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Essays on Sectoral Interconnections, Technical Change and Economic Growth

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Essays on Sectoral Interconnections, Technical Change and Economic Growth

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ABSTRACT

This thesis is composed by three chapters organized as three independent articles interconnected by a common line. Although interconnected by a common thread, which stands by the endeavour to investigate and understand sectoral interrelations, productivity and economic growth, each chapter focuses on distinct points. Our inquiry in the first chapter offers an analysis of 'stagnationist' unbalanced growth through making use of a Structural Economic Dynamic (SED) approach. What we intend to establish are some of the advantages associated with treating William Baumol's unbalanced growth model as a particular case of Luigi Pasinetti's framework. One such advantage involves challenging assumptions behind the model by scrutinising the conditions under which Baumol's result is valid. Another advantage consists of extending the analysis to an arbitrary number of sectors to consider quasi-proportional growth and full dynamics – cases considered by Pasinetti. As suggested by Nicholas Oulton and drawing from the SED framework, we then expand the model to consider intermediate inputs, through using the concept of vertical integration. This leads us to confirm the remark advanced by Oulton; namely, that in the presence of intermediate goods, the aggregate growth rate of productivity might not slow down sufficiently to converge towards the lower productivity growth sector, as advocated by Baumol. In sum, the point we seek to establish is that the 'stagnationist' outcome depends on an intricate relation between supply and demand. The multi-sectoral approach that considers intermediate inputs suggests that a disaggregated analysis of well-established results can indeed offer novel insights. The second chapter, in its turn, it is used the Domar aggregation approach to study the evolution of productivity growth in Brazil from 2000 to 2014, thus allowing us a disaggregated assessment of the issue. We found that the overall performance of the Brazilian economy can be explained not only by the poor performance of its sectors but also in terms of diminishing industrial density, with fewer backward and forward connections amongst industries in terms of chains of intermediate inputs. Besides, despite the relatively high density of the manufacturing sector, it performed a negative role concerning aggregate productivity growth both directly and indirectly. Directly insofar as that sector had negatives productivity growths during the period under consideration, and indirectly due to its high interconnection, which spread negative rather than positive productivity gains across the economy. Therefore, to improve the poor performance of the Brazilian economy, it is mandatory to restore the capability of the manufacturing sector of yielding and spreading productivity gains. Finally, in the third and last chaper, we start from the Pasinetti's (1988) structural change model of vertically integrated sectors with uneven growth, that has Sraffian inspiration, but allows the sectors to grow at different rates. The conventional hypothesis that the labour coefficients are fixed is modified, in such a way that the technical change from the industries affects the productivity growth of the vertically integrated sectors in an explicit and formal way. This represents a formal derivation that takes into account the criticisms of authors such as Schefold (1987), Lavoie (1997) and Hagemann (2007). They argue that the productivity of vertically integrated sectors should not be seen as being independent from one another, since productivity growth takes place from the industries level. In this vein, we deliver a formal measure of sectoral productivity growth, in which the labour productivity growth that accrues from the industries affects the vertically integrated sectors composed by them. This measure of productivity growth can be also considered as a substitute for the inverse of the maximum eigenvalue of the Sraffian system, which is no longer valid in the case of uneven growth among sectors. In addition, an equation for the price growth rate of vertically integrated sectors is derived, in such a way that prices rise inversely in proportion to technical progress.

Keywords: intermediate inputs, structural change, unbalanced growth, vertical integration.

***JEL* Classification Codes**: E12; E13; O41

INTRODUCTION

The main contribution of this thesis is presented in three chapters, organized as three independent papers. Although interconnected by a common thread, which stands by the endeavour to investigate and understand sectoral interrelations, productivity and economic growth, each chapter focuses on distinct points.

The ultimate goal of the demand of goods and services is to deliver well-being to those who acquire them, whether consumption destined to the most basic human needs and even to the most sophisticated or superfluous consumption. In order to yield goods and services, the modern economic system requires inputs of the most varied types, ranging from a countless variety of produced inputs to human labour.

In this vein, the evolution of the production technique of such goods and services towards efficiency gains in the use of inputs is essential to deliver increasing goods and services with lower prices. The evolution of productivity in the production process is therefore essential to make the best use of the available resources in the economy. Furthermore, not only the productivity of direct labour employed in different industries is crucial to the evolution of the technical progress, but also the produced inputs, that can be produced with variable efficiency.

To fully understand the importance of sectoral interconnections to productivity growth is essential to bear in mind that industries do not only benefit from a productivity increase in its own production process, but also from increased productivity in other sectors from which it acquires inputs, and this also generates impacts for aggregate productivity. Intermediate inputs are produced goods that link industrial sectors and have a unique role in spreading productivity growth amongst sectors [see, e.g. Aulin-Ahmavaara (1999)]. According to Jones (2011), intermediate inputs provide links between industries that create a multiplier. He argues that high productivity in an industry requires a high level of performance along many dimensions. The author proposes that linkages are a crucial part of the explanation by delivering a noteworthy example:

> *"(...) intermediate goods provide links between sectors that create a multiplier. Low productivity in electric power generation - for example, because of theft, inferior technology, or misallocation - makes electricity more costly,*

which reduces output in banking and construction. But this in turn makes it harder to finance and build new dams and therefore further hinders electric power generation." Jones (2011, p. 1-2)

The device of vertical integration as advanced by Pasinetti (1981) building on the Sraffas initial insight enables us to realize the necessary interconnections among industries, and thus all the required direct and indirect chains of primary inputs that generate the sub-systems. Pasinetti's (1993) SED approach investigates the economic system as truly multisectoral and delivers a structural model which is a valuable tool to investigate structural change due to its high quality and adaptability of hypotheses.

The fruitful investigation advanced by Baumol (1967), which access structural change within a two sectors model, gave rise to an extensive branch of research concerning long-run productivity (and economic) growth with its foundations on patterns of structural change for both manufacturing and services sectors. Baumol assumed that whilst the progressive (manufacturing) sector was the one with persistent productivity improvement, the stagnant (services) sector would be condemned to grips with low or even null productivity growth.

The structural difference concerning the technical progress of each segment is due to the nature of labour used up in each of them. While in the progressive sector, labour is only a mean, or a tool, of obtaining the final product, in the stagnant sector, labour is the product itself. Moreover, the progressive sector would be exposed to several technical improvements, as opposed to the stagnant one. The overall consequences of Baumol's model are the following. If one assumes that no sector vanishes in the long run, as times go up more and more labour goes towards the stagnant sector, and the relative price of the progressive sector tends to decrease relative to that of the stagnant one. As a result, the stagnant segment increasingly absorbs the average outlays relatively to the progressive sector and the overall economic growth leans toward stagnation.

In the first chapter, therefore, Baumol's two-sector model result is analyzed upon a Structural Economic Dynamics (SED) based on Pasinetti's (1993) approach seeking to generalize the result previously obtained by Notarangelo (1999). Therefore, it is shown that there are some advantages associated with treating Baumol's unbalanced growth model as a particular case of Pasinetti's framework. In doing so, and mainly treating the economy as truly disaggregated, it is possible to scrutinize the conditions under which Baumol's result is valid and challenging the assumptions behind the model.

Hence, Baumol's result is explored under Pasinetti's quasi-proportional growth and full dynamics cases. It is also considered a case where the services sector yields intermediate inputs to the goods sector, using the device of vertically integrated sectors and following Oulton's (2001) investigation. In that case, both Baumol's and Oulton's results could be relativized. In short, the point we seek to establish is that the 'stagnationist' outcome depends on an intricate relation between supply and demand.

The second chapter's is motivated by the fact that despite the Brazilian economy had been one of the fastest growing economy in word between the 1930s and 1980s. This phenomenahas converted the Brazilian landscape from a vast rural and backward country to an urban and somehow industrialized one, it looks like afterwards it lost its way. According to Nassif et al. (2020) after an initial and consistent period of productivity growth during the second half of the last century, the productivity of the Brazilian economy remained stagnant since the eighties and, therefore, it seems that Brazil has been stuck in the middle-income trap.

Indeed, the second chapter aims to evaluate the sectoral and aggregate Brazilian productivity advance, between 2000 and 2014, using 48, 10 and 3 levels of aggregation. Furthermore, the contribution which each of these sectors has made to the overall growth of productivity is thereby accessed, using the information provided by a series of input-output tables. But not only the direct effects of productivity change are accessed, but also the indirect impacts of technical advance, where increasing productivity of industries can affect some of the others through the provision of cheaper inputs to the economic system [see Aulin-Ahmavaara (1999)].

To perform that task, it is used the Domar aggregation method to aggregate sectoral (or industrial) productivity advance, which allows assessing the twofold impact of technical change on the efficiency of producing each output. That twofold effect occurs since productivity advance in a given industry impacts overall productivity growth squarely, but also indirectly, by changing the efficiency in which it is produced the output sold to other sectors as intermediate inputs.

That is, each industry's technical change helps to lower (or increase) circulating capital costs spent by all downstream sectors. Or, putting it oppositely, the productivity advance of a given industry is not limited for its own direct productivity change, but also on productivity gains/losses attained in the production of all intermediate inputs produced by upstream sectors. In sum, the methodology captures both direct and indirect effects of industrial productivity change, and the denser the interconnections among sectors, the greater the indirect channel's potential impact of technical advance.

A glimpse of the found results is that during the period under consideration the Brazilian economy's density, which stands by the degree of interconnection of its industries, decreased, especially after 2008, lowering its productive complexity chain and thus sinking the potential indirect effect of productivity spill.

Furthermore, Brazilian's productivity environment was harmed during the given period because while the macro sector with higher potential to spread productivity advance – the manufacturing sector – suffered due to several years of negative productivity change. Besides, the sector with higher productivity advance – the primary industries sector – is the sector with lowest interconnections and therefore a small capacity to generate productivity spillovers. The highly heterogeneous services macro sector, in its turn, is a macro sector that, in aggregate, delivered a relevant capacity of propagating technical change. Still, it carried out a timid productivity advance, and thus it was not capable of boosting aggregate productivity growth.

As an aggregate, however, the Brazilian economy showed a relevant productivity advance until 2008, of almost 15% accumulated. Still, after that its cumulative productivity advance started to shrink and it nearly lost all of its advances in 2014, which is the last year of the period under analysis.

The third and ultimate chapter delivers an investigation concerning vertically integrated sectors. The device of vertically integrated sectors allows one to focus on distinct aspects in comparison with standard input-output models, and to perceive the economic system in terms of the final commodities produced, that are the final purpose of production and social welfare.

Pasinetti (1988) relativizes some usual assumptions and delivers an economic system with growing subsystems, allowing them and their demand to grow at an uneven pace. Despite the Pasinettian efforts to generalize the result toward the case in which sectors grow at particular or uneven rates, which are given by the sum of the populational growth and the particular growth rate of per capita demand, he just managed to provide a sector outlook. His analysis also does not deal with technical progress. Indeed, at the end of Pasinetti's (1988) work, he hints of further developments of his model in the case of technical progress.

Therefore, we build an alternative approach of vertically (growing) hyper-integrated sectors with technical progress. Several criticisms have been made concerning the treatment of technical progress within vertically integrated sub-systems. For instance, Lavoie (1997) argues that technical progress in vertically integrated sectors is often supposed exogenous and independent among sectors. Still, it indeed takes place at the industry level and therefore cannot be thought as being independent of each other. A fruitful discussion about this subject is also made by Hagemann (2012).

We aim to deliver an approach of treating labour productivity that takes place at the industry's level and that are transmitted to the subsystems. Thereby we build a measure of both sectoral and aggregate technical change growth rate that explicitly takes place at the industry level and sectoral and aggregate estimates of economic growth that takes into account the contribution of the productivity advance. It is shown explicitly that the technical change of the subsystems accrues from the direct and indirect labour used up in its production, which follows the interconnection among the economic system and therefore the productivity transmission made possible by it.

In this ultimate chapter, we also explore the duality of the technical change process. The impact of the technical change in each sub-system directly impacts the growth rate of prices which is a well-known result which is explicitly explored there. Following our assumptions of the industry's profit rate, the impact of the sectoral technical change in diminishing prices is the higher the fewer the share of profit of each vertically integrated sector.

Finally, we investigate the differences and behaviours of three of the most important measures of productivity growth regarding input-output systems, namely the neoclassical multifactor productivity, the effective productivity advanced by Cas and Rymes (1991) and an estimate of productivity growth from vertically integrated sectors. We show that the last two deal with technical change, taking into account the interconnections and transmission of productivity among industries in shaping the productivity of sectors producing final commodities. The multifactor productivity growth measure, however, does not treat circulating capital as produced and misses productivity transmission among industries, at least at the industrial level.

With this analysis, we emphasize the role of a multisectoral approach to economic and productivity growth and notably the transmition of productivity advance among sectors and industries due to sectoral interconnections and circulating capital. Taking the device of three independent chapters in this thesis it was possible to seek to deepen three particular aspects, focusing on both empirical and theoretical subjects, though with a common underlying subject.

Chapter One: A Structural Economic Dynamics Approach to 'Stagnationist' Unbalanced Growth

1.1. Introduction

> 'The valuable approach that is referred to as "structural dynamics" is clearly a major contribution of Luigi Pasinetti (…). I have encountered no formal definition of the analysis, but I take it to mean that it is one whose approach brings out the pertinent relationships explicitly and in a way that makes it possible to discern clearly the implications for policy and for the prospects for development. It is a very indeed fruitful development, one that has usefully been taken in Keynesian directions, but whose spirit can also take us far along other significant avenues' (Baumol, 2012, p. 125).

Seminal contributions advanced by the Italian economist Luigi Pasinetti (1981, 1993) and the American economist William Baumol (1967) laid the foundation for a burgeoning literature dealing with some of the effects of structural change on economic growth. While these two economists focused on the same issues related to structural change and economic growth, their approaches remained distinct. While also dealing with unbalanced growth, Baumol's research reflects the Neoclassical tradition in Economic Science, and his model fits in with the traditions advanced by Solow (1956), Uzawa (1961) and Frankel (1962). For Baumol, the notion of steady growth places particular emphasis upon the supply side while neglecting the importance of demand. One of Baumol's underlying assumptions is that the dynamics associated with technological change can indeed lead to unbalanced growth.

Through neglecting the potential roles played by demand in his model, Baumol's approach contrasts with the Structural Economic Dynamics (hereafter SED[1]) framework advanced by Pasinetti (1981, 1993). In the SED approach, growth is not only supply-

[1] The SED approach is outcome of contribution of many authors that following the Pasinettian seminal contribution investigated and extended the model in a number of directions [see. e.g. Baranzini and Scazzieri (1990) and Landesmann and Scazzieri (1996)]. For an extensive survey focused upon the SED framework, please see Silva and Teixeira (2008).

constrained but is also a demand-driven process in which the saturation of demand for particular consumption goods plays a central role. This involves not only shaping the structure of the economy but also determining the pattern for growth. Accordingly, in Pasinetti's theory of consumption, an income-driven rule of non-proportional expansion of demand delivers unbalanced growth as the outcome of income, and relative-price effects are considered simultaneously. This emphasises the many channels whereby demand plays a decisive role in the growth process, with supply adapting within certain limits. According to this SED approach, interactions between supply and demand within a multi-sectoral model give rise to the particular dynamics of sectoral output, as well as prices and the structural transformation of economies at different stages of development.

Notarangelo (1999) attempted to combine Pasinetti's and Baumol's analyses into a single framework. Reformulating the Baumol model and highlighting its similarities with the Pasinettian pure labour schema, Notarangelo's research suggests that the former may be a particular case of the latter when considering a more inclusive role given to the demand side. However, Notarangelo's analysis remains limited to a two-sector framework, as she focuses mainly on bringing the demand side into Baumol's model under proportional dynamics. However, as authors, we think that adequately considering unbalanced growth requires a framework more accurately accounting for the role of preferences and not leaving preferences as homothetic. The first part of our contribution involves extending Notarangelo's analysis in order to consider what happens to the Baumol model under quasi-proportional growth and full dynamics. These were also recognised by Pasinetti (1981, 1993).

Although the process of long-run economic growth might seem stable at higher levels of aggregation, saturation is undeniable for goods with particular elasticities of demand, and we associate this with Engel's law. To fully grasp some of the effects of demand on growth, we introduce a multisector model[2], assigning to each sector a particular growth rate of demand as well as rates of productivity. To understand this accomplishment, the reader should note that the assumption that the consumption ratio of services to manufacturing remains constant is crucial for deriving some of Baumol's essential results. Although there is some empirical support for this hypothesis, there is no

[2] The closing statement of Notarangelo's (1999, p. 222) paper is 'Finally, it would be of interest to extend the results of the present paper to a general multisectoral model.'

consensus because some studies find results suggesting the opposite is taking place. This is considered in some detail in the following section. The introduction of intermediate goods also challenges Baumol's conclusion.

Oulton's (2001) research suggests that the stagnation outcome is valid only when all sectors under consideration produce final products. In the presence of an intermediate service sector, the shift of resources to this tertiary sector might well enhance rather than decrease aggregate productivity growth. To demonstrate this result, Oulton relies upon the research of Domar (1961) that sought to explain that in an economy with intermediate goods, the sum of the sector's share in national income exceeds unity. If this is the case, the productivity growth converges towards the sum of the productivity growth of the final and the intermediate goods sectors. This is related to Domar's aggregation. The rationale is that the productivity growth of an industry that delivers inputs to other sectors generates a twofold effect: it increases not only the productivity growth of the industry but also affects the productive capacity of the final goods sector that purchases goods as an intermediate input. Research advanced by Jorgenson (2018, p. 881) and Hulten (1979) confirmed this result, which came to be known in the literature as 'Domar aggregation'.

Although Oulton's result poses another challenge to Baumol's efforts, his extension completely overlooks the demand side[3] through considering the existence of only one final good. The present inquiry seeks to fill this gap in the literature by extending Oulton's results to economies with an arbitrary number of sectors, thus allowing us to consider a more inclusive and prominent role for demand. To accomplish this task within the SED approach, we advance a derivation of the 'Domar aggregation' that precludes the use of a production function but relies upon the concept of vertical integration [see Pasinetti (1973)]. Following this route, we take Oulton's analysis one step further by showing that a multisector framework with intermediate goods could indeed affect[4] Baumol's results.

[3] This shortcoming was addressed by Hiroaki Sasaki (2007, p. 439).

[4] The fact that considering intermediate goods may affect established results is somewhat widespread. Blecker and Ibarra (2013) and Araujo et al. (2019) for instance have considered the effects of intermediate inputs within a balance-of-payments constrained growth model.

With this extended framework, Oulton's remark appears to hold. Namely, when dealing with intermediate goods, the productivity of the economy might not necessarily converge down towards the lower productivity sectors. That would happen, for instance, if the intermediate industry of a vertically integrated segment generates lower productivity growth amongst all branches. Still, the corresponding final goods industry displays relatively higher productivity. When this proves the case, the productivity growth of the economy converges towards one of these vertically integrated sectors, which is not necessarily the sector with lower productivity growth. This suggests that a 'stagnationist' result might not hold. Then, the approach presented with this inquiry provides a disaggregated perspective on unbalanced growth to the extent that it generalises and extends existing results. Our approach also yields a framework in which both supply and demand are treated together with multiple sectors.

Our analysis unfolds as follows. The subsequent section presents a review of the literature that focuses on relevant stylised facts supporting the theoretical analysis. Section 3 follows Notarangelo's approach to Baumol as a particular case of Pasinetti by considering quasi-proportional growth and full dynamics. Section 4 extends these results to an arbitrary number of sectors. Section 5 shows that the introduction of intermediate goods changes the main conclusion of Baumol. But with vertically integrated sectors, we can still challenge Oulton's result. Section 6 offers concluding remarks.

1.2. Literature Review and Stylised Facts

As formalised in the next section, Baumol (1967) builds up his model using two final sectors. He assigns to the industrial sector the role of being the progressive one concerning productivity growth while the service sector remains stagnant. He then considers two main cases. In the first one, the ratio of outlays of the two segments is constant over time. This means that the output ratio of the stagnant and the progressive sectors tends towards zero. In the second and most important case, no sector vanishes in the long run as long as real outputs (or volume shares) of both stagnant and progressive industries grow at the same rate. There is, however, an unbalanced labour allocation in which the stagnant sector tends to absorb all of the economy's labour. As a consequence, the productivity of the whole economy tends to stagnate. This outcome came to be known in the literature as the 'Baumol cost disease'.

The critical assumption of real output shares (expressed in constant prices) as roughly constant seems to be corroborated by most data when considering the two or three macro sectors. For example, Baumol (2001) found a slight decrease in the share of services when expressed in constant prices associated with an increase in real GDP per capita. However, Herrendorf et al. (2014) observed a slightly upward trend for real value-added in services and a slight downward trend for real value-added both in manufacturing and agriculture, a topic that concerns real income in most developed countries. In sum, those findings are supportive of the evidence that the ratio of the outputs tends to remain roughly constant with two or three macro sectors in the long run, reinforcing the relevance of Baumol's second case. But this view is disputed when we consider higher levels of disaggregation.

Research advanced by Appelbaum and Schettkat (1999) has shown restrictive conditions in which real output shares remain constant but with changing relative prices within multiple sectors. Their findings suggest that such proportionality proves difficult to maintain indefinitely. Still, it seems to be even more difficult if we think in terms of a genuinely multi-sectoral framework. Considering sixty-seven sectors within the U.S. economy, the research of Nordhaus (2008) found that real sectoral outputs were not constant insofar as each industry grows at a particular rate even in real terms[5]. Baumol (2001) recognises that both services and manufacturing industries also display a wide range of patterns concerning rates of productivity growth and growth rates in aggregate output. Yet, there are more subtle and compelling reasons for considering further effects of demand on unbalanced growth than those analysed in his model. This will be presented in the next sections.

In summary, Baumol and Pasinetti present robust analyses consistent with the level of disaggregation employed. Baumol's approach focuses on changing the shares of the two broadly defined sectors (manufacturing and services), and that approach allowed him to explain some important facts. For instance, Sasaki (2007) and Herrendorf et al. (2014) confirmed the trend of labour migrating from the progressive (manufacturing) sector to the stagnant (services) segments. Moreover, Appelbaum and Schettkat (1999)

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[5] Nordhaus (2008) also concluded that the technologically stagnant sectors have rising relative prices and declining relative real outputs.

as well as Nordhaus[6] (2008) found that the labour migration towards the service sector was related to lower growth rates of productivity. These authors also show that prices in the service sector have increased compared with the rest of the economy. Nonetheless, as Oulton (2001) points out, it is not clear that there is a stagnation trend due to reallocating labour. Indeed, Baumol (2001, 2012) agreed with Oulton's point, especially regarding services as intermediate inputs.

Then, there is a more in-depth unbalanced growth that encompasses changes in the structure of production and employment within and between all industries of the economy. In this vein, Pasinetti provides a more overarching perspective of structural change when he treats the economy as disaggregated into multiple sectors insofar as a more complex set of the growth rate of sectoral output arises. Author Ulrich Witt (2001) emphasises the importance of this kind of analysis by considering that saturation serves as an essential property of demand, with different sectors growing at particular rates depending on their income elasticity of demand. The share of income spent on any specific consumption good is never constant as personal income increases but tends to reach saturation.

Non-homothetic preferences yield results that contrast with those obtained by the standard growth models[7]. One of the main differences is that in the presence of homothetic tastes, multi-sectoral models expand uniformly in all sectors. This is equivalent to the growth pattern in a one-sector model. The SED approach avoids that insofar as Pasinetti's model connects exogenous technological progress to increases in real per capita income, which translates into higher and uneven consumption of final goods. Those goods with a higher income elasticity of demand receive more top shares of consumer expenditures, and this process gives rise to structural changes. In the next section, the relevance of this analysis will be highlighted.

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[6] As a result, he concludes that the employment shift towards services lowers real GDP growth. In contrast, Maroto-Sanchez and Cuadrado-Roura (2009) investigating 37 OECD countries during the period 1980–2005, concludes that the employment shift toward services has a positive effect on the per capita real GDP growth.

[7] According to Syrquin (2012, p. 72), "Once we abandon the fictional world of homothetic preferences, neutral productivity growth with no systematic sectoral effects, perfect mobility, and markets that adjust instantaneously, Structural change emerges as a central feature of the process of development and an essential element in accounting for the rate and pattern of growth."

1.3. Baumol's Analysis Within a Pasinettian Setup

In this section, we shall present a two-sector version of the SED framework focusing on a pure labour economy, with only two final goods. Taking this approach, the model has a structure similar to Baumol's (1967) two-sector model as advanced by Notarangelo (1999), who was the first to emphasise this kind of connection between the two approaches. Accordingly, we can consider that sector 1 is the service (stagnant) sector, and sector 2 is the manufacturing (progressive) sector. These two sectors each produce output using labour as the only input, which is provided by the household sector. Accordingly, we can write the equilibrium in the physical quantity system as:

$$\begin{cases} Q_1 - c_1 L = 0 \\ Q_2 - c_2 L = 0 \\ L - \sum_{i=1}^{2} l_i Q_i = 0 \end{cases} \tag{1}$$

where $Q_i(t)$ stands for the production of the i-th sector, i = 1, 2 and $L(t)$ is the population, assumed to be equal to the labour force. The coefficients $c_i(t)$ and $l_i(t)$ stand for the per capita demand coefficient and the technical coefficient for each of the i-th sectors, i = 1, 2. The first two equations of the system (1) stand for the equilibrium in both segments in physical terms. The third equation refers to the balance in the labour market. We can write the equilibrium in the price system as follows:

$$\begin{cases} p_1 Q_1 - L_1 w = 0 \\ p_2 Q_2 - L_2 w = 0 \\ w - \sum_{i=1}^{2} c_i p_i = 0 \end{cases} \tag{2}$$

where $p_i(t)$ is the price of the i-th final good. As pointed out by Notarangelo (1999, p. 211), both Pasinetti and Baumol consider homogeneous wage, namely w, in the two sectors, albeit due to different reasons. One of the differences between the Pasinetti and Baumol approaches is that the latter assumes the economy is always in equilibrium, which entails a neoclassical view of the growth process. The former considers the possibility of disequilibrium both in the goods and in the labour markets. With this approach, Pasinetti aims at showing that the structure of production varies when income increases as long as technological change endows consumers with the possibility of acquiring not only more significant amounts of goods but also new and better products. According to him, we should observe a macroeconomic condition. The fulfilment of such a condition ensures the equilibrium both in quantity and the price systems, namely[8]:

$$\sum_{i=1}^{2} l_i c_i = 1 \qquad (3)$$

Pasinetti (1981, 1993) refers to equation (3) as the full effective demand condition, which allows the economy to reproduce itself with full employment and full expenditure of national income. Then, the full equilibrium requires not only the fulfilment of the sectoral equilibrium conditions but also consideration of a macroeconomic condition that is related to the coordination of the goods and labour markets. In his words: “Yet, the ‘natural’ economic system requires both the recomposition of sectoral imbalances and the achievement and maintenance of full employment in the economic system as a whole.” [Pasinetti (2007, p. 286)].

Thus, if (3) is satisfied, all produced goods are consumed with the full expenditure of income and full employment. The terms $l_i c_i$ represent either the share of the *i*-th sector in the national income or the labour share of the *i*-th sector in total labour. In what follows, we consider that: $s_i = l_i c_i$, where s_i denotes the share of the *i*-th sector in national income. Within such a framework, the shares of each sector in the national income and the labour force are equal, namely:

$$s_i = \frac{p_i Q_i}{\sum_{i=1}^{2} p_i Q_i} = \frac{L_i}{\sum_{i=1}^{2} L_i} = l_i c_i \qquad (4)$$

If fulfilled, condition (3) yields the following solutions for the physical and price systems:

$$Q_i = c_i L, \; i = 1,2 \qquad (5)$$

$$p_i = l_i w, \; i = 1,2 \qquad (6)$$

Equation (5) advances the solution for the physical quantities for each sector and shows that goods production relies exclusively on demand since they are proportional to consumption coefficients. In turn, (6) presents the solution for the price system, showing that prices are directly proportional to the amount of labour required for production, which is consistent with Baumol’s view of prices as equivalent to the cost per unit of

[8] Pasinetti (1993) noted that both systems (1) and (2) are linear and homogeneous. Thus, to yield a non-trivial solution (quantities and prices different from zero), the determinant of the coefficient matrix has to be equal to zero, which is nothing but expression (3).

production [see Notarangelo (1999, p. 211) and Baumol (1967)]. Let us assume, as in Pasinetti (1993), that the labour coefficients follow an exponential rule where ρ_i is the rate of productivity growth:

$$l_{it} = l_{i(0)} e^{-\rho_i t} \quad (7)$$

From the definition of sectoral labour productivity, namely $q_i = \frac{Q_i}{L_i}$, we define the labour productivity growth in the *i*-th sector as $\hat{q}_i = \rho_i$. Assuming that the demand coefficients follow exponential rules and that the growth rate of variation of demand is denoted[9] by r_i, it yields:

$$c_{it} = c_{i(0)} e^{r_i t} \quad (8)$$

If we substitute equation (8) into equation (5), we obtain the demand for the output of the *i*-th sector as:

$$Q_{it} = c_{i(0)} e^{r_i t} L \quad (9)$$

When dealing with the macroeconomic equilibrium concept, Pasinetti aimed at demonstrating how difficult it is to maintain full employment since there is not a once-for-all fulfilment of such an expression. Even if it holds at the beginning of the analysis, it will not be necessarily satisfied in the following periods. In general, the interaction of the evolution of labour productivity, on the one hand, and of per capita per demand on the other is responsible for the non-fulfilment of condition (3) over time. This gives rise to structural dynamics in which the most probable outcome is $\sum_{i=1}^{2} c_i(0) l_i(0) \exp{(r_i - \rho_i)t} < 1$, meaning that structural unemployment prevails in the long run. This is a central result of the Pasinettian analysis and challenges the neoclassical view of full employment. By considering the dynamic path of demand and labour coefficients, given by equations (7) and (8), the share of the *i*-th sector, namely $s_i(t) = l_i(t) c_i(t)$, has the following dynamics:

$$s_i(t) = c_i(0) l_i(0) \exp{(r_i - \rho_i)t} \quad (10)$$

[9] Note that r_i denotes productivity growth in Baumol while in Pasinetti (1993) it stands for the sectoral growth rate of demand.

which yield: $\hat{s}_i = r_i - \rho_i$. From the definition of the labour coefficient $l_i = \frac{L_i}{Q_i}$, we can write the production of each sector as: $Q_i = \frac{L_i}{l_i}$. By introducing (7) in this equation, one obtains:

$$Q_{it} = a_i L_{it} e^{\rho_i t} \tag{11}$$

where $a_i = \frac{1}{l_{i(0)}}$. Note that the equilibrium between the sectoral demand and supply requires the equalisation of (9) and (11), which yields:

$$L_{it} = L_{i0} e^{(r_i - \rho_i)t} \tag{12}$$

Equation (12) shows that the dynamics of employment in the *i*-th sector depend on two counteracting influences. While labour-saving productivity reduces the sectoral employment level, the increase in demand plays a positive role. The outcome depends on which of these effects is stronger. Baumol (1967) considers that $\hat{q}_1 = \rho_1 = 0$ and $\hat{q}_2 = \rho_2 > 0$. Here, we assume that ρ_1 can be different from zero but with $\rho_1 < \rho_2$. In this setup, the aggregate productivity growth is a weighted average in the two productivity growth sectors, where the weights are the employment shares of sectors 1 and 2, namely:

$$\rho = \sum_{i=1}^{2} s_i \rho_i \tag{13}$$

Besides, the wage grows according to the following exponential rule:

$$w_t = w_0 e^{\rho t} \tag{14}$$

By inserting (7) and (14) into equation (6), and considering that $p_{i0} = l_{i0} w_0$, we obtain after some algebraic manipulation that the dynamics of prices for sectors 1 and 2 are given respectively by:

$$p_{1t} = p_{10} e^{s_2(\rho_2 - \rho_1)t} \tag{15}$$

$$p_{2t} = p_{20}(0) e^{s_1(\rho_1 - \rho_2)t} \tag{16}$$

By dividing p_{1t} by p_{2t}, one obtains:

$$\frac{p_{1t}}{p_{2t}} = \frac{a_2}{a_1} e^{(\rho_2 - \rho_1)t} \tag{17}$$

As we are assuming that $\rho_1 < \rho_2$, taking limits when *t* tends to infinity allows us to conclude that:

$$\lim_{t \to \infty} \frac{p_{1t}}{p_{2t}} = \infty \tag{18}$$

Equation (18) illustrates what Baumol called the 'cost disease'. The price or the cost of the sector with low productivity (service) tends towards infinity when compared with the cost of the high-productivity (manufacturing) sector. Note that we obtained this result without any assumptions regarding the demand paths of the sectors 1 and 2. From equation (14), we know that wages grow at ρ. Assuming full employment, this percentage increase in income stimulates growth in income demand for services output at the percentage rate r_1, whereas the demand for the manufacturing sector grows at r_2. If we further consider that all outputs must rise at ρ, then we have the following results:

i) The service share decreases if $r_1 < r_2$

ii) The service share remains if $r_1 = r_2$

iii) The service share rises if $r_1 > r_2$

The first case was considered by Baumol when he assumed that the ratio of outlays on the two commodities is constant over time, namely:

$$\frac{p_{1t}\, Q_{1t}}{p_{2t} Q_{2t}} = A \tag{19}$$

where A is constant. It is easy to show that this hypothesis is equivalent to considering that:

$$\frac{L_{1t}}{L_{2t}} = A \tag{20}$$

By inserting equation (12) into (20), we obtain:

$$\frac{L_{10} e^{(r_1 - \rho_1)t}}{L_{20} e^{(r_2 - \rho_2)t}} = A \tag{21}$$

Then, a requirement for equation (21) to hold is that $r_2 - \rho_2 = r_1 - \rho_1$. As we are assuming that $\rho_1 < \rho_2$, then necessarily $r_1 < r_2$. This means that the elasticity of demand for the second sector has to be higher than that for the first sector, which corresponds to case i) in which the service share decreases. We can verify this fact from both the supply and demand sides. From the supply side, the ratio of sector 1 production to sector 2 production is:

$$\frac{Q_{1t}}{Q_{2t}} = A \frac{a_1}{a_2} e^{-(\rho_2 - \rho_1)t} \tag{22}$$

From the demand side, this ratio is given by:

$$\frac{Q_{1t}}{Q_{2t}} = \frac{c_1}{c_2} e^{-(r_2 - r_1)t} \tag{23}$$

Either considering (22) or (23), we conclude that:

$$\lim_{t \to \infty} \frac{Q_{1t}}{Q_{2t}} = 0 \tag{24}$$

This case corresponds to the quasi-proportional growth considered by Pasinetti (1993, p. 33), in which $r_1 \neq r_2$, but $r_1 = \rho_1$ and $r_2 = \rho_2$, and it is the first approximation for more complex structural dynamics. It entails no sectoral labour reallocation and furnishes us with an analysis of Baumol's first case, in which $\lim_{t \to \infty} \frac{Q_{1t}}{Q_{2t}} = 0$ but $\lim_{t \to \infty} \frac{L_{1t}}{L_{2t}} = A$. The fact that the ratio of the production of sector 1 to sector 2 tends towards zero occurs because the growth rate of demand in the second sector is higher than in the first one. But the higher productivity in the second sector compensates for the higher growth of per capita demand, which makes the ratios of the labour force employed in sectors 1 and 2 constant. Although of some interest as a first approximation, such a situation does not find empirical support.

As discussed in the previous section, the most crucial case highlighted by Baumol (2001) is the one where the service sector output remains roughly constant as a share of total production over time. Based on this evidence, he then considers that in the long run $\frac{Q_{1t}}{Q_{2t}} = K$, where K is constant. Note that this result is compatible with the SED proportional growth, in which the per capita demand of coefficients grows at the same rate for both sectors, namely $r_1 = r_2$. This scenario corresponds to case ii) in which the share of the service sector remains in real terms. Then, the elasticity of demand for both the manufacturing and the service sector are equal. In Baumol, this assumption is implicit [see Oulton (2001)]. From the supply side, it is equivalent to assuming that:

$$\frac{L_{1t}}{L_{2t}} = \frac{a_2}{a_1} K e^{(\rho_2 - \rho_1)t} \tag{25}$$

Since $\rho_1 < \rho_2$, we obtain $\lim_{t \to \infty} \frac{L_{1t}}{L_{2t}} = \infty$. The share of the *i*-th sector in total employment is growing, which is easy to see if considering, like Baumol, that $\rho_1 = 0$. In such a case, $\hat{s}_1 = r_1 - \rho_1 = r_1 > 0$. Besides, $\hat{s}_2 = r_2 - \rho_2 < r_1 = \hat{s}_1$, which in the long run yields the result $\lim_{t \to \infty} s_1 = 1$ and $\lim_{t \to \infty} s_2 = 0$. From equation (13), we conclude that:

$\lim_{t\to\infty}\rho = \rho_1$. The same result is valid if one assumes that $0 < \rho_1 < \rho_2$, which yields $\hat{s}_2 = r_2 - \rho_2 < r_1 - \rho_1 = \hat{s}_1$, confirming the Baumol 'stagnationist' result in which the economy is doomed to converge towards the productivity growth of the stagnant sector. This result holds under particular assumptions. Firstly, preferences should be homothetic as long as the growth rates of demand for both segments are equal and do not change over time. Secondly, it is implicit in this result that $0 < r_2 - \rho_2 < r_1 - \rho_1$; otherwise, neither of the sectors is absorbing labour[10].

Let us proceed to case iii). It is quite clear that if $r_1 > r_2$, then $r_1 - \rho_1 > r_2 - \rho_2 > 0$, since $\rho_1 < \rho_2$. Consequently, the share of the first sector grows faster than the that of the second both in national income and total employment. Then Baumol's result is still valid since the overall productivity of the economy converges towards the low productivity sector, namely $\lim_{t\to\infty}\rho = \rho_1$ since $\lim_{t\to\infty} s_1 = 1$ and $\lim_{t\to\infty} s_2 = 0$. But another possibility should be considered. If $r_1 < \rho_1$ and $r_2 < \rho_2$, implying that $r_1 - \rho_1 < 0$ and $r_2 - \rho_2 < 0$, neither of the sectors absorbs labour insofar as in both of them, the net effect of demand increase and technological change is unemployment. This possibility was considered by Notarangelo (1999), who showed that structural unemployment is one of the outcomes of structural change, following the Pasinettian insight. Thus, the stagnant industry cannot absorb the labour unemployed in the advanced one. Baumol did not consider such a case, which shows us that the dynamics of the sectoral share in total labour depends not only on productivity change but also on the balance between technological change and the growth rate of demand. However, as Notarangelo noted, this analysis considers the existence of only two final sectors, and a generalisation for an arbitrary number of sectors is welcome. That is the task of the next section.

1.4. Extending This Analysis for an Arbitrary Number of Sectors

To extend the analysis to an arbitrary number of sectors, let us keep the hypothesis of the last section but consider that there are *n* sectors that produce outputs using labour

[10] This possibility was considered by Notarangelo (1999) who has shown that structural unemployment is one of the outcomes of structural change following the Pasinettian insight.

as the only input, which is provided by the household sector, denoted by m. Regarding physical quantities, the pure labour Pasinettian model can be represented by:

$$\begin{bmatrix} \mathbf{I} & -\mathbf{c} \\ -\mathbf{l} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{Q} \\ L \end{bmatrix} = \begin{bmatrix} \mathbf{O} \\ 0 \end{bmatrix} \quad (26)$$

Where **I** is the *n*x*n* identity matrix, **c** is the column vector of the sectoral consumption coefficients, namely c_i, **l** is the row vector of the sectoral labour coefficients, namely l_i, **Q** is the column vector of sectoral production, namely Q_i, **O** stands for the null column vector and *L* stands for the employed labour force. As we are dealing with a homogeneous system, a necessary condition for a non-trivial solution is given by:

$$\mathbf{lc} = 1 \quad (27)$$

Equation (27) is a generalisation of equation (3) for an arbitrary number of sectors and, as such, is a full employment condition that, if fulfilled, guarantees that the system is in an equilibrium position. If such a requirement is satisfied, there is a solution for the system of physical quantities regarding an exogenous variable, namely the available labour force denoted by $\bar{L}$, which may be expressed as:

$$\begin{bmatrix} \mathbf{Q} \\ L \end{bmatrix} = \begin{bmatrix} \mathbf{c}\bar{L} \\ \bar{L} \end{bmatrix} \quad (28)$$

From the first *n* lines of (28), we conclude that in equilibrium, the quantity of each tradable commodity is given by the amount of labour employed in its production, that is, $Q_i = c_i L$, $i = 1, \ldots, n$. We can also carry out the analysis regarding the price system. In this case, we can write the model as:

$$[\mathbf{p} \quad w] \begin{bmatrix} \mathbf{I} & -\mathbf{c} \\ -\mathbf{l} & 1 \end{bmatrix} = [\mathbf{O} \quad 0] \quad (29)$$

where **p** is the *n*x1 row vector of sectoral prices, and *w* is the wage. In this setup, equation (27) is the condition of full expenditure of the national income. If (27) holds[11], the solution for the price system is given by:

$$[\mathbf{p} \quad w] = [\mathbf{l}w \quad \bar{w}] \quad (30)$$

From the first *n* lines of (30), we conclude that in equilibrium, the price of each tradable commodity is given by the amount of labour employed in its production, that is, $p_i = l_i w$, $i = 1, \ldots, n$. To extend the analysis of the previous section, consider that total

[11] Note that the coefficient matrix is equal for both systems.

labour productivity is given by $q = \frac{Q}{L}$, where Q is total output and $L = \sum_{i=1}^{n} L_i$. By taking logs and differentiating, we obtain:

$$\hat{q} = \hat{Q} - \hat{L} \tag{31}$$

But we know that $\hat{Q} = \sum_{i=1}^{n} s_i \hat{Q}_i$ and $\hat{L} = \sum_{i=1}^{n} s_i \hat{L}_i$. Besides, $\hat{q}_i = \hat{Q}_i - \hat{L}_i$, which allows us to conclude that:

$$\hat{q} = \sum_{i=1}^{n} s_i \hat{q}_i \tag{32}$$

Equation (32) shows that the overall growth rate of productivity is an average mean of sectoral productivities, with the weight of each sector being the share of the correspondent sector. We know from the first n lines of (30) that:

$$p_i Q_i = w L_i \tag{33}$$

Summing up the sectors, we obtain:

$$Q = wL \tag{34}$$

Taking logs and differentiating equations (33) and (34), we get respectively:

$$\hat{p}_i + \hat{Q}_i = \hat{w} + \hat{L}_i \tag{35}$$

$$\hat{Q} = \hat{w} + \hat{L} \tag{36}$$

Subtracting (35) from (36), and considering that $\hat{q}_i = \hat{Q}_i - \hat{L}_i$ and $\hat{q} = \hat{Q} - \hat{L}$, after some algebraic manipulation, it follows that:

$$\hat{q} - \hat{q}_i = \hat{p}_i \tag{37}$$

We know that the share of the i-th sector in national income is given by:

$$s_i = \frac{p_i Q_i}{Q} \tag{38}$$

Taking logs and differentiating (38), we conclude that:

$$\hat{s}_i = \hat{p}_i + \hat{Q}_i - \hat{Q} \tag{39}$$

Inserting (37) into (39), and assuming balanced growth in all sectors as Baumol did, namely $\hat{Q}_i = \hat{Q}$, we obtain:

$$\hat{s}_i = \hat{q} - \hat{q}_i = \rho - \rho_i \tag{40}$$

where $\rho = \sum_{i=1}^{N} s_i \rho_i$. In what follows, let us consider for the sake of simplicity only that $\rho_1 < \rho_2 < \cdots < \rho_{n-1} < \rho_n$. A property of the weighted mean is that $\sum_{i=1}^{n} s_i \rho_i \geq \min\{\rho_i\} = \rho_1$. This means that for the sector with the lowest productivity growth, namely sector 1, we have $\hat{s}_1 > 0$, while for other sectors aside from 1, let us say $j \neq 1$, $\hat{s}_j$ will equal zero at some moment and eventually become negative. Then, in the long run, $\lim_{t\to\infty} s_1 = 1$ and $\lim_{t\to\infty} s_j = 0$. As a result, $\lim_{t\to\infty} \rho = \rho_1$. This is nothing but a generalisation of the Baumol result for an arbitrary number of sectors. Such a conclusion considering that all sectors grow at the same rate, namely $r_i = r_j$ and there is unemployment[12] of the labour force.

Such a hypothesis turns out to be nothing but the well-known concept of steady-state growth, which is ubiquitous in the Neoclassical tradition. As discussed in Section 2, there is some empirical support for the fact that the production of the manufacturing and the service sectors keeps proportionality in the long run [see, e.g., Appelbaum and Schettkat (1999)]. But within a truly multi-sectoral setup, why should we expect that particular sectors would continue growing at the same rate? Although useful for closing the model, this assumption is disconnected from the demand side and contrasts with the empirical regularity that the share of income spent on any particular consumption good is never constant as personal income increases but tends to reach saturation. It is inconceivable to consider that in the long run, multiple sectors with particular income elasticity of demand would grow at the same rate.

Using the same reasoning as in the previous section, it is possible to show that if we allow for particular growth rates of demand for each of the sectors, then the result may be different from the Baumol prediction. Consider, for instance, the case in which $r_n - \rho_n > r_i - \rho_i$. This result may occur if r_n is sufficiently large when compared to the higher productivity growth in the n-th sector. In that case, $\hat{s}_n = r_n - \rho_n > \hat{s}_i = r_i - \rho_i$, and $\lim_{t\to\infty} \rho = \rho_n$. We conclude that the overall productivity growth tends towards faster productivity growth, which is precisely the opposite result from Baumol (1967). It is fair

[12] The shift of labour from the high productivity sectors to the low productivity sector is logically correct if the stagnant sector can absorb the labour force that is being dismissed from the advanced sector due to productivity growth.

to say that considering the demand side in Baumol's model may change its results if the source of structural change is in the interaction between the demand and supply sides.

That is evident when we focus on the literature that followed his contribution. Authors such as Echevarria (1997) and Ngai and Pissarides (2007), for instance, have shown that the source of unbalanced growth is on the demand side insofar as the main result may be obtained even with similar total factor productivity – hereafter TFP – growth in all sectors[13]. The distinction between the roots of structural change being either on demand or the supply side is so evident in the orthodox literature that Acemoglu (2009) classified the neoclassical models among those in which structural changes originate on either the demand side or supply side. The demand-side explanations, which rely on income effects, consider that differences in income elasticities of demand amongst sectors play the central role, with the uneven expenditures due to increases in per capita income driving the structural changes. The supply side is associated with relative price effects that accrue from particular productivity growth for each of the sectors. This distinction, although useful for didactical reasons, is somewhat awkward to the extent that we correctly understand structural change when we consider income and relative-price effects simultaneously.

That is precisely what the Pasinettian model accomplishes, considering two sources of structural change: the first one is related to differences in income elasticities of demand across sectors. This source plays an important role as long as the structure of the economy reflects the increase in the shares of sectors that produce goods with a higher income elasticity of demand when per capita income increases. The second one considers that changes in the sectoral composition accrue from differences in relative prices due to the particular rate of technical change, which induces the reallocation of factors of production. So far, our approach is based on the case of existence only of final goods. In the next section, we challenge such a simplification.

1.5. Revisiting Pasinetti and Baumol with a Focus on Intermediate Goods

In this section, following Oulton (2001), we challenge the Baumol result for another reason – the existence of intermediate goods. Oulton (2001) showed that the shift of

[13] Such outcome, found in some models in the burgeoning literature on structural change that unfolded [see Arena (2017)], led some authors to reassert the relevance of structural change in the growth process.

labour from manufacturing to services might yield an alternative outcome if the services are not the final but an intermediate sector. In this case, even if the productivity growth of the service sector is lower than that of the manufacturing sector, the overall productivity growth will not converge towards the stagnant sector. We confirm this result by using the concept of vertically integrated industries as advanced by Sraffa (1960) and Pasinetti (1973). Trying to build a bridge between vertically integrated sectors and the Domar aggregation, Cas and Rymes (1991, p. 92) considered that:

> (…) productivity measures at the aggregate level for all Pasinetti sectors will be the same for the traditional aggregate measures, using the Domar-Hulten aggregation procedure, because both measures 'net' out intermediate inputs and outputs.

This means that we can advance a derivation of the well-known Domar aggregation without recurring to the device of TFP growth. Such a derivation is essential as Pasinetti (1981, p. 201) highlights the irrelevance of concepts such as the marginal productivity of labour and capital, which are integral to the derivation of the TFP. Such a method allows us to compute the amount of labour that directly and indirectly goes into the production of each good. To carry out the analysis, let us make a slight change in the model presented in section 3. Following Oulton (2001), consider that sector 1 produces an intermediate input used as intermediate input by industry 2. Besides, sectors 1 and 2 are vertically integrated in the sense that labour and the production of sector 1 are the only inputs of sector 2, which yields the following price system:

$$\begin{cases} p_1Q_1 - L_1w = 0 \\ p_2Q_2 - L_2w - p_1Q_1 = 0 \\ wL - c_2p_2Q_2 = 0 \end{cases} \tag{41}$$

To unfold the analysis in the presence of intermediate inputs (or circulating capital), we need a definition of sectoral productivity growth in which it is not only influenced by its own productivity but also by the productivity growth of its intermediate inputs. As we are dealing with vertically integrated sectors, all inputs may be reduced to labour equivalents, which allows us to write all the inputs of the sector as a function of the direct and indirect labour embodied. Then, to overcome the aggregation issue[14], we can use the device of vertical integration to reduce the amounts of intermediate inputs in terms of labour quantities. According to Garbellini and Wirkierman (2014, p. 167), "total labour productivity in each vertically integrated sector *i* is given by the ratio of net product to total labour requirements in the corresponding self-replacing subsystem". Therefore, the productivity of the first sector is defined simply by labour productivity $q_1 = \frac{Q_1}{L_1}$. By taking logs and differentiating, we find the equation below:

$$\hat{q}_1 = \hat{Q}_1 - \hat{L}_1 = \rho_1 \tag{42}$$

However, the second sector uses as inputs not only its own labour force L_2 employed directly but also the services produced by sector 1. We can measure the latter in terms of the embodied labour used as inputs for industry 2. But this embodied labour carries the productivity gain that occurs in the production process of the first sector. Let us denote the labour force employed indirectly by L_1^E, whereby we define the productivity growth of the other sector as:

$$q_2 = \frac{Q_2}{L_2 + L_1^E} \tag{43}$$

With this approach, we decompose each vertically integrated labour requirement into its direct and indirect components. But if we consider only L_1 to reckon the productivity growth of the second industry, we would disregard the productivity gain that accrues from the use of Q_1 as an intermediate input in sector 2. Hence, while L_2 represents the direct labour employed in the production of the second sector, L_1^E stands for the indirect labour requirements to produce the final output 2, while L_1 represents the direct labour requirements to produce intermediate input 1.

Moreover, in static conditions, $L_1^E = L_1$. However, in dynamic terms, as L_1^E is labour applied to produce the circulating capital, it must carry along with it its productivity growth. Then $\hat{L}_1^E = \hat{L}_1 + \hat{q}_1$, which sums up the change in technical

[14] By reducing all variety of intermediate inputs into the same constituent element, namely a flow of labour, allow us to overcome the aggregation issue and to derive the Domar aggregation without recurring to the concept of a production function.

requirements for the reproduction of the second-sector output. Logarithmically differentiating and rearranging the equation above yields:

$$\rho_2 = \hat{q}_2 = \hat{Q}_2 - s_1\left(\hat{L}_1 + \hat{q}_1\right) - s_2\hat{L}_2 = \rho_2 \quad (44)$$

By replacing the first equation of system (41) by the second equation, we obtain:

$$p_2Q_2 - (L_2 + L_1)w \ = 0 \quad (45)$$

From equation (31) we know that $\hat{q} = \hat{Q} - \hat{L}$. Besides, using the fact that $\hat{L} = s_1\hat{L}_1 + s_2\hat{L}_2$, we obtain:

$$\rho = \hat{q} = \hat{Q} - s_1\hat{L}_1 - s_2\hat{L}_2 \quad (46)$$

Isolating $s_2\hat{L}_2$ from equation (44) and inserting it into equation (46), we obtain the overall productivity growth:

$$\rho = s_1\rho_1 + \rho_2 \quad (47)$$

Equation (47) shows that even in a case in which $\rho_1 < \rho_2$, $\lim_{t\to\infty} s_1 = 1$ and $\lim_{t\to\infty} s_2 = 0$, and consequently, $\lim_{t\to\infty}(s_1\rho_1 + \rho_2) = \rho_1 + \rho_2$. In this case, the overall productivity growth of the economy will not tend towards the productivity change of the stagnant sector. Oulton (2001) highlighted this point, which challenges Baumol's result in the presence of intermediate inputs. However, note that the case reviewed by him does not pay any heed to the demand side [see Sasaki (2007, p. 439)] if there is only one final consumption good. It is a well-known result [see Pasinetti (1981)] that industries that comprise a vertically integrated sector have to grow at the same rate in the long run.

What is implicit in Oulton's result is the hypothesis that consumption and the intermediate goods industries grow at the same rate in the long run. As already highlighted, although of some interest as a first approximation, such a hypothesis has no meaning within a truly multi-sectoral setup. To proceed to this case, for the sake of convenience only, let us consider a first step where we have an economy with two final goods and two intermediate correspondent goods. To establish the ideas, let us consider the existence of two vertically integrated segments, namely sectors *I* and *II*. Sector *I* comprises industries 1 and 2, where the first one is a service industry that produces intermediate goods for the final industry 2. In contrast, sector *II* includes industries 3 and

4, where industry 3 is a service industry that provides intermediate products for the final goods sector 4. Thus, we can say that we have two vertically integrated segments in this economy in the sense that one of them has a flow of labour and a flow of final goods. In this setup, the overall productivity growth is given by:

$$\rho = s_I \rho_I + s_{II} \rho_{II} \tag{48}$$

where ρ_I and ρ_{II} are the productivity changes of the vertically integrated sectors *I* and *II*, respectively. However, from the first example of this section, and using the concept of Domar aggregation, it is possible to show that the productivity change of the vertically integrated sector, denoted by $\rho_{I,}$ is given by $\rho_I = \sigma_1 \rho_1 + \rho_2$ and $\rho_{II} = \sigma_3 \rho_3 + \rho_4$, where $\sigma_1 = \frac{p_1 Q_1}{p_2 Q_2} = \frac{L_1}{(L_1 + L_2)}$ and $\sigma_3 = \frac{p_3 Q_3}{p_4 Q_4} = \frac{L_3}{(L_3 + L_4)}$ stands for the share of the $i - th$ intermediate goods sector concerning the vertically integrated sectors *I* and *II*, respectively. If we assume that $\rho_1 < \rho_2$ and $\rho_3 < \rho_4$, then $\lim_{t \to \infty} \sigma_1 = 1$ and $\lim_{t \to \infty} \sigma_3 = 1$, with $\lim_{t \to \infty} \sigma_2 = 0$ and $\lim_{t \to \infty} \sigma_4 = 0$. Therefore,

$$\lim_{t \to \infty} \rho_I = \lim_{t \to \infty} (\sigma_1 \rho_1 + \rho_2) = \rho_1 + \rho_2 \tag{49}$$

$$\lim_{t \to \infty} \rho_{II} = \lim_{t \to \infty} (\sigma_3 \rho_3 + \rho_4) = \rho_3 + \rho_4 \tag{50}$$

From (49) and (50), assuming that the intermediate goods industries have lower productivity growth than the corresponding final goods, namely $\rho_1 < \rho_2$ and $\rho_3 < \rho_4$, we conclude that $\rho_I = \rho_1 + \rho_2$ and $\rho_{II} = \rho_3 + \rho_4$. Assume now that $\rho_1 < \rho_3$. In this case, segment 1 is the one with lower productivity growth in the economy. Following both Oulton and Baumol, the share of the first sector in the national income grows continuously, tending to unity since labour tends to migrate to the lower productivity sector. But in this case, the share of the vertically integrated sector *I* will converge to one, namely $\underset{t \to \infty}{lim} s_I = 1$, and from equation (48), we conclude that $\lim_{t \to \infty} \rho = \rho_I$, which means that the overall productivity growth tends towards the first vertically integrated sector's productivity growth.

But from (49) and (50), it is not possible to guarantee that the vertically integrated sector *I* is the one with higher or lower productivity growth than sector *II*. The outcome also depends on the productivity growth of the final sectors, namely sectors 3 and 4. If, for instance, $\rho_2 > \rho_4$, and ρ_2 is high enough to compensate for the lower productivity growth of the corresponding intermediate goods industry, namely $\rho_2 - \rho_4 > \rho_3 - \rho_1$, therefore the overall productivity growth of the vertically integrated sector *I* is higher than the global productivity growth of the vertically integrated sector *II*. In this case, the economy's productivity growth converges towards the productivity growth of the vertically integrated sector with the highest productivity growth, contradicting Baumol.

If $\rho_2 < \rho_4$ or even if $\rho_2 > \rho_4$ but ρ_2 is not high enough to compensate for the lower productivity growth of the corresponding intermediate goods sector, namely $\rho_2 - \rho_4 < \rho_3 - \rho_1$, then the overall productivity rate of the vertically integrated sector I is lower than that of sector *II*. In this case, the economy's productivity growth will converge towards that of the vertically integrated branch with the lowest productivity growth, corroborating the 'stagnationist' view. With this in mind, it is easy to see that even if a particular vertically integrated sector has an intermediate industry with the lowest productivity growth in the economy, the overall productivity growth of the economy may not converge towards the productivity growth of that sector. That is because what matters now is the productivity growth of the vertically integrated sector. Thus, if the corresponding final goods sector has a higher productivity growth that compensates for the lower productivity change in the intermediate goods sector, the productivity growth of the vertically integrated sector may not be the slowest one.

It is essential to consider that we obtained these results under the assumption that vertically integrated sectors grow at the same rate in the long run. Besides, there is no structural unemployment. As discussed in the previous sections, while these results are of some interest within a two-sector economy, it is more difficult to explain why within a truly multi-sectoral economy, particular sectors should grow at the same rate. Hence, it is interesting to extend the analysis to multiple numbers of branches. In this vein, consider then the following system, which is a generalisation of (41):

$$\begin{cases} p_i Q_i - L_i w = 0 \\ p_j Q_j - L_j w - p_i Q_i = 0 \\ wL - \sum_{i=1}^{N} c_i p_i Q_i = 0 \end{cases} \tag{51}$$

We are assuming that sector i produces the intermediate inputs for the final goods sector *j* only, which means that sectors i and j are vertically integrated. Let us consider that the vertically integrated sector composed of sectors i and *j* is denoted by I. In this case, we have the vertically integrated sector I composed of sectors 1 and 2, vertically integrated sector *II* composed of sectors 3 and 4, and vertically integrated sector *N* composed of sectors *n*-1 and *n*, where *n* is an even number.

From the Oulton analysis, we know that the productivity growth of the vertically integrated sector is given by: $\rho_I = \sigma_1\rho_1 + \rho_2$, $\rho_{II} = \sigma_3\rho_3 + \rho_4$, …, $\rho_N = \sigma_n\rho_n + \rho_{n+1}$, where $\sigma_i = \frac{p_iQ_i}{p_jQ_j} = \frac{L_i}{(L_i+L_j)}$ is the share of the intermediate goods sector in the vertically integrated sector. By assuming that $\rho_i < \rho_j$, for all sectors, we conclude that: $\lim_{t\to\infty} \rho_I = \rho_1 + \rho_2$, $\lim_{t\to\infty} \rho_{II} = \rho_3 + \rho_4$, …, $\lim_{t\to\infty} \rho_N = \rho_n + \rho_{n+1}$. In addition, assuming that $\rho_1 < \rho_3 < \cdots < \rho_n$, and following the rationale advanced by Oulton, we conclude that $\lim_{t\to\infty} s_I = 1$ and $\lim_{t\to\infty} s_{II} = \cdots = \lim_{t\to\infty} s_N = 0$ insofar as $\sum_{i=1}^{N} s_i\rho_i \geq \min\{\rho_i\} = \rho_I$. But also note in the long run that $\rho_I = \rho_1 + \rho_2$. Then, the outcome critically depends on the value of ρ_1. If, for instance, ρ_2 is the lowest productivity rate amongst the productivity rates of the final goods sectors, namely $\rho_2 < \rho_4 < \cdots < \rho_n$, then the existence of intermediate goods will not change the Baumol prediction that the overall productivity rate will tend towards the productivity of the lowest vertically integrated sector.

But there is no reason to assume that the pattern of sectoral productivities in the final sectors will mimic those in the intermediate industries. Thus, in general, ρ_1 is not the lowest productivity growth. If, for instance, we consider the symmetrical case in which $\rho_1 > \rho_3 > \cdots > \rho_n$ and, if $\rho_1 + \rho_2 > \rho_n + \rho_{n+1}$, then the overall productivity will converge towards the productivity of the vertically integrated branch with the highest productivity, which is the opposite of the Baumol prediction. But between Baumol and Oulton, we have other possibilities, which shows that in general, the outcome depends on the balance between the productivity of the final and the intermediate goods sectors. What we know for sure is that the overall productivity growth will converge towards the productivity growth of the vertically integrated branch that has the intermediate industry with the lowest productivity. Whether this is the vertically integrated sector with the lowest productivity is another story. It can be the sector with the highest productivity, depending on the productivity of the final goods sectors.

1.6. Concluding Remarks

With this inquiry, we revisit Notarangelo's (1999) approach, reinforcing that there are indeed similarities between the models while confirming that Baumol (1967) offers a particular case of Pasinetti (1993). Our approach allows us to deliver the Baumol analysis in terms of the SED approach, thus providing a truly disaggregated assessment of unbalanced growth. Besides, we alleviated the passive role played by demand in the Baumol model by considering a more subtle and inclusive approach for it − as is found in the contributions of Pasinetti. With this study, we show that even in the case in which there are no intermediate goods, the Baumol result holds if the productions of all sectors grow at the same rate, and the economy maintains equilibrium.

The additional advantage of our approach is related to the fact that the Pasinetti model considers not only final goods but also intermediate goods. With this in mind, we have advanced the extension of Oulton (2001) to the Baumol model within the SED framework, to consider multiple sectors. Next, by using the concept of vertical integration, we obtained a result that summarises Oulton and Baumol's contributions. On the one hand, Oulton's point still holds, namely that in the presence of intermediate goods, the productivity of the economy will not necessarily converge towards the lower productivity sector. On the other hand, the Baumol perspective may also hold as one of the outcomes. This exercise shows that the result depends on comparing not only the productivity growth of isolated industries – as Baumol did – but also the productivity growth of vertically integrated sectors.

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Chapter Two: Productivity Growth and Sectoral Interactions under Domar Aggregation: A Study for the Brazilian Economy from 2000 to 2014

2.1. Introduction

At least since Adam Smith, economists acknowledge productivity growth as the primary source of the wealth of nations. Solow's (1956) growth model highlights that the growth rate of per capita output, capital and consumption is given by the exogenous growth rate of technological change. But within that framework, productivity, or the total factor productivity (TFP), is calculated as a residual, which is somewhat unsettling. Since then, several authors have worked on what became known as growth accounting [see, e.g. Hulten (2009) and Jorgeson et al. (1987)]. Notwithstanding the considerable literature that followed the Solow's (1957) first attempt to measure TFP, most of the estimates underestimate the contribution of intermediate inputs when tackling productivity growth insofar as they do not consider them explicitly. In the present work, we fill this gap paying particular attention to the role of intermediate inputs in analysing productivity growth[15] of the Brazilian economy from 2000 to 2014.

After an initial and consistent period of productivity growth during the second half of the last century, the productivity of the Brazilian economy remained stagnant since the eighties [see e. g. Nassif et al. (2020)]. Barbosa-Filho & Pessôa (2014) and de Souza & da Cunha (2018) registered a resurge of productivity growth at the beginning of the last decade. Still, it lasted until the 2008 crisis, with both mostly sectoral and aggregate productivity growth declining after that[16]. Some factors help us to disentangle this path.

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[15] What generally differs among distinct methodologies is the definition of output and inputs. Concerning outputs, it is usually used some description of value-added or gross production. Among inputs, normally, some set involving labour, types of capital and, sometimes, intermediate goods are applied. Concerning different theoretical ways to measure productivity growth see e.g. Baumol & Wolff (1989), Ten Raa & Shestalova (2011), Wolff (2013). See also Fox (2012) about dis(aggregating) productivity growth and De Juan & Eladio (2000) for a survey on productivity growth within a input-output framework.

[16] Overal productivity advanced almost 15% between 2000 and 2009, and then it has decreased until 2014, offsetting most of the previous increase.

A crucial one is related to the intense deindustrialisation process registered in the last decades. The wane of manufacturing share in the national income share is not just the outcome of a faster decline in the price of manufacturing goods when compared to the cost of services. Even if one calculates the shares of different sectors in terms of constant prices, as opposed to current prices, will conclude that manufacturing value-added is decreasing. Besides, the deindustrialisation in Brazil is premature, happening at lower levels of per capita income than the average of industrialised countries. And the migration of the labour force is occurring towards final services, which have lower productivity than the business services. The outcome is a reduction in overall productivity gains.

Another factor that can explain the difficulties faced by the Brazilian industry is a decrease in the density of the economy, as defined by the existing interconnection amongst sectors. A higher density means the existence of more forwarding and backward linkages amongst the industries, which is essential to spread productivity gains through them. One could argue that such a decrease is the outcome of integration to the system of global value creation. As the global economy is structured around global value chains (GVCs), [see, e.g. Gereffi and Fernandez-Stark (2011)] the extent of participation in those chains seems to be an important explanatory variable to the decrease[17] in domestic density. But Brazil, like other Latin America's economies, remains poorly integrated in terms of GVCs [see, e.g., and Andreoni and Tregena (2020)], which does not explain the density reduction.

To fully understand the importance of density to productivity growth is essential to bear in mind that industries do not only benefit from a productivity increase in its own production process, but also from increased productivity in other sectors from which it acquires inputs, and this also generates impacts for aggregate productivity. Intermediate inputs are produced goods that link industrial sectors and have a unique role in spreading productivity growth amongst sectors [see, e.g. Aulin-Ahmavaara (1999)]. According to Jones (2011), intermediate goods[18] provide links between industries that create a

[17] Andreoni and Tregena (2020, p. 327) highlightes this trade-off by reporting that "(…) in a number of cases, middle-income countries that have attempted to integrate globally have also ended up 'de-linking domestically' and hollowing out the domestic manufacturing sector".

[18] As pointed out by Amit and Konings (2007), and Goldberg et al. (2010), such goods allow for quality improvement in final products and broader participation of a country in international trade. Besides, its increased availability may facilitate product diversification and trigger pro-competition effects, inducing

multiplier. He argues that high productivity in an industry requires a high level of performance along many dimensions. The author proposes that linkages are a crucial part of the explanation by delivering a noteworthy example:

> *"(...) intermediate goods provide links between sectors that create a multiplier. Low productivity in electric power generation - for example, because of theft, inferior technology, or misallocation - makes electricity more costly, which reduces output in banking and construction. But this in turn makes it harder to finance and build new dams and therefore further hinders electric power generation.*" Jones (2011, p. 1-2)

Then to provide a more in-depth analysis of the behaviour of both sectoral and aggregate Brazilian productivity and economic growth between 2000 and 2014, we use here the Domar aggregation approach. For the best of our knowledge, this is the first time this method is adopted for studying the Brazilian economy. The advantage of this method is that it can capture not only the productivity growth contributions of individual sectors [19]but those gains that accrue from the intermediate goods. A characteristic of Domar aggregation is that it is not a weighted average, but a weighted sum of sectoral productivity growths. Moreover, the sum of its weights is higher than unity in economies with intermediate inputs. The added Domar weights then represent the potential for interconnection and linkages between sectors, capable of propagating productivity growth throughout the economy.

After Domar (1961), several authors improved the method theoretically[20] and used it empirically[21] to perform growth accounting. Some essential theoretical works are

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cost reductions and improved diversification, with the creation of productive linkages and spillover effects. The notion that linkages across industries can be crucial to economic performance dates back at least to Leontief (1936), which introduced the field of input-output economics. Hirschman (1958) emphasised the role of forwarding and backward linkages to economic development.

[19] We split the sectors into a 48, 10 and 3 levels of aggregation to structurally analyze the Brazilian economy and to deliver a sectoral and aggregate productivity growth evaluation and its interactions over the given period.

[20] See also Hulten (2009) for a complete survey on growth accounting and its relationship with Domar aggregation and other methods.

[21] Some interesting empirical works are e.g. Oulton & O'Mahony (1994) about productivity growth in United Kingdon manufacturing industries, Jorgenson & Stiroh (2000) concerning United States, Timmer

Hulten (1978) which related the Domar aggregation with a macro production possibility frontier. Jorgeson et al. (1987) is a seminal work about the usage of Domar aggregation method with several theoretical improvements, while Aulin-Ahmavaara (1999) formulated explicitly the output price reductions caused by the productivity in upstream sectors. More recently, Ten Raa & Shestalova (2011) and Balk (2019) also have delivered essential contributions[22]. Given the usefulness of Domar aggregation, particular fields of research have used it as a tool to calculate and decompose productivity growth under several theoretical fields[23].

With this approach, we confirm some result and find new ones. Besides, after decomposing the total productivity growth for three macro sectors, we confirmed the results found by de Souza & da Cunha (2018). Services and primary industries macro sectors had a positive impact on average for productivity growth, albeit manufacturing macro sector usually contributed with negative productivity growth.

However our study allows us to consider another dimension that was not studied by those authors, which is the multiplicative effect of propagating productivity growth due to Domar weights. With this approach, we conclude that manufacturing had a high sectoral density compared with the remaining two macrosectors, but it typically produced negative productivity advance, which have contributed negatively to overal productivity growth both directly and indirectly. Regarding the services and primary macro sectors, the former presented relatively high density but low technical advance while the latter yield relatively high productivity growth, but with very low sectoral density, that held its produtctivity advance when thinking in contributions to overall productivity during the

& van Ark (2005) about Europe Union and focusing on Information and Communication Technology sectors, Gu & Yan (2016) about China and Cao, et. al. (2019) regarding several developed countries.

[22] Ten Raa & Shestalova (2011) built the Domar aggregation by theoretically relating it with other variants of productivity decompositions in the literature creating a common framework. Balk (2019) advanced in a new formulation of Domar aggregation dispensing with some usual assumptions, making them more flexible.

[23] It has been useful for instance to study the implications of the Baumol Cost Disease within input-output frameworks [e.g. Oulton (2001), Sasaki (2007), Baumol (2011), Hartwig and Krämer (2019) and Sasaki (2020)]. It has also been adopted to study production networks and shock propagation channels as a mechanism for transforming microeconomic shocks into macroeconomic fluctuations [e.g. Acemoglu et al. (2012), Carvalho (2014), Carvalho and Salehi (2019) and Baqaee and Farhi (2019)].

period under consideration. Moreover, the overal Brazilian economy's density suffered from a declining density amog sectors. These findings reassert the importance of the manufacturing sector as one of the main drivers of growth. Had this sector presented a better performance during the time under consideration, the overall productivity growth of the Brazilian economy would be better both by the direct and indirect channels.

We organise this paper as follows: besides this brief introduction, in the next section, there is a theoretical review of Domar aggregation and its usage. Then, in the third section, we use the methodology as the analytical basis for the empirical analysis. The fourth section delivers the data analysis for the Brazilian economy using WIOD data and both sectorial and aggregated productivity calculations, segmented in a 48, 10 and 3 sector levels of aggregation. Finally, section 5 concludes the paper.

2.2. The Database

We analyse the Brazilian economy between 2000 and 2014 by using the Domar aggregation approach. To do that, we use the Socio-Economic Accounts (SEA) data from the World Input-Output Database (WIOD). The SEA tables provide us with all the necessary data[24] and are organised in a directly compatible[25] way, as shown by Dietzenbacher et al. (2013) and Timmer et al. (2015).

The data comprises the period between 2000 and 2014, and 48 sectors for the Brazilian economy. Aiming to improve the visualisation results, we have split the sectors, besides the original 48 levels of aggregation from the data[26], to 10 and 3 levels of aggregation as can be seen in detail in the appendix.

2.3 Methodology

[24] We use, from SEA tables, sectorial capital stocks, labor expenditures, hours worked, gross output and value added at current and constant prices. The only necessary data that is not explicitly in SEA tables is sectoral capital stock growth rate in constant prices. We have used an appropriated deflator to calculate it from nominal capital stock.

[25] The (SEA) WIOD data is built in a way that the value added per sector is equal to the sum of expenses of labor and capital inputs in one hand and equal to the difference between sectorial gross output and intermediate input in other hand, just like in the model provided.

[26] The original subdivision of sectors is given by the ISIC (International Standard Industrial Classification of All Economic Activities) revision n. 4, from the United Nations Statistics Division, which can be found at https://unstats.un.org/unsd/classifications/Econ/ISIC#isic1 .

Following the methodology proposed by Jorgeson *et al.* (1987) and Jorgenson and Stiroh (2000), consider an economy with n discinct sectors in which each of them can sell its products both to final demand and intermediate demand from other industries. The expression below shows that the nominal gross output production of the ith sector (P_iQ_i) is sold both to final demand (P_iY_i) and to intermediate demand ($\sum_{j=1}^{n} P_iQ_{ij}$) from all j sectors that require the good or service produced by i as an intermediate input to its production:

$$P_iQ_i = P_iY_i + \sum_{j=1}^{n} P_iQ_{ij} \qquad (1)$$

where P_i represents the selling price of sector i goods, both to final and intermediate demand. Moreover, Q_i, Y_i and Q_{ij} are, respectively, real gross output, real final demand and real intermediate demand produced by the ith sector. Symmetrically, consider that the nominal gross output of all i sectors can also be described from its inputs side. It means that each sector i yields a homogeneous good or service that requires, for its production, an intermediate input set bought from other sectors $\sum_{j=1}^{n} P_jQ_{ji}$, as well as a set of capital and labour inputs, respectively defined as $P_{Ki}K_i$ and $P_{Li}L_i$, as shown by the equation below, where all the P terms represent prices and the accompanying terms real quantities:

$$P_iQ_i = P_{Li}L_i + P_{Ki}K_i + \sum_{j=1}^{n} P_jQ_{ji} \qquad (1')$$

The sectoral nominal value-added ($P_i^VV_i$), or net output, is, therefore, the difference between their respective gross production and intermediate demand. In our model, it is precisely equal to the sum of sectoral primary inputs expenditures, as shown by next expression:

$$P_i^VV_i = P_iQ_i - \sum_{j=1}^{n} P_jQ_{ji} = P_{Li}L_i + P_{Ki}K_i \qquad (2)$$

Equalising (1) to (1'), and summing up for all the i sectors, we find the definition of the economy gross domestic product (GDP). It can be measured both from the sum of all final demands and value-added. It is worth noting that in that process the intermediate inputs demand and supply cancel out each other avoiding double counting.

$$\sum_{i=1}^{n} P_iY_i = \sum_{i=1}^{n} P_i^VV_i = GDP \qquad (1'')$$

Assume that sectoral production technology is described, in a more general form, as a sectoral production function that relates time and its inputs – both primary and intermediate – with the gross sectoral product. The Hicks-neutral type of this function is:

$$Q_i = Q_i(L_i, K_i, X_{ji}, t) \tag{3}$$

Differentiating totally (3) with respect to time, using (1') and considering that a hat (^) denotes growth rate, we find the next equation that describes the i −th sector multifactor productivity growth. For the sake of notation simplicity, the sectoral inputs to gross output shares are denoted[27] by $\upsilon_{Li} = \frac{P_{Li}L_i}{P_iQ_i}$, $\upsilon_{ki} = \frac{P_{Ki}K_i}{P_iQ_i}$ and $\upsilon_{Qji} = \sum_{j=1}^{n} \frac{P_jQ_{ji}}{P_iQ_i}$.

$$\hat{q}_i = \hat{Q}_i - \upsilon_{Li}\hat{L}_i - \upsilon_{ki}\hat{K}_i - \upsilon_{Qji}\hat{Q}_{ji} \tag{4}$$

The term $\hat{q}_i$ denotes the multifactor productivity advance achieved by the ith sector. The multifactor productivity growth – MFP growth hereafter – is defined as the residual of the difference between the growth rate of the gross product and the growth rate of the inputs, weighted by the share of the input's value in the value of the gross product [see, e.g. Cas & Rymes (1991)]. One of the first authors to formalise the concept of MPF[28] growth was Hulten (1978). Note that the equation above can be written in discrete time using a Törnquist[29] or translog discrete-time approximation, where the Δ term is the difference between the variable in the current and previous time:

$$\Delta lnq_{it} = \Delta lnQ_{it} - \frac{(\upsilon_{Lit}+\upsilon_{Lit-1})}{2}\Delta lnL_{it} - \frac{(\upsilon_{kit}+\upsilon_{kit-1})}{2}\Delta lnK_{it} - \frac{(\upsilon_{Qjit}+\upsilon_{Qjit-1})}{2}\Delta lnQ_{jit} \tag{4'}$$

[27] Using (1') it's easy to see that $\upsilon_{Li} + \upsilon_{ki} + \upsilon_{Qji} = 1$.

[28] According to Oulton & O'Mahony (1994), the MPF growth is, theoretically speaking, the rate at which output would have increased in some period if all inputs had remained constant. Furthermore, it is noteworthy that if we calculate MPF growth over some period and it turns out to be about zero, then we can at least say that any eventual growth in labor productivity must have been due to increased use of other inputs.

[29] See, for example, Diewert (1976), Ten Raa & Shestalov (2011) and Hulten (2009) about the use of Törnquist index for discrete time aproximations and uses in productivty growth theory. The nickname Translog index is due to Diewert (1976), who has shown that the approximation is exact for the translog production function.

We can describe the sectoral gross output growth rate as the average mean of the growth rates of both real net output and intermediate inputs, weighted by its respective shares of the gross production. In the equation below, the term υ_{Vi} equals to $\upsilon_{Li} + \upsilon_{ki}$.

$$\hat{Q}_i = \upsilon_{Vi}\hat{V}_i + \upsilon_{Qji}\hat{Q}_{ji} \tag{5}$$

Using (4) and (5) and after some algebraic manipulations, it is possible to find the following expression, that relates the growth rate of the sectoral value-added with the growth rate of physical capital stock, labour force and productivity:

$$\upsilon_{Vi}\hat{V}_i = \upsilon_{ki}\hat{K}_i + \upsilon_{Li}\hat{L}_i + \hat{q}_i \tag{6}$$

From an aggregate point of view, the economy's GDP is described as the sum of all sectoral values added (or amount of all sectoral final demand). That is, being the nominal GDP of the whole economy PY, we have that $PY = P_v V = \sum_{i=1}^{n} P_i^V V_i$. We use a general function that relates the aggregated value added with the relevant inputs and time[30]:

$$V = f(L, K, t) \tag{7}$$

When differentiating totally (7) with respect to time, and after some algebraic manipulations, one find an expression that connects the growth rate of aggregate productivity, defined as $\hat{q}$, with the growth rate of the total value added of the economy and the weighted sum of the sectorial primary inputs capital and labour:

$$\hat{q} = \hat{V} - \sum_{i=1}^{n} \frac{P_{Li}L_i}{\sum_{i=1}^{n} P_i^V V_i}\hat{L}_i - \sum_{i=1}^{n} \frac{P_{Ki}K_i}{\sum_{i=1}^{n} P_i^V V_i}\hat{K}_i \tag{8}$$

Aiming to unearth an equation that relates the productivity growth rate of the whole economy with the growth rates of sectoral productivity – the Domar aggregation – we combine (6) and (8) to obtain:

$$\hat{q} = \sum_{i=1}^{n} \frac{P_i Q_i}{\sum_{i=1}^{n} P_i^V V_i}\hat{q}_i \tag{9}$$

Expression (9) is known as the *Domar aggregation* of sectoral MPF growth. Although Domar (1961) was the first to find this relationship formally, other authors such

[30] This can be explicitly found using equations (1) and (1'), as in (1'').

as Hulten (1978) and Jorgeson *et al.* (1987) later improved it. In discrete-time, it is possible to write the expression (9) as:

$$\Delta lnq = \sum_{i=1}^{n} \frac{1}{2}\left(\frac{P_{it}Q_{it}}{\sum_{i=1}^{n} P_{it}^{V} V_{it}} + \frac{P_{it-1}Q_{it-1}}{\sum_{i=1}^{n} P_{it-1}^{V} V_{it-1}}\right)\Delta lnq_{it} \qquad (9')$$

Note that the weighted sum of sectoral MFP has the striking feature that it sums to more than unity[31] in economies with intermediate goods. The higher the participation of intermediate inputs in the economy, the higher the sum of the weightings. Regarding the *'sum to more than the unity'* of Domar aggregation and its intuition, Jorgenson (2018, p. 881) considers that:

> "*A distinctive feature of Domar weights is that they sum to more than one, reflecting the fact that an increase in the growth of the industry's productivity has two effects: the first is a direct effect on the industry's output and the second an indirect effect via the output delivered to other industries as intermediate inputs.*"

Similarly, Oulton & O'Mahony (1994, p. 14) explains the intuition behind the role of intermediate inputs in the aggregated productivity growth and the Domar weights behaviour:

> "*The intuitive justification for the sum of the weights exceeding one is that an industry contributes not only directly to aggregate productivity growth but also indirectly, through helping lower costs elsewhere in the economy when other industries buy its product*".

Domar aggregation method establishes a link between the sectoral level productivity growth and aggregate productivity growth. Productivity benefits of the aggregate economy may exceed the average productivity gains across sectors given that flows of intermediate inputs among sectors contribute to total productivity growth by allowing productivity gains – or losses – in successive industries to augment one another. Moreover, the contribution of an industry to the overall productivity growth depends

[31] Accordingly to Ten Raa & Shestalova (2011) more common productivity aggregation in the literature, like aggregating sectoral TFP-growth (without explicitly dealing with intermediate inputs in sectoral production functions), can be represented as a simple weighted average of sectoral productivity growth. However, the aggregation of sectoral multifactor productivity growth comprises a tricky aggregation issue, when dealing with input-output economies, which has been analysed by Domar (1961). The point is that the national product of an economy does not comprise the sum of all gross output, but only the sum of net outputs. Avoiding for double counting, the Domar aggregation spawns an aggregation where the weights sum to more than one.

(besides the direct productivity growth in this sector) on the efficiency changes in the production of its intermediate inputs. To clarify the mechanism in which the direct and indirect effects above mentioned behave within the model, we substitute equations (1') and (2) into the numerator of (9) to obtain:

$$\hat{q} = \sum_{i=1}^{n} \frac{P_{Vi}V_i}{\sum_{i=1}^{n} P_i^V V_i} \hat{q}_i + \sum_{i=1}^{n} \frac{\sum_{j=1}^{n} P_j Q_{ji}}{\sum_{i=1}^{n} P_i^V V_i} \hat{q}_i \quad (10)$$

Disaggregating the second term of the expression above for all sectors, we get:

$$\hat{q} = \sum_{i=1}^{n} \frac{P_{Vi}V_i}{\sum_{i=1}^{n} P_i^V V_i} \hat{q}_i + \frac{\sum_{j=1}^{n} P_j Q_{j1}}{\sum_{i=1}^{n} P_i^V V_i} \hat{q}_1 + \frac{\sum_{j=1}^{n} P_j Q_{j2}}{\sum_{i=1}^{n} P_i^V V_i} \hat{q}_2 + \ldots + \frac{\sum_{j=1}^{n} P_j Q_{jn}}{\sum_{i=1}^{n} P_i^V V_i} \hat{q}_n \quad (11)$$

Note that the sum of the value-added weights, in the first term of the equation above right-hand side, is precisely one. The terms on the right, however, depict the sectoral productivity impacts from intermediate inputs deliveries. Therefore, the weights on the right are the ones that exceed the unity considering the overall aggregation. From the equation above it must be clear that the higher the degree of interconnection, or density of the economy in terms of intermediate inputs deliveries, the higher the potential of productivity growth augmenting given the growth of sectoral productivities.

To visualise the mechanism involved, assume that $\theta_{ij} = \frac{\sum_{j=1}^{n} P_j Q_{ji}}{\sum_{i=1}^{n} P_i^V V_i}$ is the share of aggregate demand for intermediate inputs in the economy, which measures the degree of sectoral density or sectorial interconnection. Substituting θ_{ij} into equation (10) we find the equation below:

$$\hat{q} = \sum_{i=1}^{n} \frac{P_{Vi}V_i}{\sum_{i=1}^{n} P_i^V V_i} \hat{q}_i + \sum_{i=1}^{n} \theta_{ij} \hat{q}_i \quad (12)$$

The term $\sum_{i=1}^{n} \theta_{ij}$ measures the degree of interconnection, or density, of the economy since it defines the relative importance of sectoral technological interactions. The greater the term θ_{ij} is in each ith sector, the more significant is the sectoral capability to spread productivity and to augment the sum of the whole economy due to Domar weights. Let us suppose that, for some reason, the density θ_{ij} of some sector i increases, due to a more significant share of intermediate demand by the given sector in the economy's GDP. Then, by differentiating the aggregate productivity growth with respect to θ_{ij} in (12) we have that:

$$\frac{\partial \hat{q}}{\partial \theta_{ij}} = \hat{q}_i > 0 \qquad (13)$$

Hence, if the sectoral productivity growth in the given sector is positive, then an increase[32] in θ_{ij} leads, by itself, to a higher aggregate productivity growth, *given* all sectoral productivity growth. In this vein, if the share of intermediate goods in the economy increases, the sum of Domar weights increases as well. In that case, the economy is subject to a higher density[33] that generates an augmented potential of aggregate productivity growth. Finally using equations (4) and (5) and summing up for all sectors, it is possible to find an expression concerning the interactions between aggregate productivity growth and economic (GDP) growth:

$$\hat{v} = \sum_{i=1}^{n} \frac{p_i v_i}{\sum_{i=1}^{n} P_i^V V_i} \hat{v}_i = \sum_{i=1}^{n} \frac{p_i Q_i}{\sum_{i=1}^{n} P_i^V V_i} \hat{q}_i + \frac{p_{li} L_i}{\sum_{i=1}^{n} P_i^V V_i} \hat{L}_i + \frac{p_{ki} K_i}{\sum_{i=1}^{n} P_i^V V_i} \hat{K}_i \qquad (14)$$

Thus, the aggregate value-added growth rate can be viewed as a weighted sum of labour, capital and Domar productivity growth contributions. In the next section, we use expressions (4') and (9') to calculate, respectively, the sectorial and aggregate productivity growth.

2.4. Results and Discussion

Figure 1 below shows, on the top side, that the manufacturing sectors were the ones that used intermediate inputs the most as a proportion of its gross output. Moreover, albeit the service sectors were very heterogeneous compared with primary industries, it still had, on average, a more substantial share of intermediate inputs than primary sectors.

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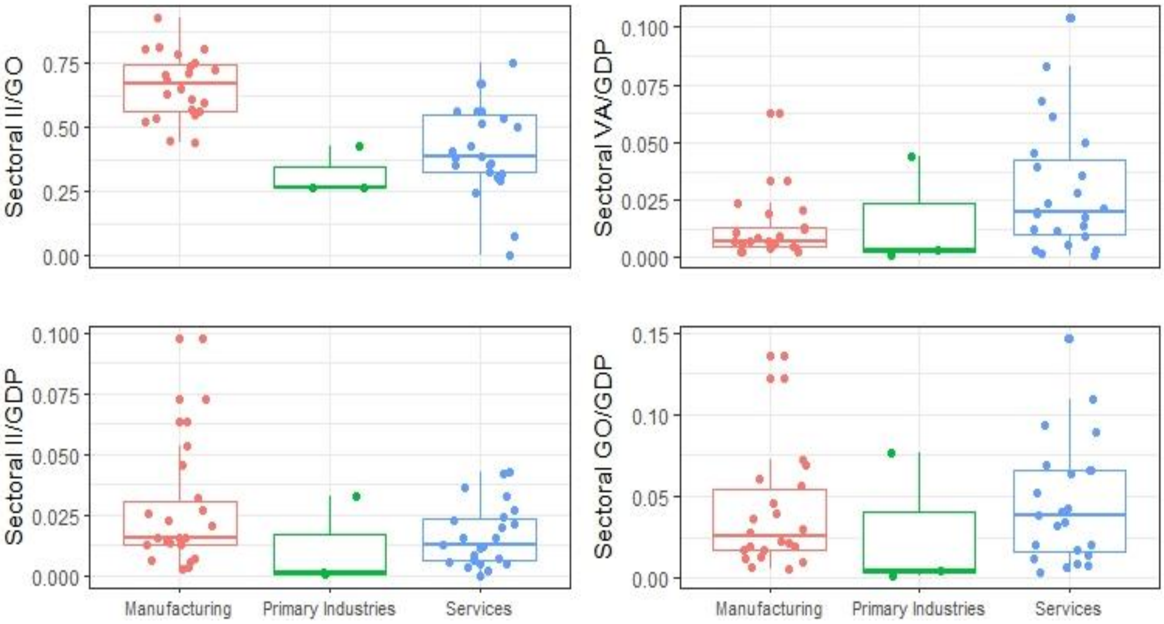
[32] Indeed, there are more than one possibly way that can lead to an augmented sum of Domar weights, or density of the economy. It can happen both if one or more sectors start to be more integrated, demanding higher shares of intermediate inputs for its production, or if one or more sectors with a structurally high share of intermediate inputs in its gross output increases its share in the whole economy in a way that led to a higher sum of Domar weights.

[33] An increase of density as a source of better growth performance is highlighted by the complex literature advanced by Hausmann and Klinger (2006). According to this view, industries with higher 'implied productivity' are those whose are well connected with other industries of the economy, being this connection made by the supply of intermediate inputs. Hidalgo & Hausmann (2009) went a step further and concluded that the ease in which a country moves from the production of one good to another depends on its position in the 'product space', which is the network connections between various sectors.

Concerning value-added share in GDP, when summing up all service sectors, they represented the majority share in GDP compared with the other two macro sectors, as expected. However, the manufacturing sectors were the ones with higher average intermediates inputs to GDP share – or density as defined in the last section – compared to services and primary industries macro sectors. Whereas the services sectors have had extensive heterogeneity regarding intermediate inputs to gross output shares, it did not happen concerning sectoral density.

Furthermore, both the manufacturing and services sectors presented relevant shares of Domar Weights, and therefore more potential to spread productivity growth than primary industries sectors.

Figure 1. Characteristics and dispersion of the sectors of the Brazilian economy

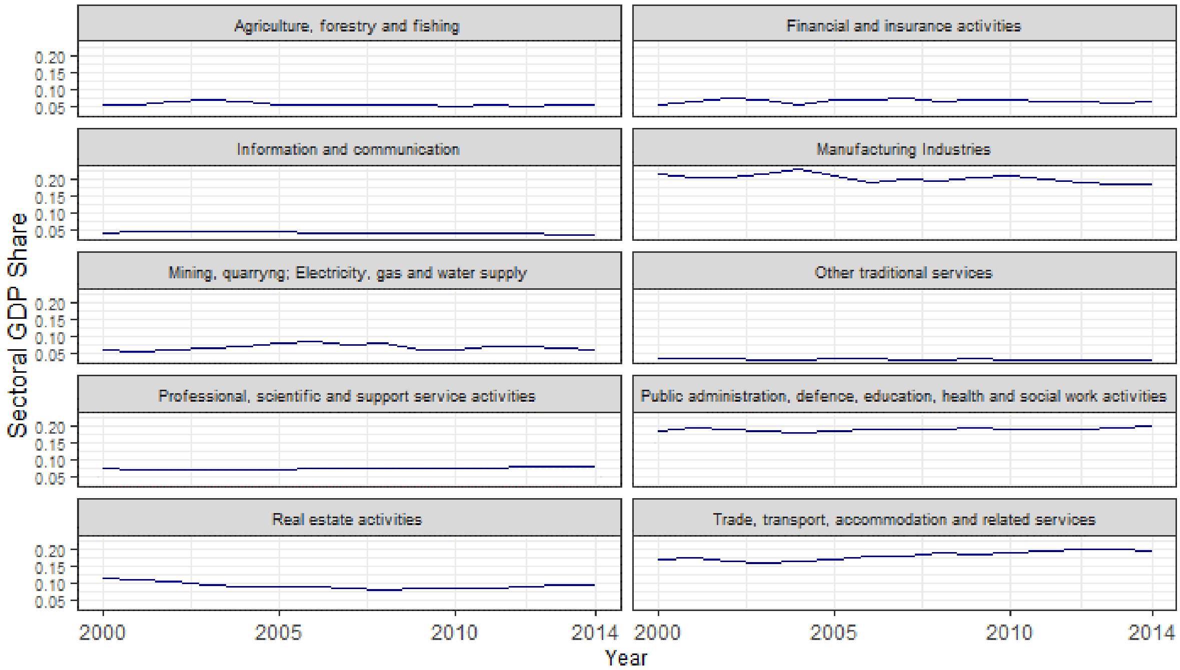


Note: Sectoral dispersion for 48 and 3 levels of aggregation: on the top side there are, respectively, the share of sectoral intermediate inputs to gross output and the share of sectoral value added in the GDP. On the bottom side there are the share of sectoral intermediate input to GDP (sectoral density) and sectoral gross output to GDP (Domar Weight), in that order. Authors calculations using WIOD data for 2010.

Figures 2 and 3 below show a time series analysis using the ten sectors level of aggregation regarding both sectorial values added to GDP share and intermediate inputs to gross output share.

Notice that, in figure 2, while agriculture, forestry and fishing sectors have shown some stability in the GDP share, manufacturing industries have had a slight decline during the given period. Most services sectors have shown an increase in its share, with the exceptions of the information and communication sector and other traditional services sector.

Figure 2. 10 sector value added share in GDP

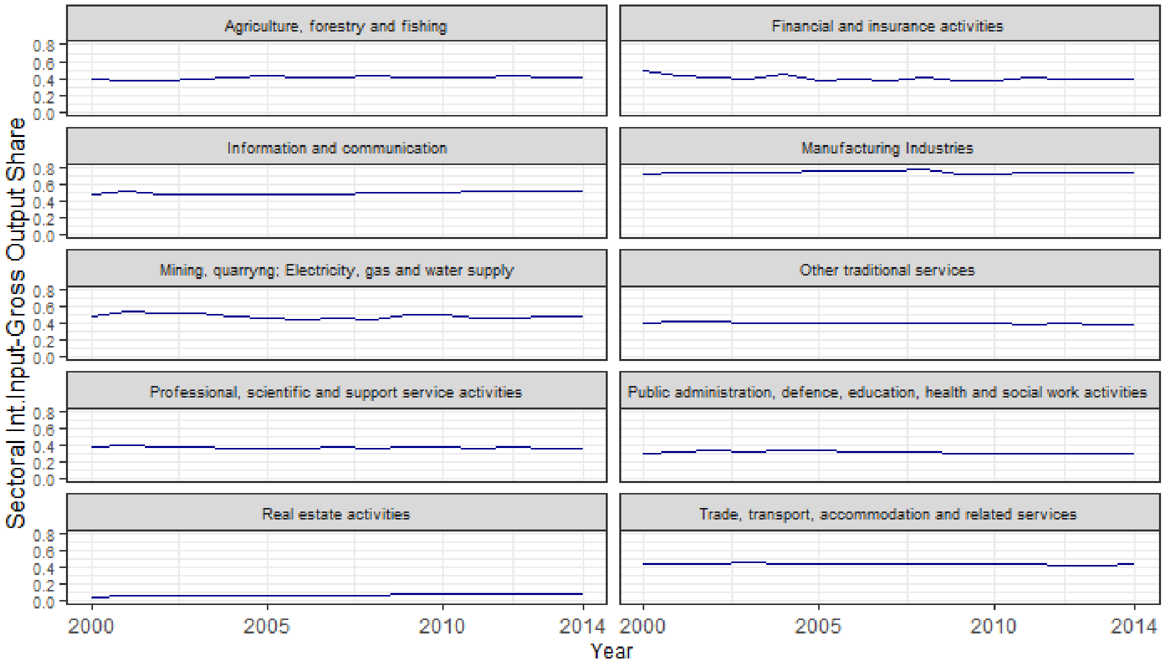


Note: Authors calculations using WIOD Brazilian data

Concerning the sectoral intermediate input to gross output share behaviour, displayed by Figure 3, notice that the manufacturing industries have been the sector with the highest demand for intermediate inputs compared to its gross output with something around sixty to eighty per cent during the given period. Agriculture, forestry and fishing experienced a slight increase to something above forty per cent of intermediate inputs to gross output share in the end of the period under analysis. Mining, quarrying, electricity, gas and water supply has had a significant share as well, above most services sectors.

Additionally, notice from figure 3 that the services sectors, as expected from the previous analysis, have exhibited a heterogeneous pattern, with a relatively high share in sectors such as financial and insurance activities and relatively small share in real estate activities.

Figure 3: 10 sector intermediate input relative to gross output share



Note: Authors calculations using WIOD Brazilian data

It is provided, in table 1 below, calculations for sectoral density (intermediate input to GDP share), value added share in GDP, sectoral Domar weights (gross output to GDP share) and sectoral multifactor productivity growth for both 10 and 3 sector levels of aggregation.

By analysing the time series, we found an inflection point on the patterns of change in most variables studied before and after 2008 confirming the findings of de Souza & da Cunha (2018) who identified the same pattern by using an alternative methodology. Hence, we decided to split the analysis for two distinct periods: from 2000 to 2008 and from 2009 to 2014. Considering the average of the entire period analysed - 2000 to 2014 - the sum of Domar weights for the whole economy is 2,042. This means that in addition to the sum of the shares of the values added to GDP, which add up to the unit, there is an extra weight of 1.042 relative to the sectorial densities. This additional portion of the sums of the sectorial weights refers to the degree of sectorial interconnection. It is clear from table 1 that the macro sector with the highest Domar weight is the services one, with 52%, followed by the manufacturing and primary sector macro sectors, with 43.5% and 4.5%, respectively.

Table 1: Average value-added share, Domar weights, multifactor productivity growth rate and density per sector and selected periods.

Sectors	2000 to 2008				2009 to 2014				2000 to 2014			
	II-GDP (sectoral density)	VA-GDP share	Sectoral Domar Weight	Productivity Growth (MPF)	II-GDP (sectoral density)	VA-GDP share	Sectoral Domar Weight	Productivity Growth (MPF)	II-GDP (sectoral density)	VA-GDP share	Sectoral Domar Weight	Productivity Growth (MPF)
Agriculture, forestry and fishing	0,039	0,057	0,096	0,022	0,037	0,051	0,088	0,014	0,038	0,055	0,093	0,018
Primary Industries	0,039	0,057	0,096	0,022	0,037	0,051	0,088	0,014	0,038	0,055	0,093	0,018
Mining, quarryng; Electricity, gas and water supply	0,062	0,068	0,130	0,011	0,058	0,065	0,123	-0,031	0,060	0,067	0,127	-0,007
Manufacturing Industries	0,588	0,207	0,795	-0,004	0,532	0,198	0,730	-0,005	0,564	0,203	0,767	-0,004
Manufacturing	0,650	0,275	0,925	-0,002	0,590	0,263	0,853	-0,009	0,624	0,270	0,894	-0,005
Trade, transport, accommodation and related services	0,136	0,174	0,309	0,007	0,147	0,195	0,343	-0,002	0,141	0,183	0,324	0,003
Information and communication	0,038	0,041	0,079	0,009	0,037	0,037	0,074	-0,022	0,038	0,039	0,077	-0,004
Financial and insurance activities	0,045	0,066	0,111	0,060	0,042	0,064	0,106	-0,045	0,044	0,065	0,109	0,015
Real estate activities	0,006	0,095	0,101	0,048	0,007	0,087	0,095	0,006	0,006	0,092	0,098	0,031
Professional, scientific and support service activities	0,043	0,072	0,114	0,004	0,045	0,077	0,122	-0,024	0,043	0,074	0,118	-0,008
Public administration, defence, education, health and social work activities	0,090	0,188	0,278	0,007	0,084	0,194	0,279	-0,012	0,087	0,191	0,278	-0,001
Other traditional services	0,021	0,032	0,053	0,024	0,019	0,030	0,049	0,004	0,020	0,031	0,051	0,016
Services	0,378	0,668	1,046	0,018	0,381	0,686	1,067	-0,012	0,379	0,676	1,055	0,005

Source: Authors elaboration based on WIOD data.

Although the manufacturing sector is the one with the most capacity to propagate productivity growth in the economy, due to its high sectoral density, it seems to have suffered from a decrease in its productivity, due to its average -0.5% annual MFP growth. That has a double impact - direct and indirect - in decreasing the aggregate productivity of the economy, especially after 2008. The primary sector was the one that generated the highest average annual productivity growth, with an average of 1.8% per year. However, the sector showed low interconnection potential and, therefore, low capacity to propagate productivity growth. The services sector, on the other hand, although it had a vital ability to spread productivity growth, presented a modest average of MFP growth, with an annual average of 0.5%. In addition, it showed a heterogeneous behaviour when the sector is observed in more disaggregated terms.

Regarding sectoral Domar weights, considering the whole period yearly average, the service sector had a 1.05 Domar weight, followed by 0.89 in manufacturing and only 0.093 in the primary industries. It is worth noting that although the services sector was the macro sector with higher Domar weight, with about 52%, it had almost 68% of the total value-added. The manufacturing sector presented 43.5% of the average Brazilian Domar weight but something around 27% of total value-added. This fact shows that the impact of intermediate inputs in the manufacturing sector generated a boost in its Domar weight compared to its value-added share[34]. The primary industries sector, in its turn, had only 4.5% of total average Domar weight, with almost 5% of the value-added share, on average.

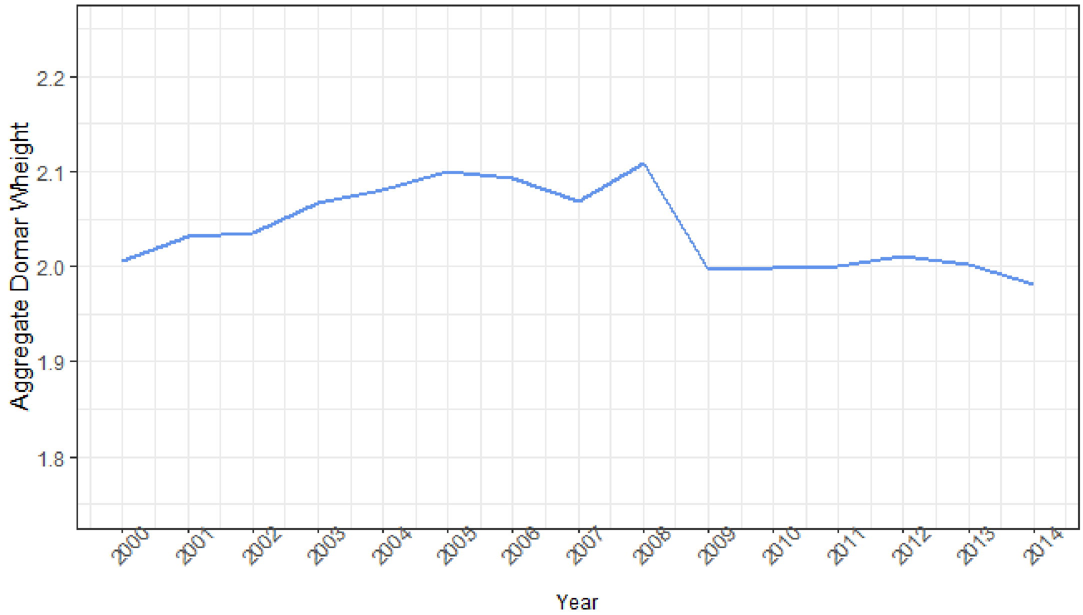
Despite both macro sectors – manufacturing and services – have had the high capacity on potentialising productivity growth throughout the economy, due to its Domar weights, many manufacturing industries showed negative productivity growth during the period. Thus the high density of the manufacturing macro sector acted in a negative way concerning the aggregate productivity growth, helping to spread and increase negative

[34] These findings corrobarates the view emphasized by authors such as Szirmai (2012) and Tregenna (2009), among others, that the manufacturing plays an important role in the growth process due to its forwarding and backward linkages, which are more pronounced than in the service and agricultural sectors. More recently, Gabriel et. al. (2020), using panel data and input-output matrix show that the manufacturing industry's output multipliers and employment are higher than that from the other sectors for developing countries, thus confirming also confirmed the view that productive linkages and spillover effects are stronger within manufacturing industries [Szirmai et al. (2013)].

productivity growths. It is therefore crucial to improve the productivity behaviour of the manufacturing macro sector, since it has an high inpact in the whole economy. Regarding possible reasons to poor MFP performance, Cas and Rymes (1991, pp. 12) argue that a possible reason is a lack of demand "*When Keynesian problems of insufficient aggregate demand are experienced, the waiting or saving of owners of capital is largely spilled onto the sands, and this shows up as a decline in multifactor productivity measures*".

The services macro sector, in its turn, showed a positive average multifactor productivity growth and then its relatively high Domar weight has performed a positive effect on potentialising productivity growth. The primary industries macro sector, albeit it was the sector with higher productivity growth average, it had the lowest sectoral Domar weight, with a relatively limited capacity of boost aggregate productivity growth.

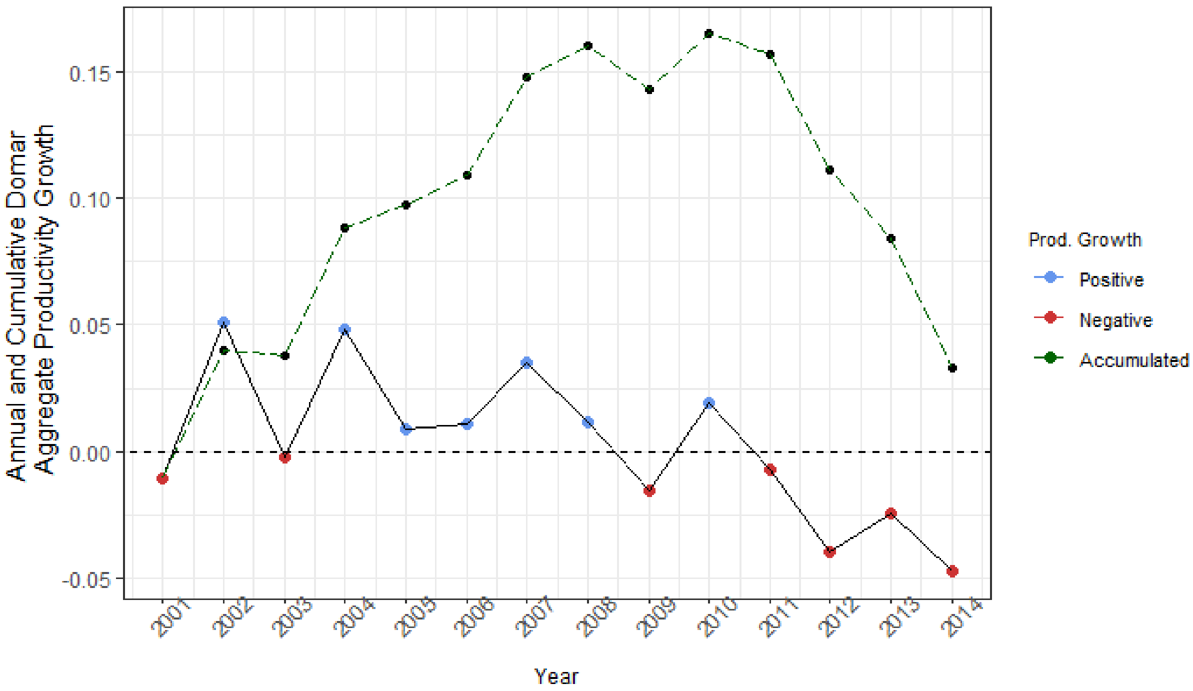
Figure 4: Sum of Brazilian Domar Wheights 2000 - 2014



Note: Author calculations using WIOD data

Figure 4 above shows the aggregate Brazilian Domar weight behaviour between 2000 and 2014. It presented a slightly upward trend until 2008, of almost 5%. After 2008, the pattern has reverted and has more than compensated previous growth. This fact indeed spawned a decrease of average Brazilian sectoral density and, therefore, a decline in both sum of Domar weights and structural capacity in potentialising sectoral productivity growth at the aggregate level. That fact is easier to see in figures 5 and 6 below, which show the aggregate productivity growth measured by the Domar aggregation method for the whole economy and decomposed by macro sectors, respectively. Both figures show the yearly and cumulative Domar aggregate productivity growth.

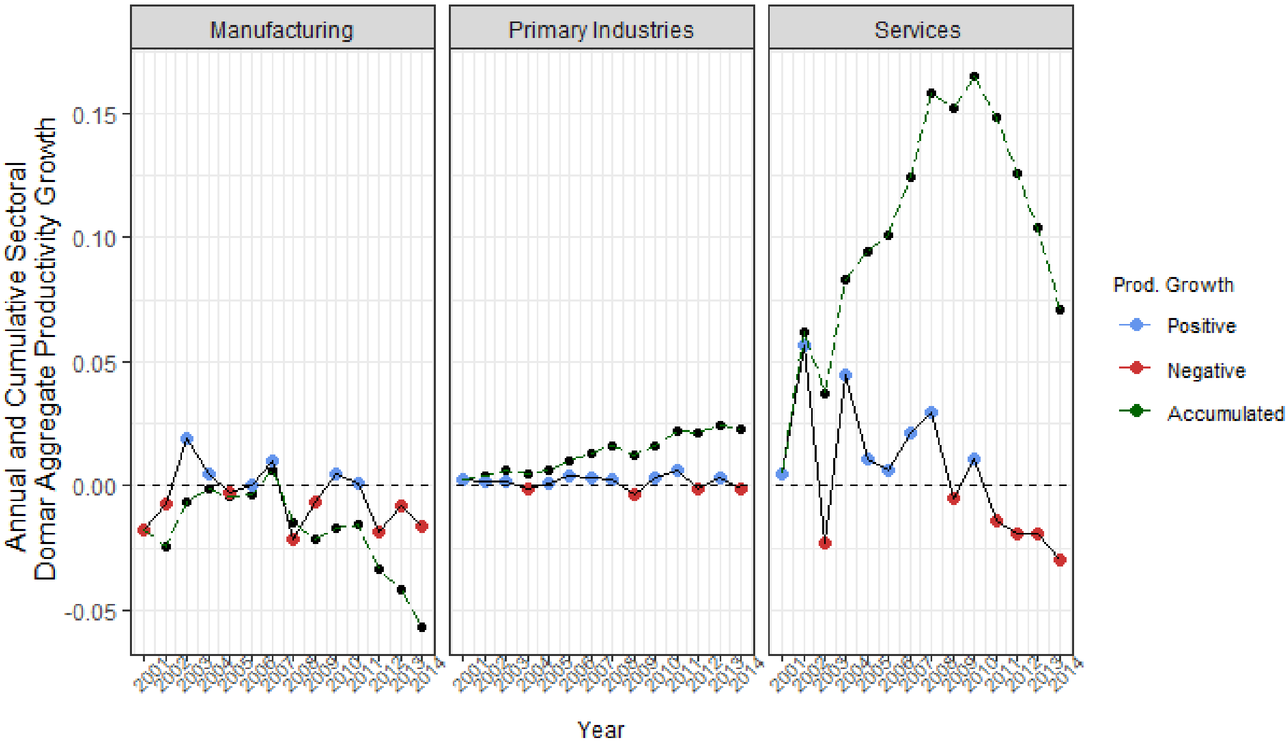
Figure 5: Domar aggregation: yearly and cumulative productivity growth



Note: Author calculations using WIOD Brazilian data

Using equation (9'), it is possible to calculate the yearly and aggregate cumulative productivity growth using the Domar aggregation method. Indeed, given sectoral productivity growth and sectoral densities, the Brazilian aggregate productivity growth increased, from a cumulative point of view, from 2000 to 2010. However, despite that behaviour, the aggregate productivity growth was negative in 2001, 2003 and 2009. However, after that and despite 2010, the yearly aggregate productivity growth was negative in all years, which led to an almost complete reversal of cumulative productivity growth previously undergone, from nearly 17% cumulative growth in 2010 to roughly 3% in 2014.

Figure 6: Domar aggregation: yearly and cumulative productivity growth decomposed by macro sectors



Note: Author calculations using WIOD Brazilian data

The behavior of aggregate productivity growth decomposed by macro sectors and is shown in figure 6. Although the cumulative productivity growth in services and primary industries macro sectors were positive, that of the manufacturing sector was consistently negative due to its negative (MFP) growth potentialised by its high Domar weight and sectoral density.

It is interesting to note that although the Primary industries macro sector was the one with more consistent yearly MPF growth, its positive contribution to overall productivity was limited due to low density and Domar weight, which is portrayed in the figure above. The services macro sector presented a relatively high variance in its annual productivity growth, but it still delivered most of the productivity growth in the economy thinking as an aggregate. After 2009, like aggregate productivity, the services macro sector showed a decrease in both cumulative and average yearly productivity growth.

As pointed out by Wolff (2014), there are two ways of increasing economic growth. The first one is by augmenting the factors available for production (‘factor augmentation’), while the second one is by raising the rate of productivity growth. Table

2 below reveals the contribution of each input and decomposed Domar aggregate productivity growth for each unity of value added for all the three macro sectors and the whole economy, considering the average of 2000 – 2014 period.

Table 2: Average sectoral contribution to aggregate value-added growth split by inputs and Domar productivity growth contributions

Sectors	Sectoral Value Added	Labor Input share	Capital Input share	Productivity Contribution share
Agriculture, forestry and fishing	1,000	-0,185	0,449	0,736
Primary Industries	1,000	-0,185	0,449	0,736
Mining, quarryng; Electricity, gas and water supply	1,000	0,068	1,346	-0,414
Manufacturing Industries	1,000	0,852	0,892	-0,744
Manufacturing	1,000	0,614	1,030	-0,644
Trade, transport, accommodation and related services	1,000	0,499	0,248	0,253
Information and communication	1,000	0,183	1,012	-0,194
Financial and insurance activities	1,000	0,299	0,042	0,659
Real estate activities	1,000	-0,023	0,162	0,861
Professional, scientific and support service activities	1,000	0,468	0,878	-0,346
Public administration, defence, education, health and social work activities	1,000	0,979	0,082	-0,061
Other traditional services	1,000	0,347	0,198	0,454
Services	1,000	0,469	0,293	0,238
Aggregate Economy	1,000	0,452	0,459	0,089

Source: Authors elaboration based on WIOD data.

Considering the three macro sectors and the economy as a whole, the average growth rate of value added generated by the primary macro sector had a negative contribution from the labour input of -18.5%, a positive contribution of capital input of 44.9% and a vital productivity contribution of 73.6%. The manufacturing sector obtained a positive contribution from primary inputs labour and capital of 61.4% and 103%, respectively, but a considerable negative contribution of -64.4% from productivity, for each value added generated. The services sector, in turn, had a positive contribution from either both labour and capital inputs and productivity growth, with 46.9%, 29.3% and 23.8%, respectively. The average of each unit of the added value generated by the economy in the period, considering the economy as a whole, attained the contribution of 45.2% of labour input, 45.9% of capital and 8.9% of generated productivity measured by Domar aggregation. The result that the productivity growth in the service sector was higher than that of the industrial sector is somewhat surprising insofar as we would expect that the latter would have a higher productivity gain than the former[35].

2.5. Concluding Remarks

In this paper, we use the Domar aggregation approach to study the evolution of productivity growth in Brazil from 2000 to 2014. This method was adopted to approach other countries, but for the best of our knowledge, this is the first time for the Brazilian economy. That is particularly important because it allowed us a disaggregated assessment of the Brazilian productivity and growth pattern during that period. We can explain the overall productivity performance of the Brazilian economy not only in terms of the poor performance of its sectors but also in terms of diminishing industrial density, with fewer backward and forward connections amongst industries in terms of chains of intermediate inputs[36].

Besides, despite the relatively high density of the macro manufacturing sector when compared to other sectors in the Brazilian economy, it performed a negative role concerning aggregate productivity growth both directly and indirectly. Directly insofar as that sector had negatives productivity growths during the period under consideration, and indirectly due to its high interconnection, which helped to spread negative rather than positive productivity growth across the economy.

Therefore, to improve the poor performance of the Brazilian economy witnessed in recent years, it is mandatory to enhance the capability of Brazilian manufacturing macro sector to generate productivity growth. It is also important for future investigations to understand the reasons of the low productivity advance carryed out by the Brazilian economy, and the manufacturing macro sector particularly. In sum, Brazil has failed in

[35] This hypothesis is commonly associate to the Baumol's model of unbalanced growth in which he assumes that the service sector is the stagnant one due to its lower productivy gains when compared to the industrial sector. Such view was confirmed empirically by a number of authors such as Appelbaum and Schettkat (1999) and Nordhaus (2008).

[36] The complexity literature advanced by Hausmann et al. (2007) highlights an increase of density as a possible source of better growth performance. In the presence of an intermediate service sector, the shift of resources to the service sector may enhance rather than decrease aggregate productivity growth even if the productivity growth of the service sector is lower than that of the industrial sector.

its task to deepen its industrial density. As a consequence, it has witnessed a prematurely shrink in the share of the manufacturing sector in GDP, being stuck in a middle-income trap.

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2.7. Apendix

Table 2: Detailed levels of sectoral aggregation

Sector (48 levels)	Code (ISIC Rev.4)	Sector (10 levels)	Sector (3 levels)
Crop and animal production, hunting and related service activities	A01	Agriculture, forestry and fishing	Primary Industries
Forestry and logging	A02	Agriculture, forestry and fishing	Primary Industries
Fishing and aquaculture	A03	Agriculture, forestry and fishing	Primary Industries
Mining and quarrying	B	Mining, quarryng; Electricity, gas and water supply	Manufacturing
Manufacture of food products, beverages and tobacco products	C10-C12	Manufacturing Industries	Manufacturing
Manufacture of textiles, wearing apparel and leather products	C13-C15	Manufacturing Industries	Manufacturing
Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials	C16	Manufacturing Industries	Manufacturing
Manufacture of paper and paper products	C17	Manufacturing Industries	Manufacturing
Printing and reproduction of recorded media	C18	Manufacturing Industries	Manufacturing
Manufacture of coke and refined petroleum products	C19	Manufacturing Industries	Manufacturing
Manufacture of chemicals and chemical products	C20	Manufacturing Industries	Manufacturing
Manufacture of basic pharmaceutical products and pharmaceutical preparations	C21	Manufacturing Industries	Manufacturing
Manufacture of rubber and plastic products	C22	Manufacturing Industries	Manufacturing
Manufacture of other non-metallic mineral products	C23	Manufacturing Industries	Manufacturing
Manufacture of basic metals	C24	Manufacturing Industries	Manufacturing
Manufacture of fabricated metal products, except machinery and equipment	C25	Manufacturing Industries	Manufacturing
Manufacture of computer, electronic and optical products	C26	Manufacturing Industries	Manufacturing
Manufacture of electrical equipment	C27	Manufacturing Industries	Manufacturing
Manufacture of machinery and equipment n.e.c.	C28	Manufacturing Industries	Manufacturing
Manufacture of motor vehicles, trailers and semi-trailers	C29	Manufacturing Industries	Manufacturing
Manufacture of other transport equipment	C30	Manufacturing Industries	Manufacturing
Manufacture of furniture; other manufacturing	C31_C32	Manufacturing Industries	Manufacturing
Electricity, gas, steam and air conditioning supply	D35	Mining, quarryng; Electricity, gas and water supply	Manufacturing
Water collection, treatment and supply	E36	Mining, quarryng; Electricity, gas and water supply	Manufacturing
Construction	F	Manufacturing Industries	Manufacturing
Wholesale and retail trade and repair of motor vehicles and motorcycles	G45	Trade, transport, accommodation and related services	Services

Wholesale trade, except of motor vehicles and motorcycles	G46	Trade, transport, accommodation and related services	Services
Retail trade, except of motor vehicles and motorcycles	G47	Trade, transport, accommodation and related services	Services
Land transport and transport via pipelines	H49	Trade, transport, accommodation and related services	Services
Water transport	H50	Trade, transport, accommodation and related services	Services
Air transport	H51	Trade, transport, accommodation and related services	Services
Warehousing and support activities for transportation	H52	Trade, transport, accommodation and related services	Services
Accommodation and food service activities	I	Trade, transport, accommodation and related services	Services
Publishing activities	J58	Information and communication	Services
Motion picture, video and television programme production, sound recording and music publishing activities; programming and broadcasting activities	J59_J60	Information and communication	Services
Telecommunications	J61	Information and communication	Services
Computer programming, consultancy and related activities; information service activities	J62_J63	Information and communication	Services
Financial service activities, except insurance and pension funding	K64	Financial and insurance activities	Services
Real estate activities	L68	Real estate activities	Services
Legal and accounting activities; activities of head offices; management consultancy activities	M69_M70	Professional, scientific and support service activities	Services
Architectural and engineering activities; technical testing and analysis	M71	Professional, scientific and support service activities	Services
Scientific research and development	M72	Professional, scientific and support service activities	Services
Administrative and support service activities	N	Professional, scientific and support service activities	Services
Public administration and defence; compulsory social security	O84	Public administration, defence, education, health and social work activities	Services
Education	P85	Public administration, defence, education, health and social work activities	Services
Human health and social work activities	Q	Public administration, defence, education, health and social work activities	Services
Other service activities	R_S	Other traditional services	Services
Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use	T	Other traditional services	Services

Source: Authors elaboration based on WIOD data.

Chapter Three: A Reinterpretation of Vertically (Growing) Hyper-Integrated Sectors and the role of Technical Progress

3.1. Introduction

Pasinetti (1973, pp.1) states that "Very few notions in economic analysis are so seldom explicitly mentioned as the notion of vertical integration and are at the same time so widely used, implicitly or without full awareness". Since then and following the systematization of the notion of vertical integration advanced by Sraffa's and Pasinetti's works, considerable effort has been made to advance investigations concerning the theme seeking to answer former and new issues.

Treating the economic system with the point of view of vertically integrated sectors allows one to focus on distinct aspects in comparison with standard input-output models. Besides it allows us to perceive the economic system in terms of the final commodities, which are the ultimate purpose of production and social welfare. Moreover, the device of vertical integration enables us to focus on the necessary interconnections among industries, and thus all the required direct and indirect chains of inputs that gives rise to the sub-systems.

The Sraffian system approach delivers some key conclusions in that subject regarding economic growth and productivity capacity. Indeed, under assumptions such as uniform rate of profit and equal growth rate among sectors, the inverse of the maximum eigenvalue from the technical matrix can be therefore seen as a measure of maximum potential economic growth and also a measure of productivity capacity for the economy as a whole [see e.g. Pasinetti (1977) and De Juan and Febrero (2000)].

The problem with the Sraffian analysis is that it considers that all sectors grow at the same rate, meaning that all of them have the same growth rate of demand. One of the main contributions of the Pasinettian analysis is to show that the interaction between particular growth rates of demand and labor productivity are behind the structural changes that shape the economic structure. In this set up the Sraffian hypothesis that all sectors have the same growth rate of demand is unsettling. In this vein, Pasinetti (1988) relativizes some usual assumptions and delivers an economic system with growing subsystems, allowing them and their demand to grow at an uneven pace.

Despite the Pasinettian efforts to generalize the result toward the case in which sectors grow at particular or uneven rates, which are given by the sum of the populational growth and the particular growth rate of per capita demand, he just managed to provide a sectoral outlook. At the end of Pasinetti's (1988) work, he hints of further developments of his model in the case of technical progress.

We show that when one extends the Sraffian analysis in Pasinettian lines to consider that each sector has a particular growth rate of demand, the inverse of the maximum eigenvalue from the technical matrix cannot be anymore seen as a measure of maximum potential economic growth and also a measure of productivity capacity for the economy as a whole. Then, in this case, we need to consider alternative prouctivity measures.

Accordingly, we seek at delivering an approach of treating labour productivity that takes place at the industry's level and that are transmitted to the subsystems. Thereby we build a measure of both sectoral and aggregate technical change growth rate that explicitly takes place at the industry level and also sectoral and aggregate estimates of economic growth that takes into account the contribution of the productivity advance. It is shown explicitly that the technical change of the subsystems accrues from the direct and indirect labour used up in its production, which follows the interconnection among the economic system and therefore the productivity transmission made possible by it.

With this analysis, we aim to build an alternative approach of vertically (growing) hyper-integrated sectors with technical progress. Several criticisms have been made concerning the treatment of technical progress within vertically integrated sub-systems. For instance, Lavoie (1997) argues that technical progress in vertically integrated sectors is often supposed exogenous and independent among sectors. Still, it indeed takes place at the industry level and therefore cannot be thought as being independent of each other. A fruitful discussion about this subject is also made by Hagemann (2012).

It is also approached the duality of the technical change process. The impact of the technical change in each sub-system directly impacts the growth rate of prices which is a well-known result [see, e.g. De Juan and Febrero (2000)], that is explicitly explored here. Following our assumptions of the industry's profit rate, the impact of the sectoral technical change in diminishing prices is the higher, the fewer the share of profit in each sub-system.

Finally, we investigate the differences and behaviours of three of the most essential measures of productivity growth regarding input-output systems, namely the neoclassical multifactor productivity, the effective productivity advanced by Cas and Rymes (1991) and an estimate of productivity growth from vertically integrated sectors. We show that the last two deal with technical change, taking into account the interconnections and transmission of productivity among industries in shaping the productivity of sectors producing final commodities. The multifactor productivity growth measure, however, does not treat circulating capital as produced and misses productivity transmission among industries, at least at the industrial level.

Our investigation is organized as follows. In the next section, we show the well-established Sraffian measure of maximum economic and productivity growth. In the third and fourth segments, we respectively advance at building our economic system and providing a measure of sectoral technical advance. In the fifth section, it is established the relations among technical change and prices growth rate. In the sixth section, we show the relations and some implications among three distinct kinds of productivity measurement. Finally, some concluding remarks are done in the last segment.

3.2. The Sraffian system: a measure of productivity and e economic growth

The Sraffian approach to the Leontief system is well-known, and we present it here as a standpoint to the analysis we unfold. Then the starting point of our analysis is the Leontief open system, given by the following equation:

$$\mathbf{X} = \mathbf{Y} + \mathbf{AX} \qquad (1)$$

From (1) we know that $\mathbf{Y} = [\mathbf{I} - \mathbf{A}]\mathbf{X}$**,** which yields after some algebraic manipulation:

$$\mathbf{X} = [\mathbf{I} - \mathbf{A}]^{-1}\mathbf{Y} \qquad (1)'$$

Equation (1)' shows the column vector **X** of production required to produce the vector of final demand **Y**, given direct and indirect intermediate inputs necessary to carry out the production of final goods in the economy, namely $[\mathbf{I} - \mathbf{A}]^{-1}$. Alternatively, we can express the economy in terms of the price system as:

$$\mathbf{PA}(1 + \pi) + \boldsymbol{a_n} w = \mathbf{P} \qquad (2)$$

Where $\mathbf{P}$ is the column vector of prices, $\boldsymbol{a_n}$ is the vector of labour coefficients, w is the uniform wage and π is the uniform rate of profit. After some algebraic manipulation equation (2) may be written as:

$$\mathbf{P} = [\mathbf{I} - (1+\pi)\mathbf{A}]^{-1}\boldsymbol{a_n}w \tag{3}$$

In order to proceed to a Sraffian approach, an alternative way of writing equation (1) is:

$$\begin{bmatrix} a_{11} - \frac{1}{1+R_1} & \cdots & a_{1,n-1} \\ \vdots & \ddots & \vdots \\ a_{n-1,1} & \cdots & a_{n-1,n-1} - \frac{1}{1+R_{n-1}} \end{bmatrix} \begin{bmatrix} X_1 \\ \vdots \\ X_{n-1} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \tag{4}$$

Where $R_i = \frac{Y_i}{X_i - Y_i}$. If $R_i = R$ for $i = 1, \dots, n-1$, then we can write system (4) as:

$$\mathbf{AX} = \eta\mathbf{X} \tag{5}$$

Where $\eta = \frac{1}{1+R}$. Equation (5) may be written as:

$$(\eta\mathbf{I} - \mathbf{A})\mathbf{X} = 0 \qquad (5)'$$

This is a homogenous system. A necessary condition for its having solutions other than zero is that the determinant of the matrix of coefficients be zero. We can find the values of η which satisfy this condition by solving the characteristic equation:

$$det(\eta\mathbf{I} - \mathbf{A}) = 0 \tag{6}$$

The roots of the above equation are the eigenvalues of matrix $\mathbf{A}$. There are $(n-1)$ eigenvalues that satisfy equation (5)'. However, not all these eigenvalues have an economic meaning. In fact, only one of them, the maximum eigenvalue η_m is undoubtedly associated with a strictly non-negative eigenvector, whose elements represent physical quantities. Moreover $R = \frac{1}{\eta_m} - 1$ is the uniform rate of the surplus of the system.

Pasinetti (1988, p. 125) aims at extending the Sraffian analysis based on the concepts of vertically integrated sub-systems to a dynamic analysis. The departing point of his research is a dynamic version of the system (1) as presented by Pasinetti (1973, 1977). Let us consider now the case in which the population grows at an exogenous rate g, but there is no change in per capita demand for each of the goods. In this case, let us assume that:

$$\mathbf{Y}(t) = \mathbf{C}(t) + \mathbf{J}(t) \quad (7)$$

Where $\mathbf{C}(t)$ denotes the (column) vector of the physical quantities which are devoted to consumption, and $\mathbf{J}(t)$ denotes the (column) vector of the physical quantities of the same commodities which are required to increase the production of final goods. Then equation (1) should be written as:

$$\mathbf{X}(t) = \mathbf{C}(t) + \mathbf{J}(t) + \mathbf{AX}(\mathrm{t}) \quad (8)$$

Let us assume that the demand for each new consumer good increases at the percentage rate g. Then a necessary condition for the corresponding production to take place is that all the means of production increase at the same percentage rate.

$$\mathbf{J}(t) = g\mathbf{AX}(\mathrm{t}) \quad (9)$$

Substituting (9) into (8) one obtains after some algebraic manipulation:

$$\mathbf{C}(t) = [\mathbf{I} - (1+g)\mathbf{A}]\mathbf{X}(t) \quad (10)$$

It is possible to rewrite (10) as:

$$\mathbf{X}(t) = [\mathbf{I} - (1+g)\mathbf{A}]^{-1}\mathbf{C}(t) \quad (11)$$

Where $[\mathbf{I} - (1+g)\mathbf{A}]^{-1}$ is the 'Leontief dynamic inverse matrix'. From (11) we can find the maximum growth, following the Sraffian/Pasinettian analysis by setting $\mathbf{C}(\mathrm{t}) = \mathbf{0}$. In this case:

$$[\mathbf{I} - (1+G)\mathbf{A}]\mathbf{X}(t) = \mathbf{0} \quad (12)$$

Where G is the maximum growth rate, namely $g_{max} = G$. We can rewrite (12) as:

$$[\eta\mathbf{I} - \mathbf{A}]\mathbf{X}(t) = \mathbf{0} \quad (13)$$

Where $\eta_{max} = \frac{1}{(1+G)}$ is the maximum eigenvalue of **A**. According to De Juan and Febrero (2000), the inverse of the maximum eigenvalue has several important economic implications. It can be therefore seen as a measure of maximum potential economic growth and also a measure of productivity capacity for the economy as a whole since it shows the maximum growth rate given the economy's inputs. Since it is assumed that all sectors grow at the same rate, then the maximum growth rate holds both to sectoral an aggregate level.

It is, however, worth noting that this is valid under the assumptions taken so far, that is, under a uniform profit rate and all sectors growing at the same pace. Hereafter we aim to deliver measures of both sectoral and aggregate economic growth rate and technical change for growing subsystems, allowing for distinct demand and production growth rates among vertically integrated sectors.

3.3. Growing sub-systems and technical change: an alternative device of vertical integration

Let us now allow for distinct growth rates among industries and sectors and consider a slightly different growing system with technical progress. To begin with, suppose that this economy also has $n-1$ industries and that the nth sector belongs to the family's sector. Each industry uses labour input acquired from the family's sector, paying an exogenous homogenous wage $w = p_n$ for it, and uses circulating capital from the system of $j = 1, \ldots, n-1$ industries as inputs. Notice that the homogeneous wage can be seen as the numéraire of prices by fixing $w = 1$. Besides, each industry pays for each physical commodity used as circulating capital its respective production price defined by the production system. Therefore, every industry faces the following input and output equations:

$$p_j X_j = wL_j + \sum_{i=1}^{n-1} p_i x_{ij} + \pi_j \quad (14)$$

$$p_j X_j = p_j Y_j + \sum_{i=1}^{n-1} p_j x_{ji} \quad (15)$$

The first two terms on the right hand side of equation (14) stands for the production costs faced by the jth industry, which are, respectively, the costs of labour inputs and the costs of circulating capital necessary to handle its production. Additionally, each industry would enact an exogenous amount of profit, or surplus, represented by π_j which it is capable of carrying out. Hence, the production price in (14) can be viewed as the production cost, in which the jth industry has no direct control on it, plus the profit, or surplus, added to its price.

Equation (15) shows that the output produced by industry j is completely sold either to final demand $p_j Y_j$ and intermediate demand $\sum_{i=1}^{n-1} p_j x_{ji}$ from all $i = 1, \ldots, n-1$ industries. One can rewrite both equations using matrix notation by rearranging them and considering that $\mathbf{a}_{\boldsymbol{n}}$ is a vector of the j labour coefficients defined by $a_{nj} = \frac{L_j}{X_j}$ and, similarly, $\mathbf{q}$ is a vector of the rate of profit capacity for each industry, where $q_j = \frac{\pi_j}{X_j}$:

$$\mathbf{P}(t) = w\mathbf{a}_{\boldsymbol{n}}(t) + \mathbf{AP}(t) + \mathbf{q} \tag{16}$$

$$\mathbf{X}(t) = \mathbf{Y}(t) + \mathbf{AX}(\mathbf{t}) \tag{17}$$

At this point we are assuming that all the production coefficients $a_{ij} = \frac{X_{ij}}{X_j}$ of the matrix $\mathbf{A}$ are constant, which implies that[37] $dlogX_{ij} = dlogX_j$ for all i and j. However, we also assume that it may not be the case for labour coefficients $\mathbf{a}_{\boldsymbol{n}}(t)$. Thus, all industrial technical progress must be labour reducing, in which $[\mathbf{A}, \mathbf{a}_{\boldsymbol{n}}(t)]_t$ is its technique of production, which means that both technique and labour coefficients are allowed to vary with time. The labour reducing technical advance in every industry could be understood as a decrease in the labour coefficients requirements against the previous segment of time $a_{nj}(t) = a_{nj}(t-1) - \beta_j(t)\, a_{nj}(t-1)$, where $\beta_j(t)$ stands for the jth industry's labour reducing technical progress[38] in the period t.

Regarding equation (17), notice that in each period the term $\mathbf{AX}(\mathbf{t})$ must account for both the replacement of previous circulating capital and its necessity of expansion, given both the technique of production and sectoral demand requirements. Therefore, we assume that in each period $\mathbf{AX}(\mathbf{t}) = \mathbf{AX}(\mathbf{t-1}) + \mathbf{A}\hat{\boldsymbol{\gamma}}\mathbf{X}(\mathbf{t-1})$, in which the first addendum on the right-hand side stands for the necessity of replacement of produced means of production and the second term stands for the need of circulating capital expansion in a given period based on previous capacity. Besides, $\hat{\boldsymbol{\gamma}}$ is a diagonal matrix in which each term of the diagonal represents the industrial necessity of circulating capital expansion.

We also assume that, at the end of each period, the necessary expansion of circulating capital for the subsequent period, in each industry, is sourced by the moving

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[37] We adopt here the notation that $dlogz$ indicates variable's z growth rate with respect to time, defined by $dlogz = \frac{dz}{dt}\frac{1}{z}$.

[38] Notice that if $\beta_j(t)$ is positive, it means that either the same amount of commodities previously produced by the jth industry could be produced with less quantity of labour or that a higher quantity of commodities could be produced with the same quantity of labour as before.

economic system capacity of yielding profit, or surplus, which is represented by the vector $\mathbf{q}$ in equation (16). It is also possible to rewrite the previously industrial system into a moving system of vertically integrated sectors. Thus, each final consumption good gives rise to a particular vertically integrated sector, which is a composite sector of all the necessary inputs from distinct industries required to produce each consumption commodity. By manipulating equation (17), one has that:

$$\mathbf{X}(t) = [\mathbf{I} - \mathbf{A}]^{-1}\, \mathbf{Y}(\mathrm{t}) \tag{18}$$

The well-known equation (18) above shows, on the left-hand side, the vector of total industrial production necessary to deliver all final commodities produced by every single vertically integrated sector, given its requirements of production. Ought to the definition of the labour coefficients vector one can rewrite (18) as:

$$L(t) = \mathbf{a}_{\boldsymbol{n}}(t)[\mathbf{I} - \mathbf{A}]^{-1}\mathbf{Y}(\mathrm{t}) \tag{19}$$

Equation (19) has a remarkable economic meaning that all direct and indirect labour expended in all vertically integrated sectors sums up to the overall labour availability, in the given period, defined by the scalar L(t). Note that the term $[\mathbf{I} - \mathbf{A}]^{-1}$ represents the requirements of physical quantities of commodities which have been used up directly and indirectly in the economic system to obtain one unit of each final consumption good [see, e.g. Pasinetti (1977)]. Therefore, the vertically integrated labour coefficients are given by the following row vector, where each of its components $l_j^* = \sum_{j=1}^{n-1} l_{ij}^*$ denotes the sum of all direct and indirect vertically integrated labour coefficients[39]:

$$\mathbf{l}'^*(\mathrm{t}) = \mathbf{a}'_{\boldsymbol{n}}(t)[\mathbf{I} - \mathbf{A}]^{-1} \tag{20}$$

It is worth noting that the vector $\mathbf{l}'^*$ of direct and indirect labour requirements of each vertically integrated sector (hereafter VIS) can be viewed as a measure of technological interdependence among industries in the economic system, aiming to yield

[39] Indeed, notice that each component α_{ij}inside $[\mathbf{I} - \mathbf{A}]^{-1}$ represents the direct and/ or indirect requirements from the ith industry to the jth one. Thereby, each post multiplication of $\mathbf{a}'_{\boldsymbol{n}}(t)$ by a column in $[\mathbf{I} - \mathbf{A}]^{-1}$ delivers the sum of all direct and indirect labour requirements for each vertically integrated sector.

each final consumption good[40]. The row vector of total direct and indirect labour employed in each VIS is defined as below:

$$\mathbf{L'}^{\boldsymbol{vi}}(t) = \mathbf{l'}^{*}(t)\widehat{\mathbf{Y}}(\mathrm{t}) \quad (21)$$

Where $\widehat{\mathbf{Y}}(\mathrm{t})$ refers to a diagonal matrix in which the non-null elements of its diagonal stands for the final commodities produced by each VIS. Each component of the vector of a total vertically integrated labour $\mathbf{L'}^{\boldsymbol{vi}}(t)$ is defined by $l_j^{*}Y_j = L_j^{vi}$ and represents the sum of all direct (L_j^D) and indirect (L_j^I) labour used up in each jth VIS:

$$L_j^{vi} = L_j^D + L_j^I. \quad (22)$$

Referring now to the price system, we rearrange equation (16) as follows

$$\mathbf{P}'(t) = \mathbf{a'}_{\boldsymbol{n}}(t)[\mathbf{I}-\mathbf{A}]^{-1}w + \mathbf{q}'[\mathbf{I}-\mathbf{A}]^{-1}$$

Thereby, by defining $\boldsymbol{q}^{*} = [\mathbf{I}-\mathbf{A}]^{-1}\mathbf{q}$ as the vector of the vertically integrated profit rate embodied directly and indirectly in each VIS, the equation above become:

$$\mathbf{P}(t) = \mathbf{l}^{*}(t)w + \boldsymbol{q}^{*} \quad (23)$$

Equation (23) shows that the price of each commodity is composed both by the total direct and indirect labour requirements value and by an exogenous term indicating the volume of direct and indirect surplus delivered by each VIS.

3.4. Reviewing growing subsystems, technical progress and economic growth

Although each industry has its independent labour reducing technical progress rate, defined as $\beta_j(t)$ above, each VIS, as a composite sector of industries, produces their final commodities with changing efficiency due to its composite technical progress.

The total labour productivity of each VIS is thereby defined by the ratio of its final commodities production and the total direct and indirect labour used up in its productive process.

$$\rho_j^{VIS} = \frac{Y_j}{L_j^{vi}} \quad (24)$$

[40] Every final good produced by the jth vertically integrated sector contains, embodied in it, both direct labour – a portion of L_j used up in the production of the gross product of industry j - and indirect labour employed to produce all required circulating capital from other industries.

By using (21), taking logs and differentiating we get equation below:

$$dlog\rho_j^{VIS} = dlogY_j - \frac{L_j^{\boldsymbol{D}}}{L_j^{\boldsymbol{D}}+L_j^{\boldsymbol{I}}} dlogL_{\boldsymbol{j}}^{\boldsymbol{D}} - \frac{L_j^{\boldsymbol{I}}}{L_j^{\boldsymbol{D}}+L_j^{\boldsymbol{I}}} dlogL_{\boldsymbol{j}}^{\boldsymbol{I}} \qquad (25)$$

That means that the total labour productivity advance in each VIS is defined by the difference between its final production growth rate and the growth rate on the usage of labour, both directly and indirectly applied, weighted by its share in total labour used. Note that another possible way of writing (25) is by using the fact that $\rho_j^{VI} = \frac{Y_j}{L_j^{vi}} = \frac{Y_j}{l_j^* Y_j} = \frac{1}{l_j^*}$, and proceed the same steps as before. Therefore the productivity growth rate of each VIS can be alternatively written as:

$$dlog\rho_j^{VIS} = -\sum_{i=1}^{n-1} \frac{wl_{ij}^*}{wl_j^*} dlogl_{ij}^* \qquad (26)$$

It is worth noting that if one totally differentiates (21) and substitute (26) on it will find (25) as well. Also notice by (26) that the technical advance from a given VIS could be thought as the weighted decrease rate of the direct and indirect labour coefficients, which are given by the industries that compose the VIS and its technical progress. Moreover, as pointed out by Cas and Rymes (1991) and Lavoie (1997), the productivity advance of a composite sector depends upon the technical change in all suitable industries[41] that are part of the sub-system. Thus, the labour reducing technical change from each industry delivering labour either directly or indirectly, via intermediate inputs, to a given VIS, must affect its technical advance and mitigate its costs requirements.

A remarkable fact that accrues from the measure of technical advance above is that, given productivity gains at the industry level, the technical progress of the composite sectors also depends on the degree of industrial interconnection[42] necessary for the production of each final commodity. This is due to the declining requirements of indirect labour embodied in all required circulating capital, which attempt to reduce costs and

[41] This stems from the fact that, under industrial technical advance, less direct labour would be necessary to yield the same quantity of final good and less indirect labour would be needed to produce the same amount of circulating capital and as a consequence less indirect labour also would be necessary to yield the same quantity of final goods, given technical requirements.
[42] Indeed, Cas and Rymes (1991) emphasize the industry's productivity advance channels in which the composite sectors benefits via fewer requirements of primary inputs and thereby declining costs. De Juan and Febrero (2000), however, argue that the VIS's productivity growth depends on the structural relations between all industries and accounts for the productivity transfers from innovative industries to those who need their inputs, directly or indirectly.

spread technical advance in the growing sub-systems and ultimately in the whole economic system.

Aiming to give rise to an equation defining the supply changing capacity from each growing sub-system, let us manipulate (25) as below. Also note that, in equilibrium, the supply and demand of each growing sub-system's final commodity must change at the same pace.

$$dlogY_j = dlog\rho_j^{VI} + dlogL_j^{vi} = g + r_j \quad (27)$$

That is, in equilibrium the supply growth rate $dlogY_j$ must move along with the demand growth rate for the jth consumption good defined as $g + r_j$. Following Pasinetti (1988), g represents the population growth rate and r_j stands for the per capita demand growth rate for the jth consumption good. Hence, the supply of the final commodity produced by a given growing sub-system could grow both sourced by a technical advance or by an increase in the total labour used up by the composite sector either directly or indirectly.

Let us now turn our analysis to the economic growth and productivity advance achieved by the whole economy. To begin with, we define the aggregate production as the sum of all final goods produced by each VIS, namely $Y = \sum_{j=1}^{n-1} Y_j$. Regarding the labour input, by equation (19) we have that $L = \boldsymbol{l}'^{*}\mathbf{Y} = \sum_{j=1}^{n-1} \boldsymbol{L_j^{vi}} = \sum_{j=1}^{n-1} L_j$. That is, the total labour employed in the whole economy could be seen both as the sum of all labour used up in each industry and by the sum of all direct and indirect labour hired in each VIS. Additionally, we set out here the overall technical progress achieved by the economy as the ratio between the total net production and the total labour employed.

$$\rho^{VI} = \frac{Y}{L} \quad (28)$$

Indeed, by taking logs and differentiating the equation above, one can find the following equation regarding the overall economic productivity growth[43]:

$$dlog\rho^{VI} = \sum_{j=1}^{n-1} \frac{Y_j}{\sum_{j=1}^{n-1} Y_j} dlogY - \sum_{j=1}^{n-1} \frac{L_j^{vi}}{\sum_{j=1}^{n-1} L_j^{vi}} dlogL_j^{vi} \quad (29)$$

[43] Notice that it is possible to multiply the first and second addendums on the right-hand side of (16) by, respectively, $\frac{p_j}{p_j}$ and $\frac{w}{w}$ to obtain the weighted shares defined as values rather than real measures.

The overall economy's growth rate could be set out as an aggregate equivalent of equation (27) as following:

$$dlogY = dlog\rho^{VI} + \sum_{j=1}^{n-1} \frac{L_j^{vi}}{\sum_{j=1}^{n-1} L_j^{vi}} dlogL_j^{vi} \qquad (30)$$

Hence, the growth rate of the economy's net product can evolve due to a growth rate combination of the aggregate productivity advance and the total direct and indirect labour usage, along with a suitably weighing. The Sraffian system delivers the maximum growth rate of the economic system by the maximum eigenvalue from the coefficients matrix. Here we delivered both a sectoral and aggregate economic growth given uneven growing sub-systems.

3.5. Relationship between changing prices and technical progress within growing sub-systems

Let us turn again to the industry level device to discuss some aspects regarding the price equations and its relationship with the growing composite sub-systems and the technical progress carried out by them. As discussed in the second section, each industry faces its price formation as $p_j(\mathrm{t}) = w\mathrm{a}_{nj}(t) + \sum_{i=1}^{n-1} p_i(t)a_{ij} + \mathrm{q}_i$ which is portrayed in matrix notation for all economic system by equation (16). Moreover, the price equations for the vertically integrated sectors are depicted, in matrix terms, by equation (23). The following equation is the equivalent of equation (23) for the jth sector:

$$p_j(t) = l_j^*(t)w + q_j^* \qquad (31)$$

Equation (31) simply shows that the price of the final commodity produced by the jth composite sector, in each period, is defined by the value of all direct and indirect labour inputs used up in its production and by an exogenous direct and indirect surplus achieved by the jth composite sector. By taking logs and differentiating (31) with respect to time:

$$dlogp_j = \sum_{i=1}^{n-1} \frac{wl_{ij}^*}{wl_j^* + q_j^*} dlogl_{ij}^* \qquad (32)$$

Notice that the growth rate of the final commodity's price produced by a given VIS move in a similar way as the negative of its productivity advance depicted by equation (26). The only difference is that the denominator of (32) has the sectoral composite profit rate added to the sectoral vertically integrated labour coefficient. Notice that the denominator of (32) is equal to the price equation (31), which is equivalent to the denominator of (26) plus the term q_j^*.

If we multiply both sides of (32) by $\frac{wl_j^*+q_j^*}{wl_j^*} = \frac{p_j}{wl_j^*}$ and using (26) it is possible to unearth a relation that indicates the elasticity of the price's growth rate relative to the growth rate of the sectoral productivity change:

$$dlogp_j = -\frac{wl_j^*}{wl_j^*+q_j^*}\ dlog\rho_j^{VIS} \qquad (32')$$

In other words, the growth rate of the price of a given VIS moves proportionally, but in the opposite way, to the growth rate of the sectoral technical change as may be expected. The elasticity of that connection depends on the relative weight of the total labour costs, namely wl_j^* on the price equation (18). Therefore, the changing price of the composite sector moves along with the declining labour costs necessary for its production process, either directly or indirectly.

The fewer the composite profit rate, the grater the impact of a positive technical change on lowering prices, due to the impact of lowering direct and indirect labour costs. If we consider the possibility of null profit rate, that is, the existence of perfect competition and $p_j(t) = l_j^*(t)w$, then $dlogp_j = -dlog\rho_j^{VIS}$ and each percentage rate of productivity advance in a given VIS corresponds to the negative of that percentage change on prices.

3.6. The role of industries on vertically integrated sectors' technical change: comparing alternative methods of accessing productivity advance

Lavoie (1997) argues that Pasinetti (1981, 1993) often advances his analysis as if the technical progress of each VIS were independent of each other. He disagrees with the conception of entirely autarchic sub-systems, mainly concerning technical advance.

Indeed, both Schefold (1982) and Lavoie (1997) defend that the productivity changes takes place at the industry level and given that each VIS are a composite sector of industries, in which each industry can be part of several sub-systems, the rates of technical advance in different VIS cannot be seen as being independent of one another.

The point is that as productivity growth takes place at the industry level, those industries can produce intermediate commodities more efficiently, which helps to spread productivity advance across all economic system. Pasinetti[44], concerning the points razed by Schefold (1982) and Lavoie (1997), states that "any technical change, taking place at the industry level, will indeed affect the changes in many (…) vertically integrated sectors. It is however inappropriate to say that the various vertically integrated sectors are inter-dependent, because dependence does *not* run from one vertically integrated sector to another. What one should say is that they are all causally dependents (…) on the same technical changes that take place in the industry level". That is, the composite sectors would be indirectly dependent from one another (through common industry channels), but not directly dependents.

To clarify the sources of technical change that arises in each VIS and its industry origins we now turn to analyze other prominent measures of technical progress in an input-output framework and compare them with the productivity advance of composite sectors. To do so, we compare the 'traditional' mainstream multifactor productivity growth (MFP), the 'new measure', or effective productivity (EP) growth, advanced by Cas and Rymes (1991) and an adapted version of the VIS's technical progress advanced above. We define, in the industry level, the MFP and the effective productivity measures, respectively, by ρ_j^{MFP} and ρ_j^{EP}.

In this vein, we need to apply a similar hypothesis in both three measures to turn them comparable. Therefore, we assume now that there is no profit – the economy works in perfect competition. Besides, there are only labour and circulating capital as inputs, and now we allow both labour *and* technical coefficients to change with time. Hence, the relevant production's technique now becomes $[\mathbf{A}(t), \mathbf{a}_{\boldsymbol{n}}(t)]_t$.

[44] This quotation is due to Hagemman (2012), which shows a stretch of a letter from Luigi Pasinetti to Marc Lavoie, dated 16 January 1995.

The MFP measurement can be seen in a dual way, from the quantities and prices sides. The MFP growth rate is defined as the difference between the industry's production growth rate and the growth rate of all measurable inputs used up in its production, weighted by the ratio between the value of each input and the total value of production. As we are working now in a perfect competition environment, equation (1) becomes:

$$p_j X_j = wL_j + \sum_{i=1}^{n-1} p_i x_{ij} \qquad (14')$$

Taking logs and differentiating both sides of equation (1') gives us the following:

$$dlogp_j + dlogX_j = \frac{wL_j}{p_j X_j}\left(dlogL_j\right) + \frac{\sum_{i=1}^{n} p_i x_{ij}}{p_j X_j}\left(dlogp_j + dlogx_{ij}\right)$$

Separating prices and quantities and given the definition of MFP growth, one can write (23) as:

$$dlog\rho_j^{MFP} = dlogX_j - \frac{wL_j}{p_j X_j} dlogL_j - \frac{\sum_{i=1}^{n} p_i x_{ij}}{p_j X_j} dlogx_{ij} \qquad (33)$$

$$= \frac{\sum_{i=1}^{n} p_i x_{ij}}{p_j X_j} dlogp_i - dlogp_j$$

Hence, the MFP growth rate can be seen both as the rate in which the production of the jth industry can grow given its inputs and the rate in which its price can decrease given its inputs' prices, as shown by (33).

The MFP[45] measure, despite its flaws, has some degree of sophistication when compared to other mainstream traditional productivity measures [see De Juan and Febrero (2000)]. This is since the MFP considers all the measurable inputs needed to produce the gross product of each industry, including circulating capital. According to De Juan e Febrero (2000, p. 71) "By accounting for all the inputs, MFP avoids the problems of overvaluation of productivity gains, which are typical of those measures based exclusively on direct labour. Yet MFP continues treating each industry in isolation and missing the transfers of productivity among sectors".

Note that, given the definitions of both labour and technical coefficients, they can vary with time as follows[46]: $dlogx_{ij} = dloga_{ij} + dlogX_j$ and $dlogL_j = dloga_{nj} +$

[45] For details regarding MFP measurement see e.g. Jorgenson et al (1987) and Cas and Rymes (1991).

[46] This is just a consequence of taking logs and differentiating both following definitions: $a_{nj} = \frac{L_j}{X_j}$ and $a_{ij} = \frac{x_{ij}}{X_j}$.

$dlogX_j$. If one substitute it in the physical definition of MFP growth measurement given by equation's (33) first part, it is possible to rewrite it as:

$$dlog\rho_j^{MFP} = -a_{nj}\frac{w}{p_j}dloga_{nj} - \frac{\sum_{i=1}^{n} p_i a_{ij}}{p_j}dloga_{ij}$$

Rewriting the above equation in matrix form we get:

$$dlog\boldsymbol{\rho}'^{\mathbf{MFP}} = -(dlog\boldsymbol{a}'_{\boldsymbol{n}}\widehat{\boldsymbol{a}}_{\boldsymbol{n}}\mathrm{w} + \mathbf{P}'\mathbf{A} \circ dlog\mathbf{A})\widehat{\mathbf{P}}^{-1} \qquad (34)$$

Where the symbol ∘ denotes the direct product (or Schur product) that indicates an element by element matrix product. Equation (21) above is the continuous analogous of Leontief's (1953) 'measure of structural change', as shown by Domar (1961). This equation, in a discrete form, was used by Leontief to analyze changes over time in the technical coefficients, as a weighted average of the relative changes in the coefficients of inputs (primary and intermediate) [see Peterson (1978)].

Although MFP's individual measure misses the productivity transmission among industries and, therefore, the relevance of the interconnection among them in shaping productivity advance, when suitably aggregated it delivers a better understanding of the technical advance course than individual measures. The procedure of aggregating the industrial productivity growth, defined by (35), to access the productivity growth rate of the whole economy, is known as Domar aggregation[47]. The Domar aggregation procedure delivers a remarkable feature that its weights add up to more than the unity as following:

$$dlog\rho^{MFP} = \sum_{j=1}^{n}\frac{P_jX_j}{\sum_{j=1}^{n}P_jY_j}dlog\rho_j^{MFP} \qquad (35)$$

Alternatively, in matrix notation:

$$dlog\rho^{MFP} = (\mathbf{p}'\mathbf{Y})^{-1}(dlog\boldsymbol{\rho}'^{\mathbf{MFP}})\widehat{\mathbf{P}}\mathbf{X} \qquad (35')$$

When aggregating the industrial MFP's growth rate with (35), unlike the individual industrial measure, the interconnection's role between industries arises since the productivity of each industry impacts the aggregate productivity directly and indirectly, evidenced by the aggregation and the sum of the weights greater than the unit. Moreover, the greater the connectivity among industries, via circulating capital, the higher the sum of weights above, due to the higher potential of transmitting productivity among industries.

[47] Domar aggregation was pioneered by Domar (1961) and generalized by Hulten (1978).

The effective productivity measure (hereafter EP) advanced by Cass and Rymes (1991), however, acknowledge contrary to the traditional MFP measure, that the intermediate inputs are themselves produced and therefore subject to changing production's efficiency. It thus impacts all industries that acquire the growing efficiency intermediate inputs. Hence, their measure recognizes the role of technological interconnections even in the industry level [see, e.g. Gu and Yan (2016)].

Suppose, for instance, an industry that produces a commodity using as inputs only labour and the same commodity that it produces. Suppose also that this industry benefits from technical change. But notice that this technical advance will benefit that industry twice: first due to the technical advance in its production process itself and second because it will acquire its circulating capital with increasing efficiency, that is, with the fewer necessity of inputs and therefore reducing costs. Notice, however, that the same is true for all industries that acquire produced circulating capital from any industry. Hence, Cas and Rymes (1991) deliver the following productivity measure about the jth industry:

$$dlog\rho_j^{EP} = dlogX_j - \frac{wL_j}{P_jX_j}dlogL_j - \frac{\sum_{i=1}^{n} p_i x_{ij}}{p_jX_j}(dlogx_{ij} - dlog\rho_i^{EP}) \quad (36)$$

The EP's growth rate above considers that the required circulating capital would be produced with decreasing necessity of primary inputs, given technical advance. Therefore, those industries that acquire it would benefit from the declining costs of intermediate inputs due to less primary inputs embodied in it. This is the reason why productivity increase carried out in the production of all circulating capital should be excluded from the growth rate of the circulating capital consumption.

According to Cas and Rymes (1991), $dlog\rho_j^{EP}$ corresponds to the productivity growth rate of the jth industry given all the necessary primary inputs used to yield its commodity. The term $dlogx_{ij} - dlog\rho_i^{EP}$, however, measures the primary input services used indirectly by industry j, given the relevant input-output technology. This makes it possible to account for the technical progress impact on the produced inputs costs.

From (36) notice that the higher the growth rate of the technical progress from all ith industries that deliver circulating capital to industry j – $dlog\rho_i^{EP}$ – the higher the productivity growth of j. It then shows the importance of technical relationship among industries and the productivity 'spillover' among them. Moreover, from manipulating (36), one has that:

$$dlog\rho_j^{EP} = dlogX_j - \frac{wL_j}{P_jX_j} dlogL_j - \frac{\sum_{i=1}^{n} p_i x_{ij}}{p_jX_j} dlogx_{ij} + \frac{\sum_{i=1}^{n} p_i x_{ij}}{p_jX_j} dlog\rho_i^{EP}$$

From the definition of MFP growth given by (33) and using the equation above we obtain an equation that shows the relationship between the technical advance measured by both MFP and EP as following.

$$dlog\rho_j^{EP} = dlog\rho_j^{MFP} + \sum_{i=1}^{n} \frac{P_i a_{ij}}{P_j} dlog\rho_i^{EP} \qquad (37)$$

From (37) it is easy to note that in an ordinary situation $dlog\rho_j^{EP} > dlog\rho_j^{MFP}$ due to the existence of technical advance transmission embodied in the circulating capital. Putting equation (37) in matrix notation and doing some algebraic manipulations we get that:

$$dlog\boldsymbol{\rho}'^{\mathbf{EP}} = \mathrm{dlog}\boldsymbol{\rho}^{\mathbf{MFP}}\widehat{\mathbf{P}}[\mathbf{I} - \mathbf{A}]^{-\mathbf{1}}\widehat{\mathbf{P}}^{-\mathbf{1}} \qquad (38)$$

The equation above shows the relationship among both MFP and EP measures following a matrix notation. Note that they are the same in the case where there are no circulating capital and therefore no technical change transmission among industries. According to Cas and Rymes (1991), contrary to the MFP aggregation, the EP is aggregated through a weighted average of the EP's growth rate of each industry. Note that the weighting sum equals the unit:

$$dlog\rho^{EP} = \sum_{j=1}^{n} \frac{P_jY_j}{\sum_{i=1}^{n} P_jY_j} dlog\rho_j^{EP} \qquad (39)$$

Cas and Rymes (1991, p. 67) regarding the differences between EP and MFP measures argue that[48] "The individual industry *EP* measures are aggregated by the shares in final output, such shares summing to unity. The new measures take into account, however, the fact that (…) *circulating capital* are themselves being produced with increased efficiency when the capital goods industries are experiencing advances in technical knowledge. The new measures will always exceed the traditional (*MFP*) measures when such productivity advance is occurring. It will be remembered that the *productivity change* take into account the changing efficiency of not only those primary

[48] Our comments are in italics.

nonreproducible inputs measured and classified as being directly involved in industry j but also those primary inputs indirectly involved in all related industries".

The previous equation can also be written as:

$$dlog\rho^{EP} = (\mathbf{p'Y})^{-1}(dlog\boldsymbol{\rho}'^{\mathbf{EP}})\widehat{\mathbf{P}}\mathbf{Y} \qquad (40)$$

If we add (18) to (40) and using (38) we finally get:

$$dlog\rho^{EP} = (\mathbf{p'Y})^{-1}(\mathrm{dlog}\boldsymbol{\rho}^{\mathbf{MFP}})\widehat{\mathbf{P}}\mathbf{X} = dlog\rho^{MFP} \qquad (41)$$

Equation (41) shows that when measuring the productivity growth for the whole economy, both MFP and EP measures are the equivalent. This is due to, albeit they are distinct at the industry level, both measures consider the productivity change transmission among industries in the aggregate level.

Let us now turn to consider the relationship between the VIS's productivity advance and the other two measures discussed here, namely the MFP and EP measures. To do that, it is necessary to consider equivalent assumptions among the three measures and therefore we should considerate the sub-systems in a condition of perfect competition, that is without profit or surplus. Consequently equation (31) becomes now $p_j(t) = l_j^*(t)w$. By taking logs and differentiating it and post-multiplying both sides by $\left(\widehat{\boldsymbol{P}}\right)^{-1}$we get the following:

$$dlog\boldsymbol{P} = \left(\widehat{\boldsymbol{P}}\right)^{-1} w\hat{\boldsymbol{l}}^* dlog\boldsymbol{l}^* = -dlog\boldsymbol{\rho}'^{\mathbf{VIS}} \qquad (42)$$

Equation (42) is similar to the result found by Aulin-Ahmavaara (1999). It shows that, in a perfect competition condition, the growth rate of prices in composite sectors are the inverse of the growth rate of its productivity. It is also equivalent to the argument of De Juan and Febrero (2000) that the evolution of prices occurs in parallel with the evolution of the total labor requirements incorporated in the commodities, which makes it a good indicator of inherent sectoral competitiveness[49].

Let us now consider equation (16) devoid of profit, which gives us $\mathbf{P}(t) = w\mathbf{a}_{\boldsymbol{n}}(t) + \mathbf{AP}(t)$. By taking logs and fully differentiating it with respect to time:

[49] Similarly, Peterson (1979, p. 216) argues that "In this system each sector produces one type of final output, making use only of primary inputs to do so that indices of productivity change for the vertically integrated sectors do reflect the extent to which technical progress is leading to the increased availability and lower cost of individual final products".

$$dlog\mathbf{P}' = (dlog\boldsymbol{a}'_{\boldsymbol{n}}\widehat{\boldsymbol{a}}_{\boldsymbol{n}}\mathrm{w} + \mathbf{P}'\mathbf{A} \circ dlog\mathbf{A} + \mathbf{P}'\mathbf{A} \circ dlog\mathbf{P}')\left(\widehat{\boldsymbol{P}}\right)^{-\mathbf{1}}$$

By rearranging the above equation,[50] one gets the following:

$$dlog\mathbf{P}' = (dlog\boldsymbol{a}'_{\boldsymbol{n}}\widehat{\boldsymbol{a}}_{\boldsymbol{n}}\mathrm{w} + \mathbf{P}'\mathbf{A} \circ dlog\mathbf{A})[\mathbf{I} - \mathbf{A}]^{-1}\left(\widehat{\mathbf{P}}^{-1}\right) \qquad (43)$$

$$= -dlog\boldsymbol{\rho}'^{\mathbf{VIS}}$$

The connection between the change in the productivity of the VIS and MFP measures can therefore be obtained by post multiplying both sides of equation (34) by $\widehat{\boldsymbol{P}}$ and inserting the resulting equation in (43).

$$dlog\boldsymbol{\rho}'^{\mathbf{VIS}} = \mathrm{dlog}\boldsymbol{\rho}^{\mathbf{MFP}}\widehat{\mathbf{P}}[\mathbf{I} - \mathbf{A}]^{-\mathbf{1}}\widehat{\mathbf{P}}^{-\mathbf{1}} = dlog\boldsymbol{\rho}'^{\mathbf{EP}} \qquad (44)$$

The equality among equations (38) and (44) reveals that, under the assumptions here exposed, the EP's growth measure, or 'new measure' advanced by Cas and Rymes (1991), and the VIS's measurement of technical change, considering vertically integrated sub-systems, are precisely the same. It means that equation (36), which defines the industrial EP growth is also useful to understand the VIS productivity growth. Namely, the productivity growth of the industries that deliver circulating capital are crucial to understanding the productivity change mechanism of sectors producing final commodities.

Indeed, it was already realized by Cas and Rymes (1991, p.67), albeit it was not formally shown by them[51]: "The method of aggregation for the new (*EP*) measures provides a consistent aggregation measure of multifactor productivity (*MFP*) as well as a method of capturing measures of improvements in productivity of Pasinetti's vertically integrated sectors (*VIS*). Two important new conceptual additions to National Accounting, the new measures (…)*)* advanced in this study (*EP)* and Pasinetti's concept of vertically integrated sectors, are thus shown to be linked".

In short, the productivity growth rate occurred in vertically integrated sectors are exactly equivalent to the growth rate measured by the EP method in sectors producing final commodities. Therefore, it is easy to show that both are also equal in the aggregate level and consequently equal to the Domar aggregation of MFPs measure. The VIS and

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[50] Note that the equivalence of equation (43) with equation (42) can be found by fully differentiating $\boldsymbol{l}^* = \boldsymbol{a_n}[\mathbf{I} - \mathbf{A}]^{-\mathbf{1}}$, replacing it in (43) and also using the fact that $\mathbf{P} = \boldsymbol{l}^* w$.

[51] Our comments are in italics.

EP measures are greater than the MFP at the industrial/sectoral level because they already incorporate at the individual level the technological interrelation among industries and their flows of technical progress. In contrast, the MFP's measurement only deals with such interconnections at the aggregate level by using Domar aggregation.

3.7. Concluding remarks

In this investigation, we aim to provide an alternative approach to vertically (growing) hyper-integrated sectors and the role of technical progress. Following the suggestion made by the last paragraph of Pasinetti (1988), we advanced in a model of vertically integrated sectors that are allowed to grow at a distinct pace one another and that explicitly takes into account technical change taking place at the industry level and that are transmitted to the sub-systems through direct and indirect labour.

Accordingly, we also deliver here measures of both aggregate and sectoral technical change growth rates and economic growth rates. Those measures take into account both particular demand and supply sides such as input availability and technical change. They can be seen as alternative measures to the maximum eigenvalue of the coefficients matrix from the Sraffian system since our hypothesis of uneven growth among sectors and industrial technical change unviable, such result.

Moreover, it is provided with an equation that shows the relationship between the growth rate of sectoral technical progress and sectoral prices growth rate. The composite profit rate amount affects the elasticity of such relationship. This equation thereby shows the impact of technical change on changing prices of composite sectors.

Finally, we also deliver an investigation under differences and behaviours of three of the most essential measures of productivity growth regarding input-output systems, namely the neoclassical multifactor productivity, the effective productivity and an estimate of productivity growth from vertically integrated sectors. We show that the last two deal with technical change, considering the interconnections and transmission of productivity among industries in shaping the productivity of sectors producing final commodities. The multifactor productivity growth measure, however, does not treat circulating capital as produced and misses productivity transmission among industries, at least at the industrial level.

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CONCLUDING REMARKS

In the first chapter of this thesis, there is a revisit of Notarangelo's (1999) approach, reinforcing that there are indeed similarities between the models while confirming that Baumol (1967) offers a particular case of Pasinetti (1993). Our approach allows us to deliver the Baumol analysis in terms of the SED approach, thus providing a truly disaggregated assessment of unbalanced growth. Besides, we alleviated the passive role played by demand in the Baumol model by considering a more subtle and inclusive approach for it – as is found in the contributions of Pasinetti. With this study, we show that even in the case in which there are no intermediate goods, the Baumol result holds if the productions of all sectors grow at the same rate, and the economy maintains equilibrium.

The additional advantage of our approach is related to the fact that the Pasinetti model considers not only final goods but also intermediate goods. With this in mind, we have advanced the extension of Oulton (2001) to the Baumol model within the SED framework, to consider multiple sectors. Next, by using the concept of vertical integration, we obtained a result that summarises Oulton and Baumol's contributions. On the one hand, Oulton's point still holds, namely that in the presence of intermediate goods, the productivity of the economy will not necessarily converge towards the lower productivity sector. On the other hand, the Baumol perspective may also hold as one of the outcomes. This exercise shows that the result depends on comparing not only the productivity growth of isolated industries – as Baumol did – but also the productivity growth of vertically integrated sectors.

In the second chapter, it is used the Domar aggregation approach to study the evolution of productivity growth in Brazil from 2000 to 2014. This method was adopted to approach other countries, but for the best of our knowledge, this is the first time for the Brazilian economy. That is particularly important because it allowed us a disaggregated assessment of the Brazilian productivity and growth pattern during that period. We can explain the overall productivity performance of the Brazilian economy not only in terms of the poor performance of its sectors but also in terms of diminishing industrial density, with fewer backward and forward connections amongst industries in terms of chains of intermediate inputs.

Besides, despite the relatively high density of the macro manufacturing sector when compared to other sectors in the Brazilian economy, it performed a negative role concerning aggregate productivity growth both directly and indirectly. Directly insofar as that sector had negatives productivity growths during the period under consideration, and indirectly due to its high interconnection, which helped to spread negative rather than positive productivity growth across the economy.

Therefore, to improve the poor performance of the Brazilian economy witnessed in recent years, it is mandatory to enhance the capability of Brazilian manufacturing macro sector to generate productivity growth. It is also for future investigations to understand the reasons for the low productivity advance carryed out by the Brazilian economy, and the macro manufacturing sector particularly. In sum, Brazil has failed in its task to deepen its industrial density. As a consequence, it has witnessed a prematurely shrink in the share of the manufacturing sector in GDP, being stuck in a middle-income trap.

Finally, in the third and ultimate chapter, it is proposed an alternative approach to vertically integrated sectors formulation with technical progress. In this investigation, we aim to provide an alternative approach to vertically (growing) hyper-integrated sectors and the role of technical progress. Following the suggestion made by the last paragraph of Pasinetti (1988), we advanced in a model of vertically integrated sectors that are allowed to grow at a distinct pace one another and that explicitly takes into account technical change taking place at the industry level and that are transmitted to the sub-systems through direct and indirect labour.

Accordingly, we also deliver here measures of both aggregate and sectoral technical change growth rates and economic growth rates. Those measures take into account both particular demand and supply sides such as input availability and technical change. They can be seen as alternative measures to the maximum eigenvalue of the coefficients matrix from the Sraffian system since our hypothesis of uneven growth among sectors and industrial technical change unviable, such result.

Moreover, it is provided with an equation that shows the relationship between the growth rate of sectoral technical progress and sectoral prices growth rate. The composite profit rate amount influences the elasticity of such relationship. This equation thereby shows the impact of technical change on changing prices of composite sectors.

Finally, we also deliver an investigation under differences and behaviours of three of the most essential measures of productivity growth regarding input-output systems, namely the neoclassical multifactor productivity, the effective productivity and a measure of productivity growth from vertically integrated sectors. We show that the last two deal with technical change, considering the interconnections and transmission of productivity among industries in shaping the productivity of sectors producing final commodities. The multifactor productivity growth measure, however, does not treat circulating capital as produced and misses productivity transmission among industries, at least at the industrial level.

With this analysis, we emphasize the role of a multisectoral approach to economic and productivity growth and notably the transmition of productivity advance among sectors and industries due to sectoral interconnections and circulating capital. Taking the device of three independent chapters in this thesis it was possible to seek to deepen three particular aspects, focusing on both empirical and theoretical subjects, though with a common underlying subject.