60}$$

$$f(x) = a_0 + a_1x + a_2x^2 + ... \tag{2.61}$$

Replacing 2.61 into 2.59, we can have the following expression:

$$\begin{aligned} E\{f(x)\} &= E\{a_0\} + E\{a_1\ x\} + E\{a_2\ x^2\} + ... \\ &= a_0 + a_1E\{x\} + a_2E\{x^2\} + ... \end{aligned} \tag{2.62}$$

But we also know that the continuous and discrete moments can be describe in mathematical terms as beins:

$$E\{x^k\} = \sum p(x_i)x_i^k = \int x^kp(x)dx \tag{2.63}$$

By expanding equation 2.63, we can create a non linear system that can be easily solved using numerically computation:

$$p(x_1) + p(x_2) + p(x_3) = 1$$

$$p(x_1)x_1 + p(x_2)x_2 + p(x_3)x_3 = \mu$$

$$p(x_1)x_1^2 + p(x_2)x_2^2 + p(x_3)x_3^2 = E\{x^2\}$$

$$p(x_1)x_1^3 + p(x_2)x_2^3 + p(x_3)x_3^3 = E\{x^3\}$$

$$p(x_1)x_1^4 + p(x_2)x_2^4 + p(x_3)x_3^4 = E\{x^4\}$$

$$p(x_1)x_1^5 + p(x_2)x_2^5 + p(x_3)x_3^5 = E\{x^5\}$$

For this case, we have six equations and also six variables. We can call $p(x_k) = w_k$ as weighs.

Using the collated rainfall rate data from Inmet, it is possible to calculated the statistical moments and then calculate the weights to use as our input. With sigma and weights, points we can use them as input for the process that uses Mie/Weibull and also ITU-R to finally calculate the attenuation for a THz range of frequencies. Figure 2.9 shows the process of estimating the rainfall attenuation using the UT. Figure 2.10 shows how is the entire process to calculate rain fall attenuation that was used in this work.

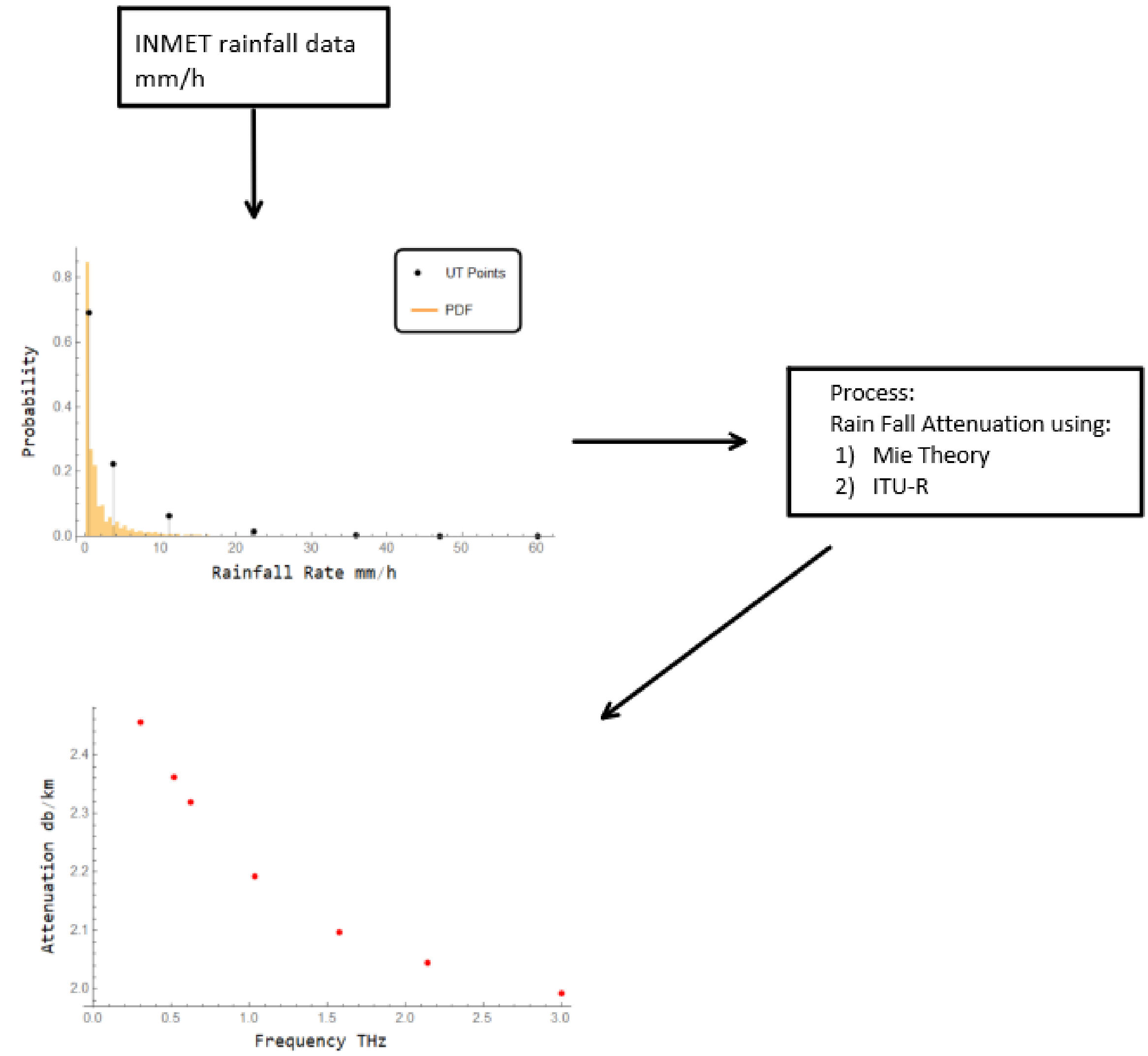


Figure 2.9 – The basic structure used to estimate attenuation by using UT.

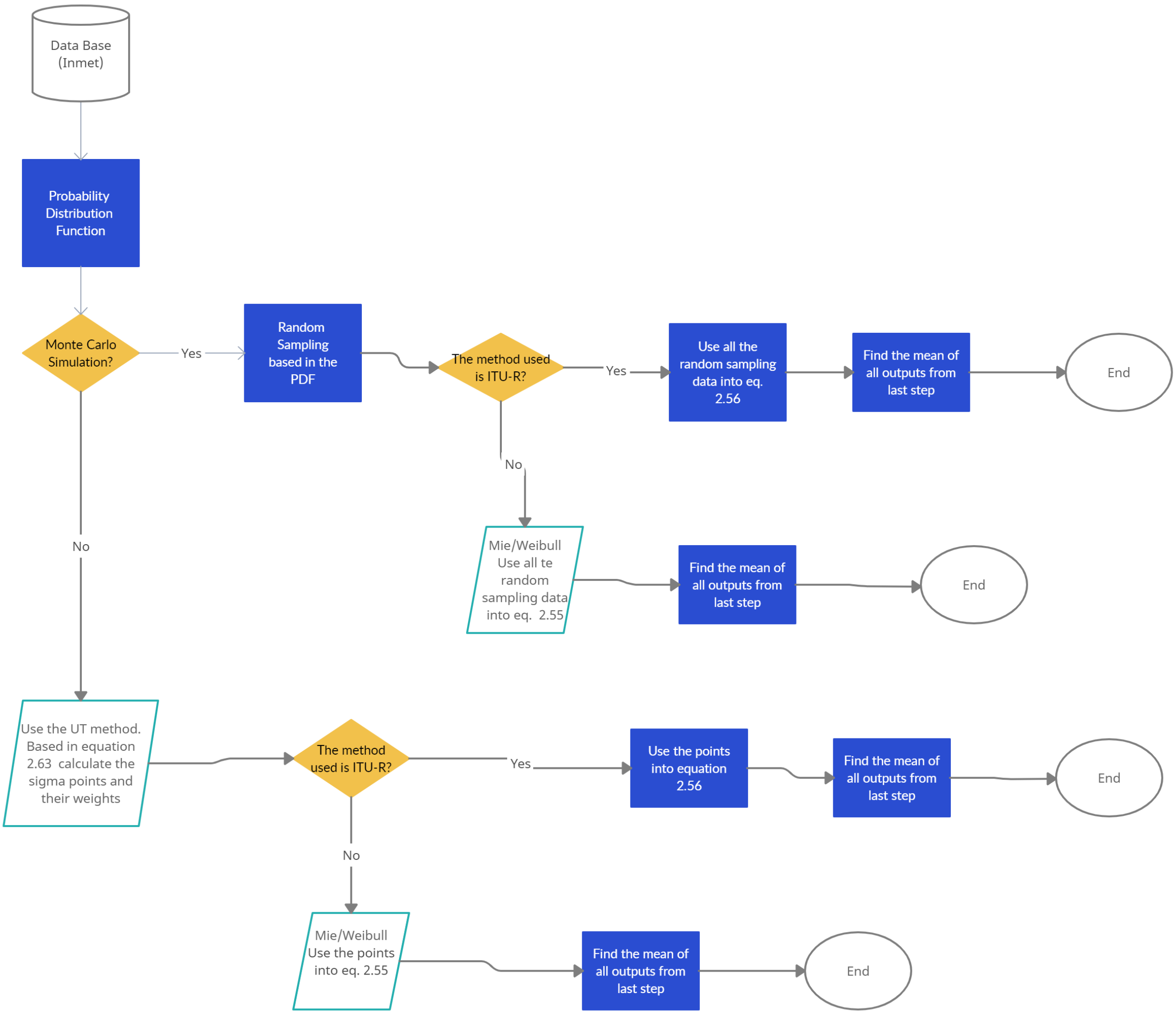


Figure 2.10 – Flowchart of the entire process to find rainfall attenuation.

As we could note, the uncertain relative to rainfall lead us to the field of statistics. The Monte Carlo and UT are the key to understand the impact of uncertain in the attenuation model. They give a prediction based in probability about how an uncertain event (rain) can affect the wave propagation in THz frequencies. Monte Carlo is a well-known method and can give very accurate results, but the computational efficiency of Monte Carlo is a disadvantage in comparison with UT. The UT uses few sigma/weights points as the input to the attenuation function, which gives a simple task to compute while Monte Carlo has to do a huge number of interactions based in the PDF to reach a good estimation for the final task.

3 RAINFALL BEHAVIOR IN BRAZIL'S MIDWEST

3.1 RAINFALL DATA COLLECTED IN BRASILIA BY INMET

It is crucial to know the rainfall behavior throughout the year and seasons to correctly analyze and predict rainfall attenuation. Therefore, a third important phase of this work is based on the analyses of data collected at the National Institute of Meteorology (INMET). The rainfall rate was collected in Brazil between 2001 and 2020 at the weather station localized in Brasilia, Brazil (latitude: -15.789343, Longitude: -47.925756, Height: 1160.9 m).

Rainfall rate intensity is determined as the average rainfall rate in mm/h for specific rainfall duration and frequency. The rainfall intensity is classified according to the rate of precipitation. The following categories are used to classify rainfall intensity:

- Light Rain: $R \leq 2.5$ mm/h
- Moderate Rain: $2.5 \leq R < 10$ mm/h
- Heavy Rain: $10 \leq R < 50$ mm/h
- Violent Rain: $R \geq 50$ mm/h

To have an entire picture of rainfall attenuation in Brasilia, we divided the data collected by INMET per month, so we could analyze how was the total volume of rain for each month in the past 20 years. Based on the collected data, it is also possible to extract the mean rate in mm/hr for each month. The regime of rain in Brasilia can be easily divided into two main types: The wet season and the dry season. The next step was to analyze the data for what is the focus of this work: the wet season.

3.1.1 Data Analysis throughout the Year

By analyzing all the data collected at INMET, it was possible to divide the total volume of rain for each month in the past twenty years. Figure 3.1 shows the total volume of rain and the mean rate of rainfall throughout 20 years of data. It is interesting to note that April is not the rainiest month by analyzing the total volume of rain, but it has the greatest mean rate in mm/hr. Also, fig. 3.1 shows that the dry season in Brasilia is from May to August. Even though there is some rain throughout the dry season, it is clear by analyzing the data that they are quite rare events.

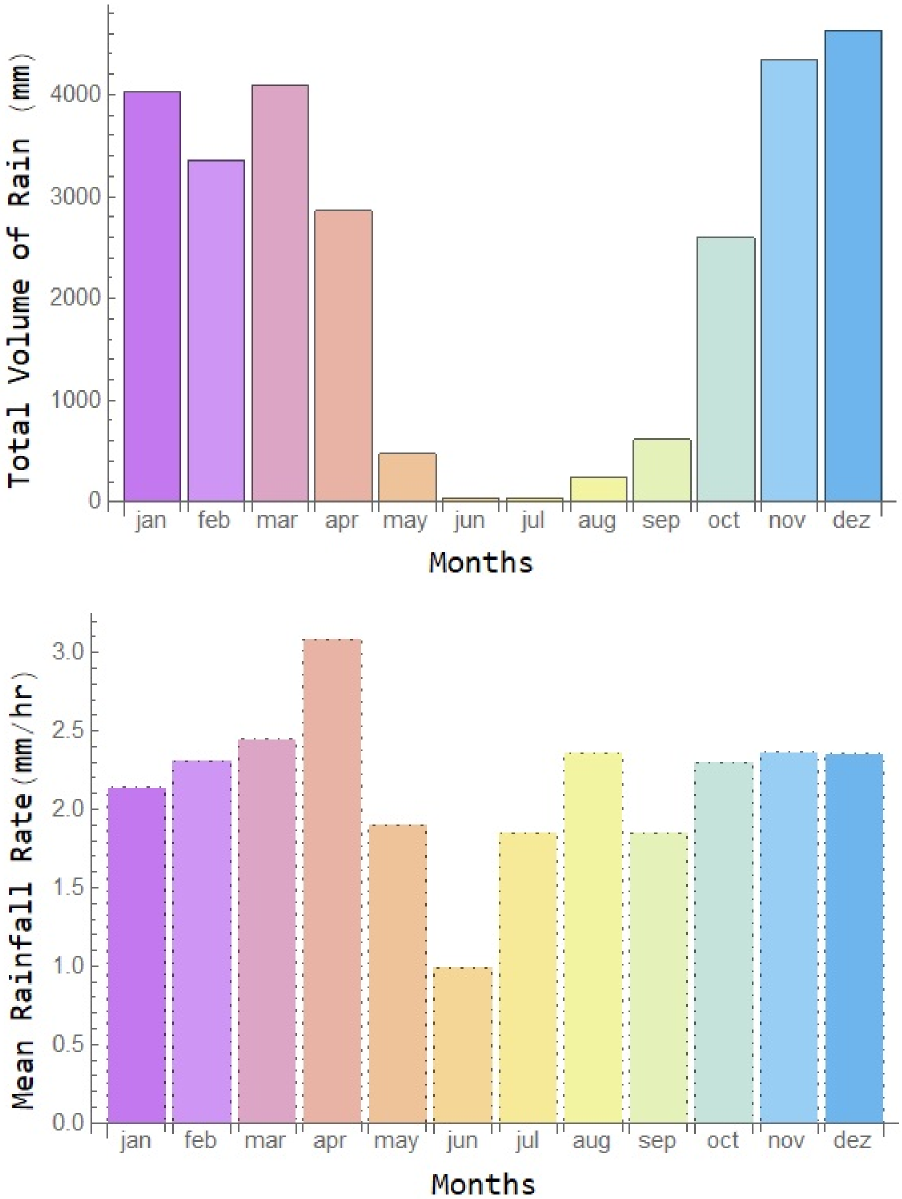


Figure 3.1 – Total Volume and Mean Rate of Rainfall Between 2001-2020

To understand the behavior of attenuation in Brasilia, it is fundamental to give close attention to the extreme events, which are rainfall rates above 10 mm/hr. Table 3.1 shows the probability of a rain event greater than 10 mm/hr in each month of the year. It is interesting to note that as was expected, April shows the greater probability of an extreme event with 7.43%. Figure 3.2 shows the PDF and the CDF with the number of rainfall events along 2001 to 2020.

Tabela 3.1 – Probability of a rain event greater than 10 mm/hr in each month

Months	Probability[%]
jan	4.25
fev	4.67
mar	5.49
apr	7.43
may	3.22
jun	–
jul	4.76
aug	3.96
sept	3.60
oct	5.14
nov	4.90
dez	5.64

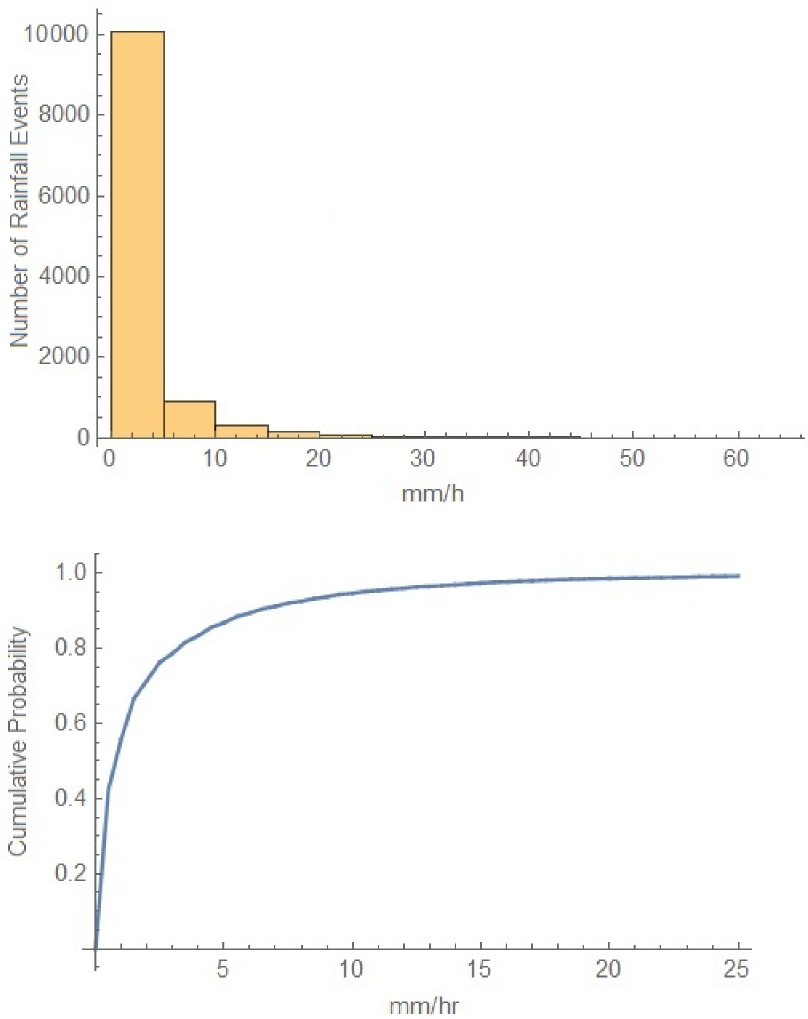


Figure 3.2 – Histogram with number of rainfall events along 2001 to 2020 and CDF of Rainfall rate in mm/hr

This subsection was dedicated to the data analysis of all rainfall events that occur between the years 2000 and 2020. By using the INMET data, it was possible to define the wet season and the dry season in the region of Brasilia, DF. Also, it was possible to calculate the probability of an extreme event with a rainfall rate above 10 mm/hr each month for the whole year. Also, the PDF of rainfall rate events was calculated and the estimation of rainfall attenuation in THz spectrum will be based on this PDF.

3.1.2 Data Analysis along Wet Season

To have an entire picture of the rainfall behavior throughout the year, we decided to analyze the data with all our focus on the wet season. As we could note, the wet season is the most important phase to be studied because it will be fundamental to have a good perspective of rainfall attenuation in waves propagating in THz frequencies. To analyze the attenuation throughout the wet season, we collected the rainfall data from September to April. Figure 3.3 shows the histogram and the cumulative distribution function (CDF) of 20 years of rainfall versus the rain rate measured in mm/h. Most events of rain, more precisely 86.62 $\%$, occur in the range of 0 to 5 mm/h.

To statistically analyze the attenuation in the wet season, we will use two approaches: Monte Carlo Simulation and Unscented Transform. Figure 3.3 shows the PDF that will be used in the Monte Carlo simulation and also shows the seven Unscented Transforms points that further will be used to estimate attenuation via UT.

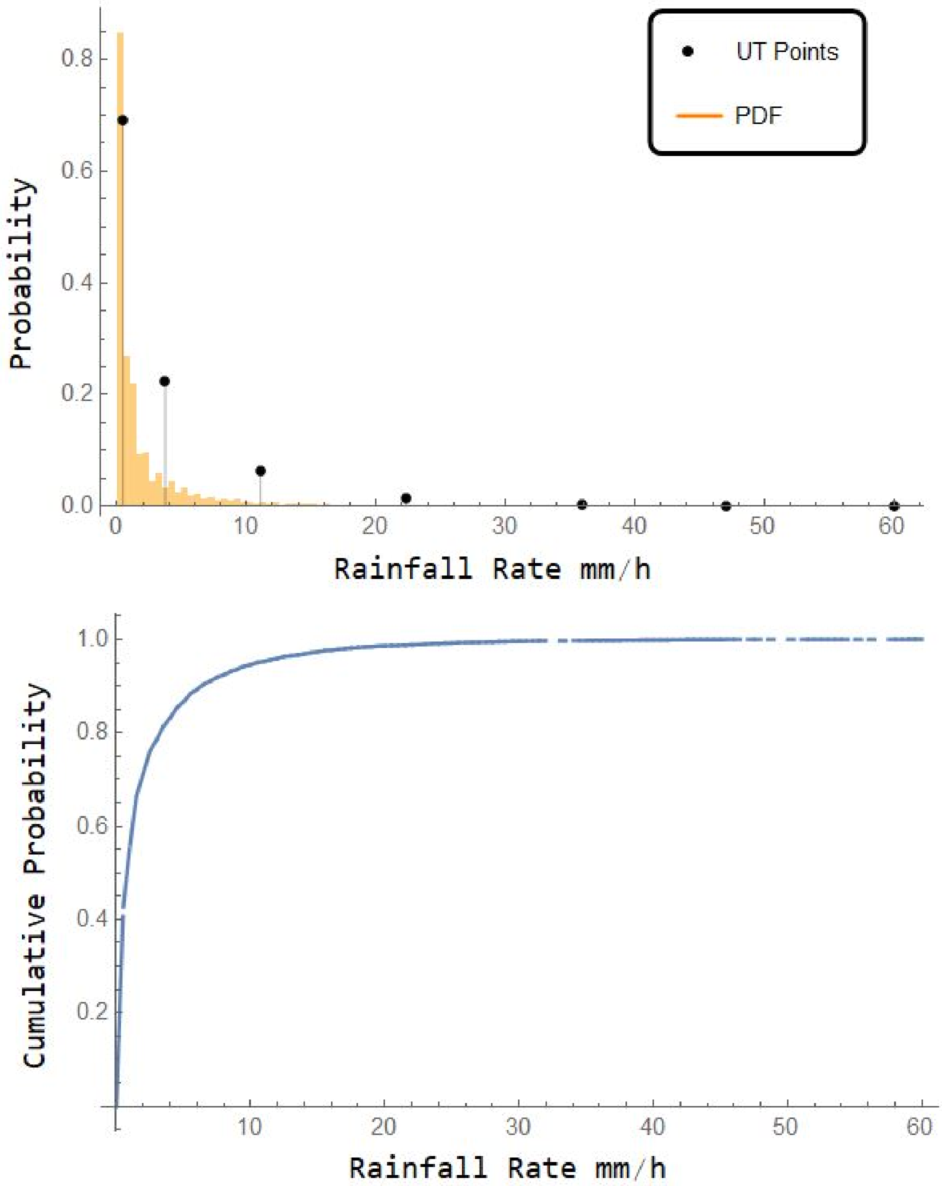


Figure 3.3 – PDF with several rainfall events throughout the wet season (October to April) along 2001 to 2020 and CDF of Rainfall rate in mm/hr. The histogram contains the Unscented Transform points (UT).

The rainfall needs to be studied in its extreme cases to predict damage attenuation, which is the rain that will strongly affect the wave propagation. Figure 3.4 shows the PDF of all rainfall with a rate above 10 mm/h that occurs from October to April throughout 20 years of data. The UT points were also plotted in Fig.3.3 and Fig.3.4. In addition, an event of heavy rain, which is rain above 10 mm/hr, has the probability of 5.41 $\%$ occurring during the wet season.

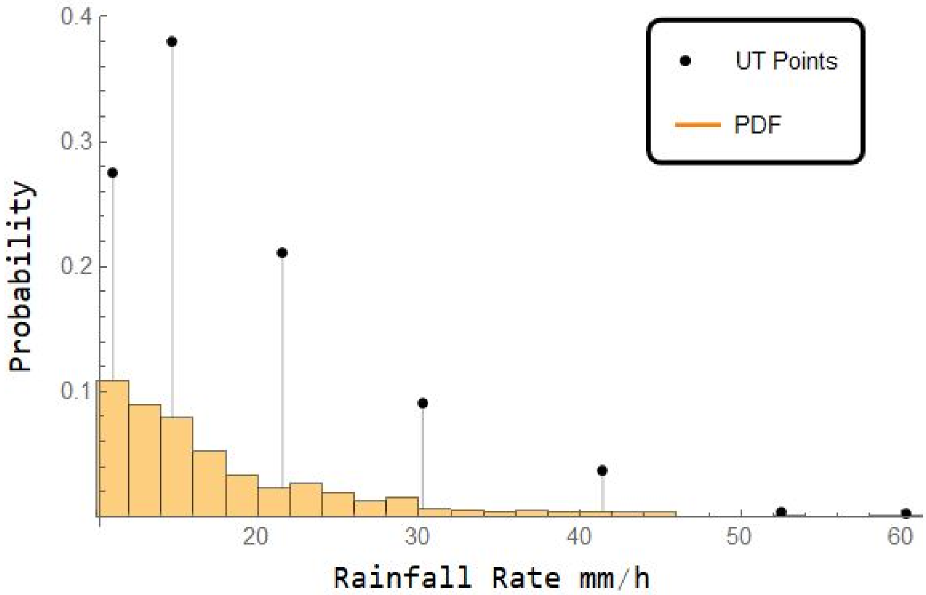


Figure 3.4 – PDF with several rainfall events throughout the wet season (October to April) along 2001 to 2020 for events of rain above 10 mm/h. The PDF contains the Unscented Transform points (UT).

4 RESULTS

The results obtained by analyzing the data collected and treated in chapter 3 were divided in two section: section 4.1 and 4.2. In section 4.1, we calculated the mean attenuation for each month in the year using the methods described in chapter 3. Also, we used a statistical approach to calculate the mean attenuation for extreme events of rainfall (rain rate above 10 mm/hr).

In section 4.2, we calculate the mean attenuation by using all the data we collected in the wet season. We also used the Unscented Transform to compare with the Monte Carlo simulation. Again, the calculation for extreme events was made, but now using all the data collected in the wet season and the UT.

4.1 MEAN ATTENUATION DIVIDED BY MONTHS

Using all the theories disposed of in this work, it is now possible to estimate the mean attenuation for each month in the year. As we could see in chapter 3, the regime of rain in the Midwest of Brazil is divided into the wet season and a dry season. The wet season usually goes from October to April. The months of May, June, July, and August are known for the gap of rain in Brasilia, DF. For having a good perspective on how rain attenuation can impact signal propagation in the THz spectrum, it is crucial to deal with the data related to the wet season. By analyzing the figure 3.1, we can see that even though the dry season has a low volume of rain, the mean rate in mm/hr can give a wrong impression of what is happening. Therefore, we decided to calculate the mean attenuation for each month inside the wet season and section 4.1.1 shows the results. As we said in chapter 3, the extreme events of rain with rates above 10 mm/hr are quite important when analyzing attenuation because they are responsible for an attenuation that can impact the THz propagation in outdoor environments. For this reason, section 4.1.2 is dedicated to showing the results of attenuation calculated by using the extreme events as a parameter to our estimations.

4.1.1 Mean Attenuation

Figure 4.1 shows the mean attenuation for each month inside the wet season in Brasilia, DF. Figure 4.1a) shows attenuation versus frequency for each month by using Mie/Weibull theory. By looking to 4.1a) it is clear that April shows the highest level for attenuation, it is interesting to note that even though April is not the rainiest month of the year, it has the highest rain mean rate, which means that April has the highest mean attenuation. Using Mie/Weibull theory, it is possible to see in Fig. 4.1a) that April has the maximum mean attenuation of 3.4 dB/Km for 0.3 THz and the minimum mean attenuation of 2.8 dB/Km for the frequency of 3 THz.

Figure 4.1b) shows the same results but now using the ITU-R theory based on equation 2.56. Unfortunately, the ITU-R has enough data to estimate attenuation up to 1 THz, so it is not possible to plot the attenuation for frequencies above that level. As a matter of example, by using ITU-R, April shows the highest mean attenuation with the maximum mean of 2.7 dB/Km and the minimum mean of 2.3 dB/Km.

This section shows the mean attenuation given in dB/Km for each month in the wet season using two models: Mie/Weibull and ITU-R. By comparing both models (ITU-R and Weibull/Mie), we can see that the levels of attenuation using ITU-R are lower in comparison with the theoretical calculations using Mie theory.

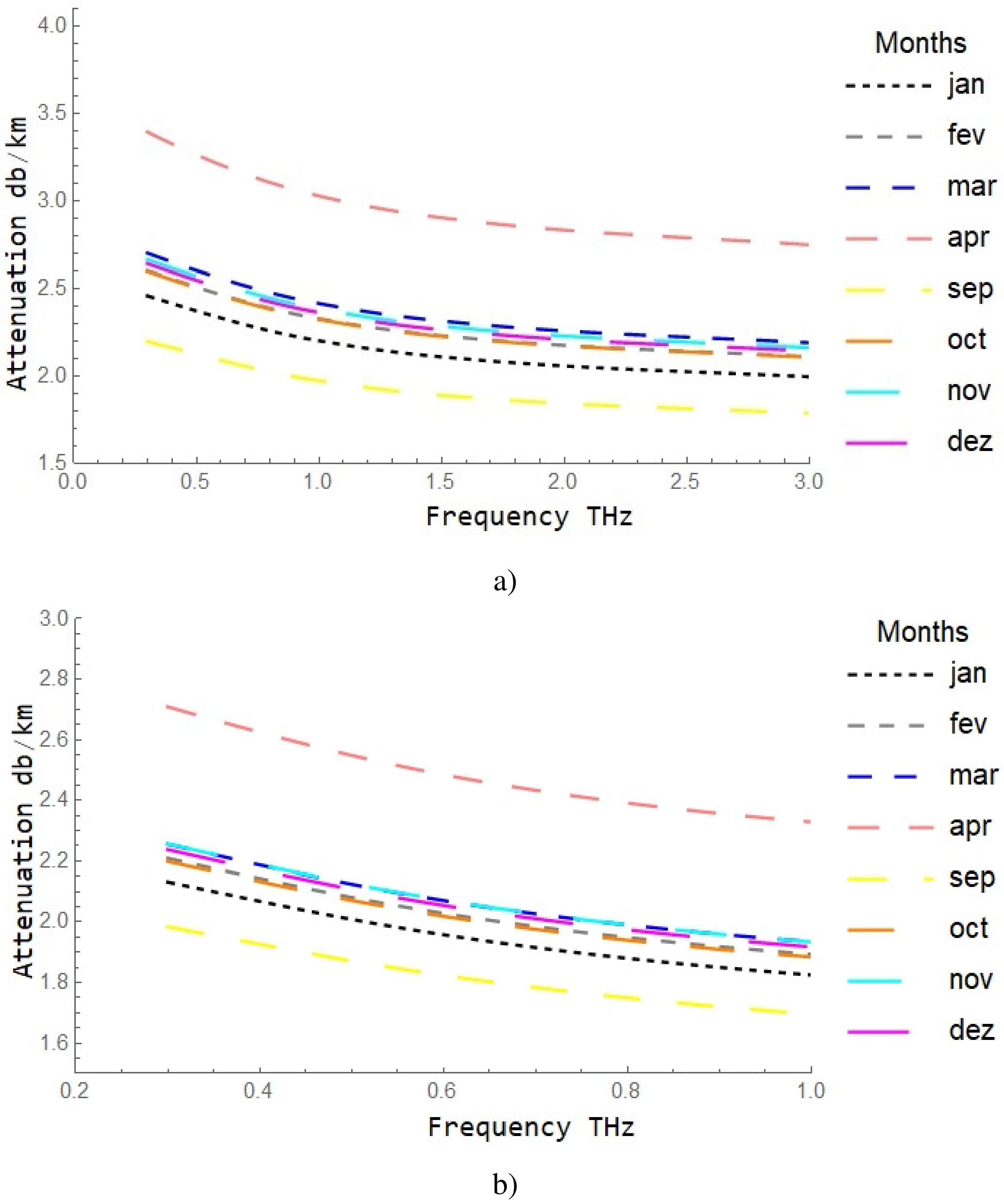


Figure 4.1 – (a) Attenuation versus frequency for each month by using Mie/Weibull theory. (b) Attenuation versus frequency for each month by using ITU-R theory.

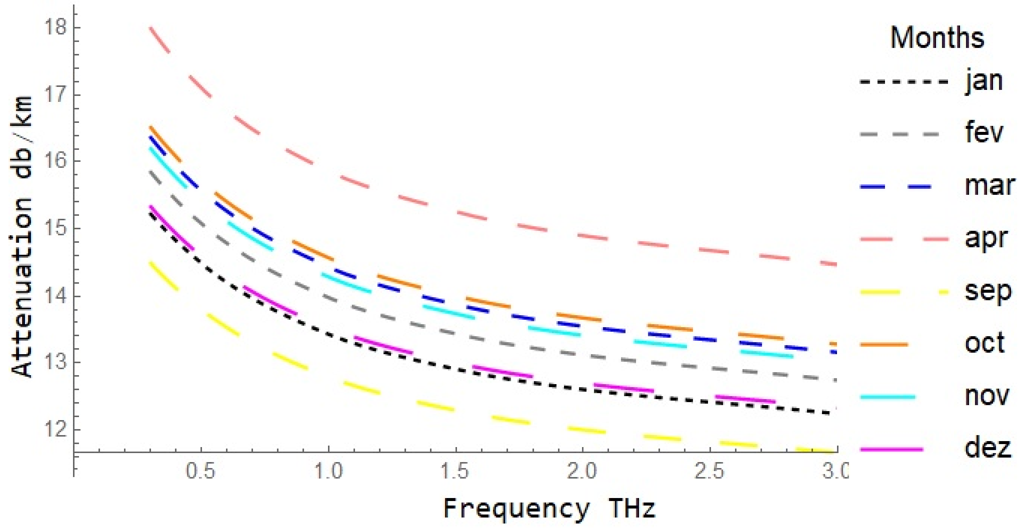
4.1.2 Mean Attenuation for Extreme Events

As we said before, the level of attenuation can reach high values when the rain rate goes up. Therefore, it is fundamental to understand the behavior of rainfall attenuation for extreme values of rainfall rate, which is the rate above 10 mm/h. Figure 4.2 shows the attenuation versus frequency for each month in the year given the assumption that will occur an event of extreme rain with rate above 10 mm/hr. In table 3.1, it is possible to see the probability of an extreme event of rain for each month in the year.

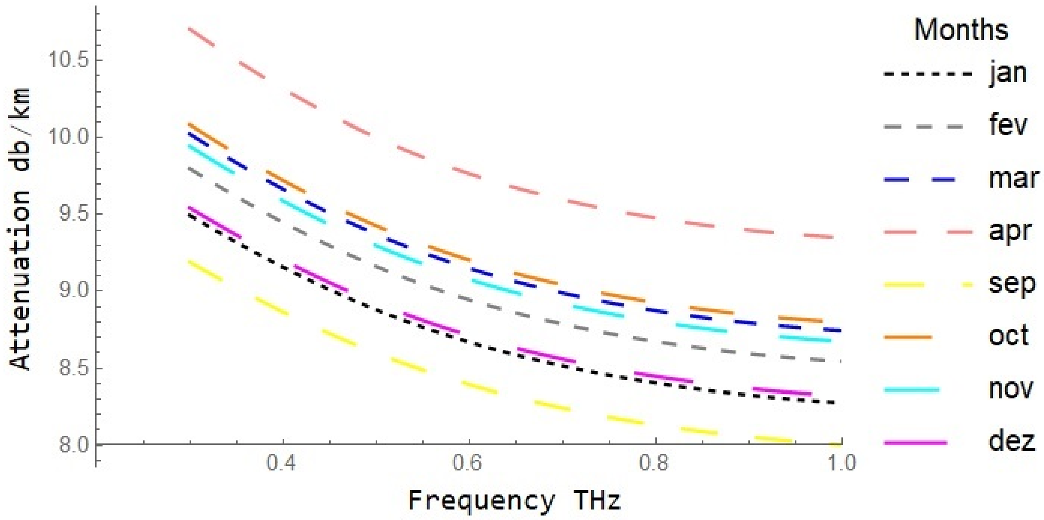
Figure 4.2a) shows the attenuation versus frequency using the theoretical approach with

Mie/Weibull and the data of extreme events of rainfall. Again, April shows the highest level of attenuation. By looking at the probability of extreme events in table 3.1, we can see that the probability of an extreme event of rainfall with a rate greater than 10 mm/hr is about 7.43 percent in April. Using the Mie/Weibull theory, the maximum mean attenuation in April is 18 dB/Km for the frequency of 0,1 THz and the minimum is 14,5 dB/Km for the frequency of 3 THz.

Figure 4.2b) shows the same results but using the empirical approach used in ITU-R table. Again April shows the highest level for attenuation. The maximum mean attenuation is 10.8 dB/Km for the frequency of 0.3 THz and a minimum mean of 9,4 dB/Km for 1 THz.



a)



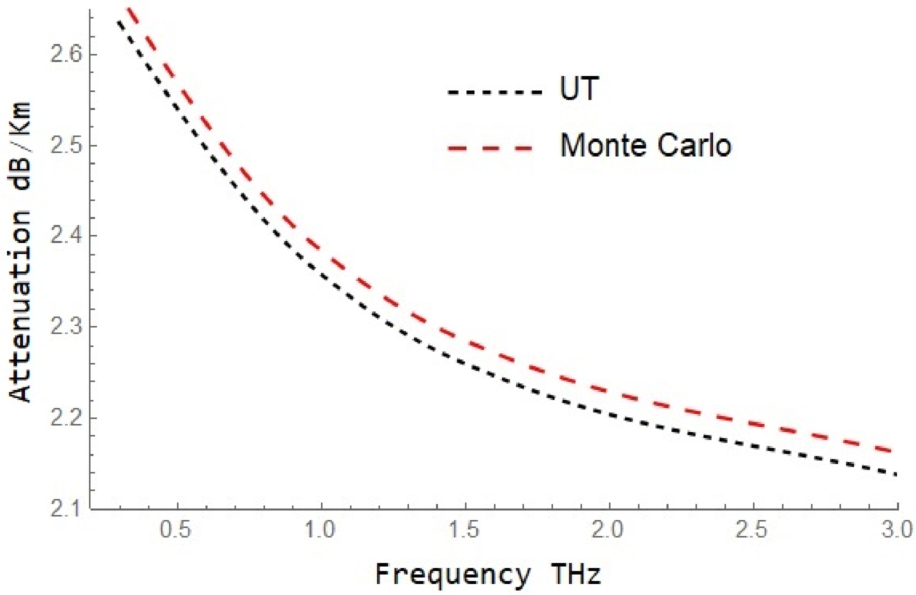
b)

Figure 4.2 – (a) Attenuation versus rainfall rate for frequencies between 300GHZ-3THz by using Mie/Weibull theory for extreme events. (b) Attenuation versus rainfall rate for frequencies between 300GHz-1THz by using ITU-R model.

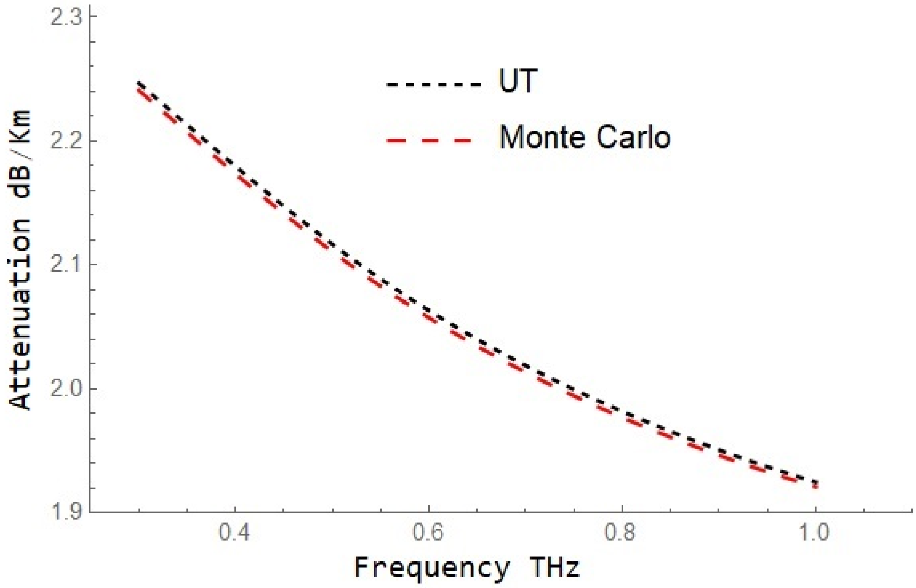
4.2 ATTENUATION IN THE WET SEASON

To have a perspective on the entire wet season behavior in the Midwest of Brasil, we decided to calculate the mean attenuation using the data collected just in the wet season. Again, the method used to estimate the attenuation was done by Monte Carlo simulation; we also used the Unscented Transform to compare with the results of MC. By using Monte Carlo, the Probability Distribution Function (PDF) of the wet season (October to April) was collected and then used as input data for the attenuation function calculated by (2.55) using Mie/Weibull theory and as an input in (2.56) that uses the ITU-R recommendation. The seven UT points based on the rain rate distribution (Fig.3.3/ Fig.3.4) were used as input and a comparison between both approaches is disposed in Fig.4.3, which shows the attenuation as a function of frequency for MC and UT methods. Fig.4.3a shows the attenuation calculated from Mie/Weibull theory and Fig.4.3b from ITU-R data. It is important to underline that for ITU-R empirical values, the range of frequency is limited to 1 THz; the frequencies using Mie/Weibull theory can go up to 3 THz. We can see the similarity between the estimation using UT and MC for both calculations: using Mie/Weibull and ITU-R.

4.2.1 Mean Attenuation



a)



b)

Figure 4.3 – (a) Mean attenuation versus frequency in the wet season by using Mie/Weibull theory. (b) Attenuation versus frequency in the wet season by using ITU-R theory.

The mean attenuation disposed in Fig.4.3 is not enough to have a complete picture of the behavior of attenuation due to rain in Brasilia, DF. The amount of dispersion in the values is also an important parameter to have a better idea of attenuation. Tab.4.1 shows a comparison for values of mean and standard deviation (SD) using MC and UT methods for attenuation values that were calculated using (2.55), which is Mie/Weibull theory. Table 4.2 shows the same information but now calculated using (2.56), which is ITU-R.

Tabela 4.1 – Mean Attenuation and SD using Mie/Weibull Theory, Monte Carlo, and UT for Terahertz Frequencies

Freq THz	Mean MC	Mean UT	Std.Dev MC	Std.Dev UT
3.	2.161	2.137	3.198	3.255
2.142	2.217	2.192	3.277	3.325
1.578	2.274	2.249	3.355	3.396
1.035	2.375	2.349	3.492	3.522
0.625	2.513	2.486	3.691	3.712
0.517	2.561	2.533	3.768	3.787
0.3	2.666	2.635	3.981	3.998

Tabela 4.2 – Mean Attenuation and SD using ITU-R, Monte Carlo, and UT for Terahertz Frequencies

Freq THz	Mean MC	Mean UT	Std.Dev MC	Std.Dev UT
1	1.918	1.925	2.084	2.058
0.9	1.944	1.951	2.096	2.068
0.8	1.974	1.982	2.114	2.085
0.7	2.011	2.019	2.141	2.111
0.6	2.055	2.063	2.179	2.148
0.5	2.108	2.116	2.231	2.200
0.4	2.171	2.179	2.302	2.269
0.3	2.238	2.246	2.388	2.355

As we could see, using Mie/Weibull approach the maximum mean attenuation is 2.666 for the frequency of 0.3 THz and the minimum mean attenuation is 2.161 for the frequency of 3 THz. Using the empirical ITU-R approach, we have the maximum mean attenuation of 2.238 for the frequency of 0.3 THz and the minimum mean attenuation is 1.918 for the frequency of 1 THz.

Fig. 4.4 is the smooth probability distribution that was calculated using MC and Mie/Weibull Theory. It shows how the probability distribution of attenuation is varying for some THz frequencies. As expected, the attenuation tends to decrease as the frequency increase for terahertz applications.

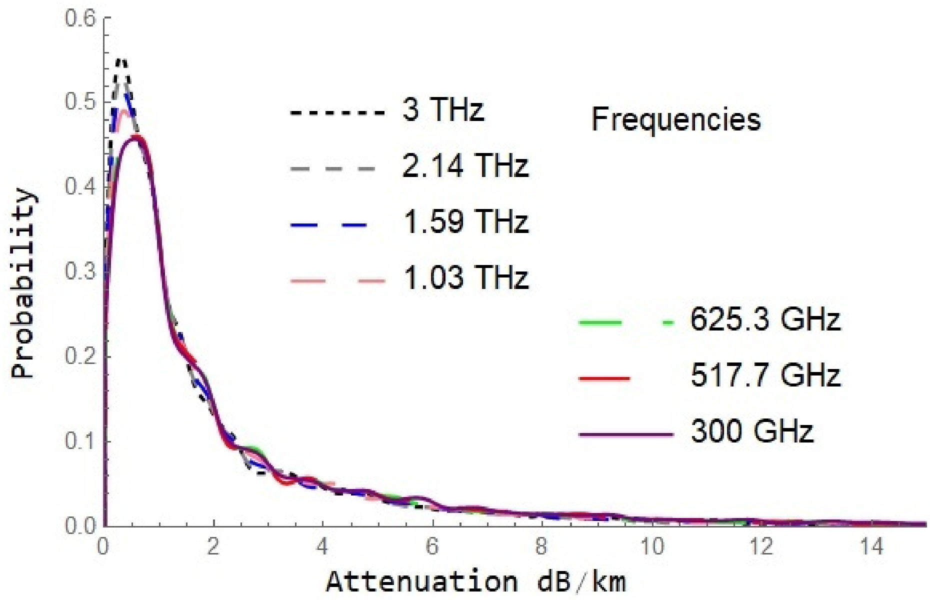


Figure 4.4 – Smooth PDF of rain attenuation for some THz frequencies based on Monte Carlo simulation.

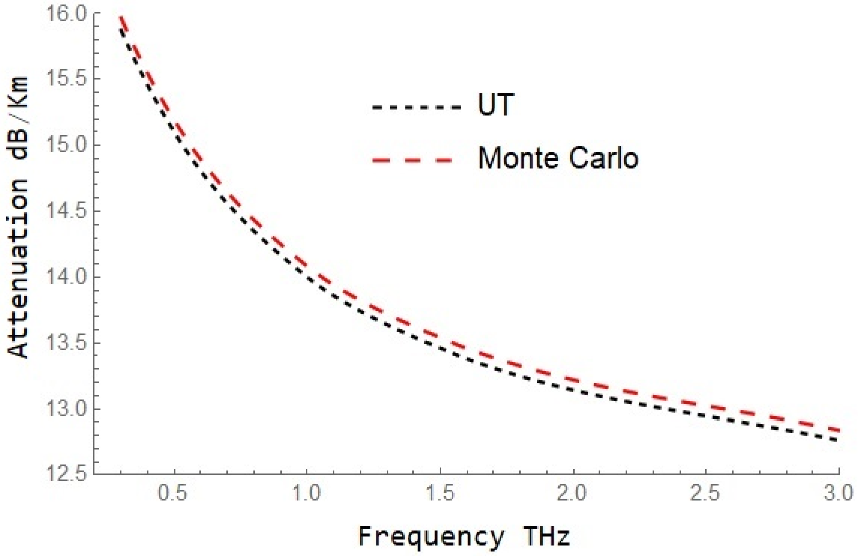
This section was based on the calculation of mean attenuation for the wet season in Brasilia, Brazil. The results show a very accurate similarity using the Unscented Transform or Monte Carlo simulation. As we could expect, the ITU-R model shows lower levels of attenuation for the same frequencies when compared with the theoretical model based on Mie/Weibull theory.

4.2.2 Mean Attenuation for Extreme Events

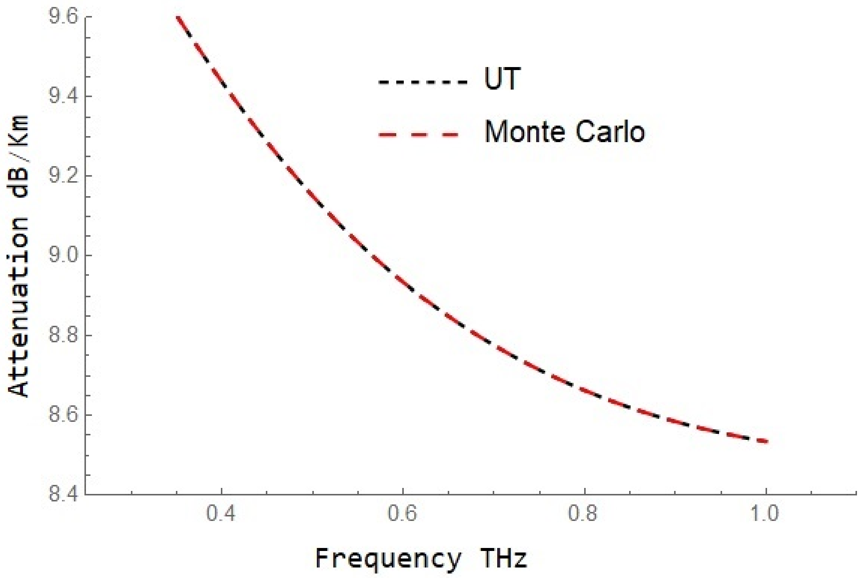
As already observed, attenuation due to scattering and absorption in THz frequencies is a major limitation to the application of these technologies. To fully understand the behavior of a system working on these frequencies, it is necessary to comprehend the attenuation response due to extreme cases. Extreme cases can be understood as events of rainfall with a rate greater than 10 mm/h. The probability of a rain event of this magnitude is around 5.41% in the wet season.

Fig. 4.2 is the mean attenuation of extreme events over the wet season using Mie/Weibull and ITU-R. As expected, the divergence between ITU-R and Mie/Weibull grows for extreme events. Table 4.3 shows the mean and SD calculated for some THz frequencies using Mie/Weibull. Also, it shows a comparison of mean and SD between MC and UT methods. Table 4.4 shows the same parameters but are now calculated using ITU-R recommendations. Figure 2.2 shows the probability distribution of attenuation for some frequencies at the THz spectrum using Mie/Weibull theory. As expected, the attenuation curve goes to the right showing that the mean attenuation is increasing as the frequency decreases.

The maximum value for mean attenuation using Monte Carlo simulation with Mie/Weibull distribution was 15.96 dB/Km for the frequency of 0.3 THz and the minimum mean attenuation was 12.83 for 3 THz. Using the empirical model proposed by ITU-R, we can find the maximum attenuation of 9.78 dB/Km for the frequency of 0.3 THz and 8.53 THz for the frequency of 1 THz. All these information are disposed in table 4.3 and 4.4.



a)



b)

Figure 4.5 – (a) Mean attenuation versus frequency for extreme cases in the wet season by using Mie/Weibull theory. (b) Attenuation versus frequency for extreme cases in the wet season by using ITU-R theory.

Tabela 4.3 – Mean Attenuation and SD using Mie/Weibull Theory, Monte Carlo, and UT for Terahertz Frequencies based on the rainfall rate above 10 mm/hr

Freq Thz	Mean MC	Mean UT	Std.Dev MC	Std.Dev UT
3.	12.838	12.7624	4.63476	4.87904
2.14286	13.1551	13.0778	4.75035	5.19163
1.57812	13.473	13.3939	4.85625	5.33217
1.03555	14.0254	13.9432	5.0383	5.74728
0.625391	14.8253	14.7386	5.31335	6.0626
0.517777	15.1318	15.0432	5.42732	6.25045
0.3	15.9675	15.8733	5.77921	6.43654

Tabela 4.4 – Mean Attenuation and SD using ITU-R, Monte Carlo, and UT for Terahertz Frequencies based on rainfall rate rate data above 10 mm/hr

Freq Thz	Mean MC	Mean UT	Std.Dev MC	Std.Dev UT
1	8.53461	8.5344	2.28378	2.05812
0.9	8.58465	8.58445	2.2799	2.06835
0.8	8.66282	8.66262	2.28529	2.08551
0.7	8.77638	8.77618	2.30256	2.11144
0.6	8.93424	8.93404	2.33481	2.14838
0.5	9.15136	9.15115	2.38772	2.20016
0.4	9.43829	9.43808	2.46654	2.26959
0.3	9.78778	9.78755	2.57338	2.35537

Figure 4.6 shows the smooth PDF for attenuation with rainfall rate greater than 10 mm/h at THz frequencies based on Monte Carlo simulation. The probability versus attenuation in dB/Km is given for some specifics frequencies in the range of THz. For the frequency of 3 THz, we can see that the curve tends to be on the left. Otherwise, for 0.3 THz or 300 GHz the curve tends to be dislocated to the right, which shows that as the frequencies go down in the THz spectrum, the attenuation tends to increase.

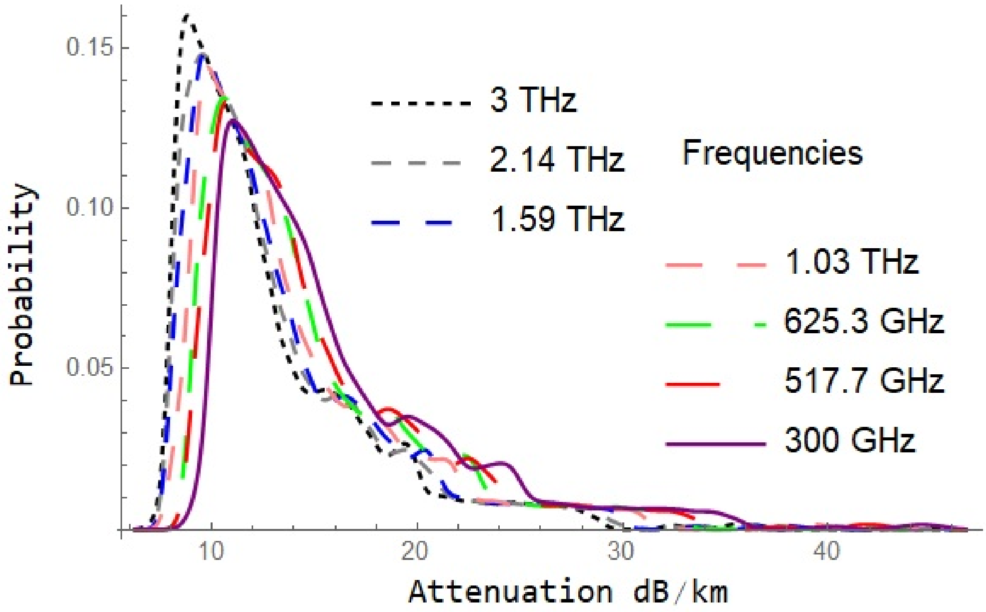


Figure 4.6 – Smooth PDF of attenuation for rainfall rate greater than 10 mm/h at Thz frequencies based on Monte Carlo simulation.

5 CONCLUSION

Electromagnetic attenuation due to rainfall at THz frequency was studied in this thesis. The first part of this work was devoted to giving a theoretical background to the calculus of electromagnetic attenuation in THz spectrum caused by drops of rain. We used two methods to calculate attenuation: The Mie theory together with a Drop Size Distribution, more specifically the Weibull distribution; the second approach was based on an empirical model that was proposed by ITU-R. In both approaches, the attenuation has a strong dependency on the rainfall rate.

That is why, the second part of this work was dedicated to the regime analysis of rainfall in the city of Brasilia, DF, Brazil. By analyzing the rain data throughout 20 years, we could see that the wet season in Brasilia goes basically from September to April. The months of May, June, July, and August historically show low levels of rainfall. Based on the data collected, it was important to analyze the extreme values of rainfall rate, which are rates above 10 mm/hr. The probability of an event of rainfall rate above 10 mm/hr is given in table 3.1. April shows the greatest probability of rainfall rates above 10 mm/hr.

The rainfall data has to be treated statically when used to calculate attenuation. That is why the statistical techniques based on Monte Carlo simulations and Unscented Transform were used to estimate the mean attenuation caused by water drops in electromagnetic waves propagating through space in the THz spectrum. First, in section 4.1 we used the Monte Carlo technique to estimate attenuation for all months inside the wet season in Brasilia. A second approach was used in 4.2 to have an entire picture by looking at the mean attenuation for the whole wet season. We add to the estimation, the Unscented Transform technique to compare with Monte Carlo results for mean attenuation in the wet season.

Finally, the results show that considering the minimum/maximum value of rainfall attenuation using ITU-R, Mie/Weibull, MC, and UT, the mean attenuation along the wet season in a year will be from 1.918 to 2.666 dB/km. For extreme rain events with a rate greater than 10 mm/h, results showed that the mean attenuation varies from 8.534 to 15.967 dB/km. Also, when analyzing the mean attenuation per month during the wet season, we can see that April shows the greatest level of attenuation.

A further step in this work would be to validate the estimations proposed by using experimental data collected using a transmitter/receiver working in THz frequencies throughout the year. This work is fundamental to understanding THz signal propagating phenomena in a rainy environment in the Midwest of Brazil to further outdoor applications.

References

1 Sekine, M.; Lind, G. Rain attenuation 0f centimeter, millimeter and submillimeter radio waves. In: *1982 12th European Microwave Conference*. [S.l.: s.n.], 1982. p. 584–589.

2 Norouzian, F. et al. Rain attenuation at millimeter wave and low-thz frequencies. *IEEE Transactions on Antennas and Propagation*, v. 68, n. 1, p. 421–431, 2020.

3 ULBRICH, C. W. Natural variations in the analytical form of the raindrop size distribution. *Journal of climate and applied meteorology*, v. 22, n. 10, p. 1764–1775, 1983.

4 WORTMANN, F.; FLÜCHTER, K. Internet of things. *Business & Information Systems Engineering*, Springer, v. 57, n. 3, p. 221–224, 2015.

5 GUNN, K.; MARSHALL, J. The distribution with size of aggregate snowflakes. *Journal of Atmospheric Sciences*, v. 15, n. 5, p. 452–461, 1958.

6 MOOLEY, D. A. Gamma distribution probability model for asian summer monsoon monthly rainfall. *Monthly Weather Review*, v. 101, n. 2, p. 160–176, 1973.

7 WILKS, D. S. Rainfall intensity, the weibull distribution, and estimation of daily surface runoff. *Journal of Applied Meteorology and Climatology*, v. 28, n. 1, p. 52–58, 1989.

8 HONG, E. et al. Validation of the mie theory for rain attenuation at 72 and 84 ghz. In: IEEE. *2016 USNC-URSI Radio Science Meeting*. [S.l.], 2016. p. 111–112.

9 ISHII, S. et al. Rain attenuation in the microwave-to-terahertz waveband. *Wireless Engineering and Technology*, Scientific Research Publishing, v. 7, n. 02, p. 59, 2016.

10 LUO, Y.; HUANG, W.-x.; LUO, Z.-y. Attenuation of terahertz transmission through rain. *Optoelectronics Letters*, Springer, v. 8, n. 4, p. 310–313, 2012.

11 HAN, C. et al. *Terahertz Communications (TeraCom): Challenges and Impact on 6G Wireless Systems*. 2019.

12 TAN, J.; DAI, L. *THz Precoding for 6G: Applications, Challenges, Solutions, and Opportunities*. 2020.

13 YANG, P. et al. 6g wireless communications: Vision and potential techniques. *IEEE Network*, IEEE, v. 33, n. 4, p. 70–75, 2019.

14 YOU, X. et al. Towards 6g wireless communication networks: Vision, enabling technologies, and new paradigm shifts. *Science China Information Sciences*, Springer, v. 64, n. 1, p. 1–74, 2021.

15 SERIES, I.-R. P. P. 838-3,". *Specific attenuation model for rain for use in prediction Methods*, 2005.

16 HAMMERSLEY, J. *Monte carlo methods*. [S.l.]: Springer Science & Business Media, 2013.

17 MENEZES, L. R. de et al. Efficient computation of stochastic electromagnetic problems using unscented transforms. *IET Science, Measurement & Technology*, IET, v. 2, n. 2, p. 88–95, 2008.

18 BOHREN, C. F.; HUFFMAN, D. R. *Absorption and scattering of light by small particles*. [S.l.]: John Wiley & Sons, 2008.

19 ISHII, S.; SAYAMA, S.; KAMEI, T. Measurement of rain attenuation in terahertz wave range. Scientific Research Publishing, 2011.

20 OLSEN, R.; ROGERS, D. V.; HODGE, D. The ar b relation in the calculation of rain attenuation. *IEEE Transactions on antennas and propagation*, IEEE, v. 26, n. 2, p. 318–329, 1978.

21 BEST, A. The size distribution of raindrops. *Quarterly Journal of the Royal Meteorological Society*, Wiley Online Library, v. 76, n. 327, p. 16–36, 1950.

22 SEGELSTEIN, D. J. *The complex refractive index of water*. Tese (Doutorado) — University of Missouri–Kansas City, 1981.

23 ZHANG, G.; VIVEKANANDAN, J.; BRANDES, E. A method for estimating rain rate and drop size distribution from polarimetric radar measurements. *IEEE Transactions on Geoscience and Remote Sensing*, IEEE, v. 39, n. 4, p. 830–841, 2001.

24 JAMESON, A.; KOSTINSKI, A. What is a raindrop size distribution? *Bulletin of the American Meteorological Society*, American Meteorological Society, v. 82, n. 6, p. 1169–1178, 2001.

25 BLANCHARD, D. C. Raindrop size-distribution in hawaiian rains. *Journal of Atmospheric Sciences*, v. 10, n. 6, p. 457–473, 1953.

26 MARSHALL, W. P. J. The distribution of raindrops with size. Journal of Meteorology, v. 5, p. 165–166, 1948.

27 THOM, H. C. A note on the gamma distribution. *Monthly weather review*, v. 86, n. 4, p. 117–122, 1958.

28 RAYCHAUDHURI, S. Introduction to monte carlo simulation. In: IEEE. *2008 Winter simulation conference*. [S.l.], 2008. p. 91–100.

29 JULIER, S. J.; UHLMANN, J. K. Consistent debiased method for converting between polar and cartesian coordinate systems. In: SPIE. *Acquisition, Tracking, and Pointing XI*. [S.l.], 1997. v. 3086, p. 110–121.

30 JULIER, S. *Comprehensive process models for high-speed navigation*. Tese (Doutorado), 1997.

31 MORAIS, L.; MENEZES, L.; MORAES, P. Rain attenuation at thz frequencies from historical data collected in brasilia, brazil. In: IEEE. *2021 USNC-URSI Radio Science Meeting (USCN-URSI RSM)*. [S.l.], 2021. p. 045–050.

32 MORAIS, L. V. de; MENEZES, L.; MORAES, P. Estimating brasilia rain attenuation at THz frequencies from historical data based in monte carlo simulation and unscented transform. In: *Anais do XXXIX Simpósio Brasileiro de Telecomunicações e Processamento de Sinais*. Sociedade Brasileira de Telecomunicações, 2021. Disponível em: <https://doi.org/10.14209/sbrt.2021.1570724130>.

33 BABKIN, Y. S. Measurement of attenuation in rain over 1 km path at a wavelength of 0.96 mm. *Rad. Eng. Elec. Phys.*, v. 15, p. 2164–2166, 1970.

APPENDIX

A RESUMO ESTENDIDO EM LÍNGUA PORTUGUESA

Título: Uma estimativa da Atenuação de Ondas Eletromagneticas propagando-se em THz Devido ao Regime de Chuvas Considerando Dados Históricos Coletados em Brasilia,DF
Autor: Lucas V. Morais
Orientador: Leonardo Menezes, Dr.
Programa de Pós-Graduação em Engenharia Elétrica
Brasília, 09 de junho de 2022

Palavras-chave: Distribuição das Gotas de Chuva, Terahertz, Teoria de Mie, Atenuação por Chuvas.

Contextualização

O fluxo de dados via sistemas wireless de comunicação vem crescendo exponencialmente nos últimos anos. A demanda por altas taxas de transmissão vem trazendo novos desafios para pesquisadores em todo o mundo. Dentre as tecnologias em ascensão, está a comunicação de telefones mobile via sexta geração (6G). Para que haja avanço na área, será imprescindível a transmissão em altas frequências, mais precisamente no espectro de Terahertz.

A transmissão de ondas eletromagnéticas no espectro de THz é fundamental para o avanço da tecnologia no desenvolvimento mobile. Nesse contexto faz-se fundamental o estudo de quais componentes naturais podem interferir no fluxo de dados no espectro THz. Um grande limitante da propagação em ambientes externos está relacionado ao espalhamento espectral e alta absorção de pequenas partículas e gases atmosféricos. Uma grande fonte de atenuação do sinal propagando-se em THz é a chuva.

A chuva pode impactar fortemente a comunicação em um canal transmissor em frequência THz. A absorção e o espalhamento espectral podem se tornar um grande inimigo da tecnologia em questão. Baseando-se no efeito negativo da chuva na propagação em altas frequências, essa dissertação de mestrado procura estimar os efeitos do regime de chuvas de Brasília, DF, Brasil na propagação de ondas eletromagnéticas em THz propagando-se em ambiente externo.

Metodologia

Esse estudo baseou-se em métodos estatísticos para projeção de valores relativos a atenuação de ondas eletromagnéticas em dB/km. Primeiramente foi realizado a coleta de dados das taxas de chuva ao longo de vinte anos. Os dados foram coletados em banco de dados do INMET (Instituto Nacional de Meteorologia) na estação de coleta localizada em Brasília, DF, Brasil em latitude de -15:789343, longitude de -47:925756 e altitude de 1160,9 m. A segunda parte desse trabalho foi embasada no tratamento dos dados relativos as taxas de chuvas, primeiramente foi realizado uma análise do total de chuvas em mm e também a média em mm/hr durante os meses de janeiro à dezembro entre os anos de 2001 até 2020. A partir destes dados foi possível traçar a função de distribuição de probabilidade das chuvas em determinada taxa em mm/hr.

Após análise de dados, partimos para o calculo da estimativa da atenuação por chuvas. Para calcular atenuação usamos dois métodos: Weibull/Mie e ITU-R. O método usando a teoria de Mie/Weibull foi escolhido pois trata-se uma maneira puramente teórica que garante uma grande gama de possibilidades de escolhas relativas a frequências e taxas de chuvas. O modelo relacionado a ITU-R é um método baseado no modelo exponencial de atenuação levando em consideração predições empíricas. Para se realizar a determinação estatística da atenuação, faz-se necessário a utilização de métodos estatísticos. Nessa dissertação de mestrado utilizamos dois métodos para se realizar a predição relativa atenuação de ondas eletromagnéticas propagando-se em THz: o método de simulações usando Monte Carlo e o modelo da Transformada de Incertezas (UT). O método de Monte Carlo é baseado na amostragem aleatória massiva com o intuito de se obter resultados numéricos. Monte Carlo Utiliza a aleatoriedade probabilística para resolução de problemas que a priori seriam determinísticos, método que exige uma capacidade computacional relativamente alta. O método da Transformada da Incerteza uma distribuição de probabilidade pode ser mapeada em pontos específicos que podem ser usados como entrada/saída em funções. Esse mapeamento produz resultados similares a Monte Carlo com um nível computacional baixo comparado ao modelo MC.

Resultados

Os resultados obtidos pela análise de dados coletados e tradados no capítulo 3 foram divididos em duas seções: seção 4.1 e 4.2. Na seção 4.1, foi calculado a atenuação média em cada mês do ano baseando-se nos métodos descritos no capítulo 3. Além disso, foi calculado a atenuação média para eventos extremos de chuva (acima de 10 mm/hr). Utilizando o modelo de Weibull/Mie, abril apresentou a maior atenuação média na frequência em THz com média máxima de aproximadamente 3.4 dB/Km para 0.3 THz e média mínima de aproximadamente 2.8 dB/km para 3 THz. Considerando o modelo ITU-R, abril apresentou maior atenuação média com máxima de 2.7 dB/Km e média mínima de aproximadamente 2.3 dB/Km.

Considerando a divisão por meses proposta na seção 4.2 para eventos extremos, novamente abril foi o mês de maior média estimada. A probabilidade de chuvas extremas em abril, considerando o modelo Weibull/Mie, gira em torno de 7,43% com média máxima de 18 dB/Km para a frequência de 0,1 THz e média mínima de 14,5 dB/Km para 3 THz. Para eventos extremos de chuva e considerando o modelo da ITU-R, abril obteve uma média máxima de 10,8 dB/km para frequência de 0.3 THz e uma média mínima de 9,4 dB/Km para 1 THz .

Visando uma estimativa média de proporção anual, na seção 4.2 foi calculado a atenuação média baseando-se nos dados coletados na estação chuvosa de Brasília, entre os meses de setembro e abril. Nessa condição, novamente foi utilizado o modelo Weibull/Mie para o cálculo da estimativa de atenuação média. Para complementar a estimativa foi utilizado a UT para se obter uma comparação com o método de Monte Carlo no que tange aos resultados estatísticos. A atenuação média máxima para a estação chuvosa em Brasília considerando mie /Weibull foi por volta de 2,66 dB/Km para uma frequência de 0.3 THz e uma atenuação mínima de aproximadamente 2,16 dB/Km para frequência de 3 THz. Usando o modelo da ITU-R a média máxima deu-se em 2,23 dB/Km para 0,3 THz e a mínima de 1,91 para 1 THz.

Novamente, foi avaliado o caso para chuvas extremas (acima de 10 mm/hr). A probabilidade de uma chuva extrema ao longo do ano é de 5,41%, considerando-se apenas a estação chuvosa de Brasília. Para esse caso, a atenuação média máxima foi de 15,96 dB/Km para a frequência de 0,1 THz e de 12,84 para 3 THz. Baseado na ITU-R o resultado da atenuação média máxima foi de 9,78 dB/Km para a frequência de 0.3 THz e de 8,56 para frequência de 1 THz.

Conclusão

A atenuação eletromagnética devido a chuvas no espectro em THz foi alvo de estudo nesta dissertação de mestrado. A primeira parte desse trabalho foi dedicada a exposição de todo aparato teórico necessário para o cálculo da atenuação. Foi usado, basicamente dois métodos para o cálculo da atenuação devido a chuvas: A teoria de Mie junto com a distribuição das gotas de chuvas, mais especificamente a distribuição de Weibull; o método empírico descrito pela norma técnica da ITU-R também foi utilizado como forma de calcular a atenuação média. Em ambas as técnicas, a atenuação tem forte influência da taxa das chuvas medidas em mm/hr.

Consequentemente, a segunda parte desse trabalho foi dedicada ao estudo do regime de chuvas na cidade de Brasília, DF, Brasil. Analisando os dados dos últimos vinte anos, foi possível afirmar que o período de chuvas na região de Brasília vai de setembro até abril. Os meses de maio, junho, julho e agosto historicamente apresentam um baixo índice de precipitação. A partir da análise desses dados, observou-se a importância da quantificação de eventos extremos (chuvas com taxas acima de 10 mm/hr). A probabilidade de um evento de chuva com taxas acima de 10 mm/hr é explicitado na tabela 3.1.

Os dados referentes as taxas de chuvas presentes no território do distrito federal precisam ser tratadas de forma estatística quando usados para o cálculo da atenuação média de ondas eletromagnéticas propagando-se em ambientes externos no espectro de THz. Consequentemente, nesse trabalho foi usado a técnica estatística baseada em simulações de Monte Carlo e também a Transformada da Incerteza para se estimar a atenuação média na região. Na seção 4.1 foi usado a simulação de Monte Carlo para a estimativa de atenuação em todos os meses que compõe a estação chuvosa. Uma segunda abordagem usada na seção 4.2 foi usada para se obter uma estimativa média anual de atenuação voltando-se para a estação chuvosa como um todo. Para essa segunda abordagem usou-se também da técnica da Transformada da Incerteza para estimativa de ordem comparativa com a técnica de Monte Carlo.

Uma sequência lógica desse trabalho seria a validação dos dados aqui descritos por um experimento que contasse com um transmissor/receptor funcionando nas frequências aqui descrita em THz para análise dos dados ao longo do ano. Esse trabalho faz-se fundamental para um primeiro contato com sinais propagando-se em THz em um território chuvoso do Distrito Federal com objetivo de aplicações futuras nessa faixa de frequência.