REFERÊNCIA

PORTILHO, O. Helium-6-Lambda-Lambda and Be-10-Lambda-Lambda hypernuclei calculations with realistic interactions. **Brazilian Journal of Physics**, São Paulo, v. 33, n. 4, p. 840-847, dez. 2003. DOI: https://doi.org/10.1590/S0103-97332003000400040. Disponível em: http://www.scielo.br/scielo.php?script=sci_arttext&pid=S0103-97332003000400040&lng=en&nrm=iso. Acesso em: 21 out. 2020.

${}^{6}_{\Lambda\Lambda}$He and ${}^{10}_{\Lambda\Lambda}$Be Hypernuclei Calculations with Realistic Interactions

O. Portilho

Instituto de Física, Universidade de Brasília, 70919-970, Brasília, DF, Brazil

Received on 22 May, 2003. Revised version received on 23 July, 2003.

We study the ${}^{6}_{\Lambda\Lambda}$He and ${}^{10}_{\Lambda\Lambda}$Be hypernuclei in the three- and four-body models, respectively, employing harmonic oscillator bases and presently most realistic α-α, α-Λ and Λ-Λ interactions. In order to improve convergence we use correlation functions in the case of ${}^{10}_{\Lambda\Lambda}$Be. Comparison is made with results obtained using similar interactions and other methods, and it is performed an analysis concerning the possibility of a unified description of both hypernuclei.

1 Introduction

It was detected recently in the KEK E373 experiment [1] a new candidate for ${}^{6}_{\Lambda\Lambda}$He, with a smaller binding energy than the old data due to Prowse [2] which are now considered under suspicion of misinterpretation. Furthermore, we have recently demonstrated [3] the applicability of Jastrow-type correlation functions to harmonic oscillator (HO) bases in order to accelerate convergence in the case of ${}^{10}_{\Lambda\Lambda}$Be, considered as a four-body $\alpha\alpha\Lambda\Lambda$ system. This lead us to apply such a technique to the same hypernucleus, now using more realistic interactions, and to review our former calculations on ${}^{6}_{\Lambda\Lambda}$He [4] in the same scope, hoping that a satisfactory and unified description of both nuclei can be made with an adequate choice of potentials. Filikhin and Gal [5] have published a calculation on the same hypernuclei by solving Faddeev-Yakubovsky equations in the s-waves approximation (for ${}^{10}_{\Lambda\Lambda}$Be) and Faddeev equations (for ${}^{6}_{\Lambda\Lambda}$He). They have also introduced ${}^{1}S_0$ $\Lambda\Lambda$ potentials as a sum of three gaussians simulating $\Lambda\Lambda$ Nijmegen soft-core [6, 7] (NSC97), extended soft-core [8] (ESC00) and hard-core [9, 10] (ND) interaction models. We make use here of these more realistic potentials, instead of the old one-gaussian attractive Dalitz potential [11, 12], employed in our former paper [3] with the purpose of comparing convergence of the HO basis without correlation functions [13].

In Section II we have a brief presentation of the formalism and in Section III we show our results for the ground-state energies, separation distances, and contributions of the partial waves to the energy, and compare them with results due to other authors. We also discuss the best choice of combination of the several potentials employed in the calculations.

2 Formalism

The method we employ in the study of ${}^{6}_{\Lambda\Lambda}$He and ${}^{10}_{\Lambda\Lambda}$Be hypernuclei in the three- ($\alpha\alpha\Lambda$) (see Ref. [4]) and four-body ($\alpha\alpha\Lambda\Lambda$) (see Ref. [3]) models, respectively, is based in a variational calculation where the α-particles are structureless. A convenient coordinate set that splits the internal coordinates from that of the center of mass is given by the Jacobi variables for unequal mass particles. In the case of ${}^{6}_{\Lambda\Lambda}$He the internal (non-dimensionless) coordinates are

$$\begin{aligned} \mathbf{r'}_a &= \sqrt{\frac{m_2 m_3}{m(m_2+m_3)}}(\mathbf{r'}_2 - \mathbf{r'}_3) = \sqrt{\frac{m_\Lambda}{2m}}(\mathbf{r'}_2 - \mathbf{r'}_3) \\ \mathbf{r'}_b &= \sqrt{\frac{m_1(m_2+m_3)}{mM}}\left(\mathbf{r'}_1 - \frac{m_2\mathbf{r'}_2 + m_3\mathbf{r'}_3}{m_2+m_3}\right) = \sqrt{\frac{2m_\alpha m_\Lambda}{mM}}\left(\mathbf{r'}_1 - \frac{\mathbf{r'}_2 + \mathbf{r'}_3}{2}\right) \end{aligned} \quad (1)$$

where

$$M = m_\alpha + 2m_\Lambda \quad (2)$$

is the total mass and

$$m = \frac{m_\Lambda(2m_\alpha + m_\Lambda)}{M} \quad (3)$$

is the reduced mass. Coordinate $\mathbf{r'}_1$ and mass m_1 refer to the α-particle while coordinates $\mathbf{r'}_2$, $\mathbf{r'}_3$ and respective masses m_2 and m_3 are related to the Λ-particles. In an analogous way, we have for ${}^{10}_{\Lambda\Lambda}$Be the coordinates

$$\begin{aligned}
\mathbf{r'}_a &= \sqrt{\frac{m_1 m_2}{m(m_1+m_2)}}(\mathbf{r'}_1 - \mathbf{r'}_2) = \sqrt{\frac{m_\alpha}{2m}}(\mathbf{r'}_1 - \mathbf{r'}_2) \\
\mathbf{r'}_b &= \sqrt{\frac{m_3 m_4}{m(m_3+m_4)}}(\mathbf{r'}_3 - \mathbf{r'}_4) = \sqrt{\frac{m_\Lambda}{2m}}(\mathbf{r'}_3 - \mathbf{r'}_4) \\
\mathbf{r'}_c &= \sqrt{\frac{(m_1+m_2)(m_3+m_4)}{mM}}\left(\frac{m_1\mathbf{r'}_1 + m_2\mathbf{r'}_2}{m_1+m_2} - \frac{m_3\mathbf{r'}_3 + m_4\mathbf{r'}_4}{m_3+m_4}\right) \\
&= \sqrt{\frac{m_\alpha m_\Lambda}{mM}}(\mathbf{r'}_1 + \mathbf{r'}_2 - \mathbf{r'}_3 - \mathbf{r'}_4)
\end{aligned} \tag{4}$$

with

$$m = \frac{1}{2}(m_\alpha + m_\Lambda) + \frac{m_\alpha m_\Lambda}{m_\alpha + m_\Lambda} \tag{5}$$

$$M = 2(m_\alpha + m_\Lambda). \tag{6}$$

Coordinates $\mathbf{r'}_1$, $\mathbf{r'}_2$ and respective masses m_1 and m_2 refer to the α-particles while coordinates $\mathbf{r'}_3$, $\mathbf{r'}_4$ and masses m_3 and m_4 are related to the Λ-particles.

We use as bases for the hypernuclei wave functions products of the HO wave functions in the internal Jacobi variables, coupled to a well defined value for the total angular momentum and its *z* projection. In the case of ${}^{6}_{\Lambda\Lambda}$He the wave function is expanded in terms of

$$|\nu; J\rangle = |n_a l_a, n_b l_b; J\rangle = \sum_{m_a} (l_a l_b m_a, M_J - m_a | J M_J)\, |n_a l_a m_a, n_b l_b, M_J - m_a\rangle, \tag{7}$$

while in the case of ${}^{10}_{\Lambda\Lambda}$Be the basis is constituted of

$$\begin{aligned}
|\nu; J\rangle &= |n_a l_a, n_b l_b, l; n_c l_c; J\rangle = \sum_{m_a, m_b} (l_a l_b m_a m_b | l, m_a + m_b) \times \\
&\times (l l_c, m_a + m_b, M_J - m_a - m_b | J M_J)\, |n_a l_a m_a, n_b l_b m_b, n_c l_c, M_J - m_a - m_b\rangle.
\end{aligned} \tag{8}$$

In the l.h.s. of above two equations we have omitted the M_J label since the physical quantities we calculate using the wave functions are independent of it. The harmonic oscillators have common quantum energy

$$\epsilon = \hbar\omega \tag{9}$$

which is changed variationally. Moreover, for this hypernucleus we introduce Jastrow-type correlation functions between the α-particles and between the Λ-particles in order to improve convergence so that the trial function becomes

$$\Psi = F_J \sum_\nu a_\nu\, |\nu; J\rangle, \tag{10}$$

with

$$F_J = f_a(r'_{12})\, f_b(r'_{34}) = f_a(r_a)\, f_b(r_b). \tag{11}$$

The correlation functions are conveniently taken as 1 + sum of two gaussians, one repulsive and one attractive

$$f_a(r'_{12}) = 1 + c_a\, \mathrm{e}^{-(r'_{12}/\rho_1)^2} + c'_a\, \mathrm{e}^{-(r'_{12}/\rho'_1)^2} = 1 + c_a\, \mathrm{e}^{-\beta_a r_a^2/\epsilon} + c'_a\, \mathrm{e}^{-\beta'_a r_a^2/\epsilon} \tag{12}$$

$$f_b(r'_{34}) = 1 + c_b\, \mathrm{e}^{-(r'_{34}/\rho_2)^2} + c'_b\, \mathrm{e}^{-(r'_{34}/\rho'_2)^2} = 1 + c_b\, \mathrm{e}^{-\beta_b r_b^2/\epsilon} + c'_b\, \mathrm{e}^{-\beta'_b r_b^2/\epsilon}. \tag{13}$$

Parameters c_a, c'_a, c_b, c'_b, are changed variationally and satisfy the conditions $c_a \geq 0$, $c_b \geq 0$, $c'_a \leq 0$, $c'_b \leq 0$, $1 + c_a + c'_a \geq 0$, $1 + c_b + c'_b \geq 0$. Unprimed coordinates mean they are dimensionless and the relation between primed and unprimed coordinates is $r' = \sqrt{\hbar/(m\omega)}\, r$. On the other hand, the parameters

$$\beta_a = \frac{2\hbar^2}{m_\alpha \rho_1^2} \quad (14)$$

$$\beta_b = \frac{2\hbar^2}{m_\Lambda \rho_2^2}. \quad (15)$$

are also changed variationally, and so are β'_a, β'_b, defined analogously, satisfying $\beta_a < \beta'_a$ and $\beta_b < \beta'_b$.

For the $\alpha\alpha$ interaction we use the Chien-Brown potential [14], which is given by

$$V_{\alpha\alpha} = V_N + V_{coul}, \quad (16)$$

with

$$V_N = V_{0N}\, \mathrm{e}^{-\mu' r'^2} + V_D(r') \quad (17)$$

$$V_D(r') = -\frac{V'_{D0}}{2} \frac{\mathrm{e}^{-\gamma r'}}{\gamma r'} \{1 + \mathrm{erf}(\lambda_-) - \mathrm{e}^{2\gamma r'}[1 - \mathrm{erf}(\lambda_+)]\} \quad (18)$$

$$V'_{D0} = 16 V_{D0}\, \mathrm{e}^{3\gamma^2/8\kappa} \quad (19)$$

$$\lambda_\pm = \sqrt{\frac{2\kappa}{3}} \left(r' \pm \frac{3\gamma}{4\kappa} \right) \quad (20)$$

$$V_{coul} = \frac{4e^2}{r'}\, \mathrm{erf}\left(\sqrt{\frac{2\kappa}{3}} r' \right). \quad (21)$$

Here erf is the error function. The potential parameters are $V_{0N} = 287.5$ MeV, $\mu' = (0.635\ \mathrm{fm}^{-1})^2$ (for $l_a = 0$), $V_{0N} = 176.5$ MeV, $\mu' = (0.620\ \mathrm{fm}^{-1})^2$ (for $l_a = 2$), $V_{D0} = 85$ MeV, $\gamma = 1.35\ \mathrm{fm}^{-1}$, $\kappa = 0.514\ \mathrm{fm}^{-2}$.

For the $\alpha\Lambda$ interaction we have two potentials. The first one is the Isle type [15] which is a sum of two gaussians such as

$$V_{\alpha\Lambda} = V_{\mathrm{rep}}\, \mathrm{e}^{-(r'/\beta_{\mathrm{rep}})^2} + V_{\mathrm{att}}\, \mathrm{e}^{-(r'/\beta_{\mathrm{att}})^2} \quad (22)$$

and that was used by Filikhin and Gal [5] as well, with parameters $V_{\mathrm{rep}} = 450.4$ MeV, $V_{\mathrm{att}} = -404.9$ MeV, $\beta_{\mathrm{rep}} = 1.25$ fm, $\beta_{\mathrm{att}} = 1.41$ fm. The second is the one-gaussian attractive potential labeled as Gibson I (ρ_{GI}) by Daskaloyannis *et al.* [16], with parameters $V_{\mathrm{att}} = -43.48$ MeV and $\beta_{\mathrm{att}} = 1.5764$ fm, which produced a reasonable ground-state energy for ${}^{9}_{\Lambda}$Be in the $\alpha\alpha\Lambda$ model [4].

In our former papers [3, 4, 13] we used as $\Lambda\Lambda$ interaction the one-gaussian attractive Dalitz potential [11, 12]. However more realistic potentials with repulsive core have been published, especially the ${}^1S_0$ channel simulations to the Nijmegen models due to Filikhin and Gal [5], conveniently expressed as a sum of three gaussians

$$V_{\Lambda\Lambda} = \sum_{i=1}^{3} V_i\, \mathrm{e}^{-(r'/\beta_i)^2}. \quad (23)$$

The parameters are the following: $V_1 = -21.49$ MeV, $V_2 = -379.1\gamma'$ MeV, $V_3 = 9324$ MeV, $\beta_1 = 1.342$ fm, $\beta_2 = 0.777$ fm, $\beta_3 = 0.350$ fm, with $\gamma' = 0.4804$ (NSC97b model), $\gamma' = 0.5463$ (NSC97e model), $\gamma' = 1$ (ND model) and $\gamma' = 1.2044$ (ESC00 model).

Using the HO bases described in this section we calculated the hamiltonian matrix elements and also, in the case of ${}^{10}_{\Lambda\Lambda}$Be, the norm matrix elements. The bases are truncated by choosing a limit to N, defined as $2(n_a + n_b)$ (for ${}^{6}_{\Lambda\Lambda}$He) or as $2(n_a + n_b + n_c)$ (for ${}^{10}_{\Lambda\Lambda}$Be). For each N corresponds a certain amount of possible combinations of n_a, n_b and n_c, together with combinations of l_a, l_b and l_c chosen appropriately, giving the basis dimension (size). After the diagonalisation process we obtained the ground-state energy and the coefficients a_ν that appear in Eq. (10). With the wave function so available we proceeded to calculating rms distances between the particles involved in each hypernucleus and expectation values for the kinetic and potential energies. More details on the formalism can be found in the former papers [3, 4].

3 Results and discussion

${}^{6}_{\Lambda\Lambda}$He

We have revisited our former calculations on ${}^{6}_{\Lambda\Lambda}$He [4] in view of the more recent $\Lambda\Lambda$ potentials with repulsive core simulating the Nijmegen model, as mentioned in the previous section. We present in Table I the 0^+ ground-state energies obtained with combinations of Gibson I (ρ_{GI}) [16]

and Isle [15] $\alpha\Lambda$ potentials with four Nijmegen model simulations of $\Lambda\Lambda$ interactions [5]: NSC97b, NSC97e, ND, ESC00. In the column labeled as ground-state energy we show the results obtained with the (0, 0)+(2, 2)+(4, 4) combination of (l_a, l_b) in the HO basis, what means dimension 408 and $N = 2(n_a + n_b)$ limited to 30. The values of ϵ were chosen in the $N = 26$ approximation and used in the larger bases. In order to check convergence, we have also shown the extrapolated values of the ground-state energy for $N = \infty$ by supposing, following Delves [17], that

$$E_N = E_\infty + AN^{-Q} + \mathcal{O}(N^{-(Q+1)}) \qquad (24)$$

is obeyed so that, to leading order,

$$\ln(-\Delta E_N) = \ln AQ - (Q+1)\ln N, \qquad (25)$$

with $\Delta E_N = E_{N+1} - E_N$, and corresponding estimates of the errors involved in the extrapolation. The extrapolation we have made for ${}^{6}_{\Lambda\Lambda}$He in our former paper [4] using the older Bando $\alpha\Lambda$ potential [12] was confirmed by Filikhin, Gal and Suslov in their recent Faddeev calculations [18]. Therefore there is no reason to doubt that the results in Table I are also reliable. We notice that results obtained following the sequence NSC97b, NSC97e, ND, ESC00 of $\Lambda\Lambda$ potentials have convergence improved in the case of the Gibson $\alpha\Lambda$ potential since the errors become smaller, while it gets worse in the case of the Isle potential. We also show in the last column the Faddeev equation calculations of Filikhin and Gal [5] who employed the same combinations of Isle $\alpha\Lambda$ potential and Nijmegen $\Lambda\Lambda$ potentials. The experimental ground-state energy is just $-B_{\Lambda\Lambda}$ from the recent results of Takahashi *et al.* [1]. By comparing the several results with the experimental value we find that the best choice of potentials is the combination Isle+NSC97e, whose extrapolated energy coincides with the upper limit of the experimental value. The ESC00 potential was constructed with the purpose of obtaining the old value of experimental ground-state energy of -10.8 ± 0.6 MeV due to Prowse [2], and our calculation with this potential combined with Isle potential shows it is successful in this aim, although these data are now discarded in favor of the results of Takahashi *et al.* [1]. Some additional results obtained using also the ND interaction, combined with G matrix methodology, are –9.23 MeV [19], –9.34 MeV [20], –9.4 MeV [21], which are a little bit smaller than our result and the one due to Filikhin and Gal, –7.25 MeV [22]; and –7.33 MeV (RPA) [23].

We have also calculated rms distances between the Λ-particles and between the α-particle and the $\Lambda\Lambda$ center-of-mass. This is shown in Table II. We notice that the distances obtained with the Isle $\alpha\Lambda$ potential are larger than the results that emerge from the Gibson potential. The results obtained by Filikhin and Gal are also a little bit larger than ours. Besides, we notice that the more bound is the nucleus the more compact it is, as it should be. Results from other authors include $\langle r'^2_{\Lambda\Lambda}\rangle^{1/2} = 3.31$ fm, $\langle r'^2_{\alpha-\Lambda\Lambda}\rangle^{1/2} = 2.14$ fm [19] and $\langle r'^2_{\Lambda\Lambda}\rangle^{1/2} = 3.20$ fm [20].

In Table III we show the contributions of the partial waves (0, 0), (2, 2), (4, 4) to the ground-state energy and to the wave function.

Table I. 0^+ ground-state energy of ${}^{6}_{\Lambda\Lambda}$He for several combinations of $\alpha\Lambda$ and $\Lambda\Lambda$ potentials. We also present the value of the corresponding variational parameter ϵ (in MeV). In the column labeled as ground-state energy we show our results with basis dimension 408, $N = 30$ approximation. We have also the extrapolated values of the ground-state energy for $N = \infty$ and corresponding errors. The results of Filikhin and Gal [5] are in the last column. The experimental ground-state energy is from Takahashi *et al.* [1]. Energies are in MeV.

Potentials ($\alpha\Lambda + \Lambda\Lambda$)	ϵ	g.s. energy	extrapolated g.s. energy	g.s. energy Filikhin and Gal
Gibson+NSC97b	32.158	–6.389	–6.8±0.2	
Gibson+NSC97e	33.519	–6.718	–7.0±0.1	
Gibson+ND	43.273	–10.290	–10.345±0.002	
Gibson+ESC00	47.526	–12.978	–13.002±0.001	
Isle+NSC97b	28.910	–6.544	–6.746±0.003	–6.60
Isle+NSC97e	30.052	–6.745	–6.947±0.005	–6.82
Isle+ND	39.206	–8.943	–9.10±0.02	–9.10
Isle+ESC00	43.961	–10.705	–10.83±0.02	–10.7
Experimental		$-7.25\pm0.19^{+0.11}_{-0.18}$		

Table II. rms values for $\Lambda\Lambda$ and α-$\Lambda\Lambda$ center-of-mass separation distances (in fm) in ${}^{6}_{\Lambda\Lambda}$He ($N = 30$ approximation), for several combinations of $\alpha\Lambda$ and $\Lambda\Lambda$ potentials. The last two columns refer to the results of Filikhin and Gal [5].

Potentials $(\alpha\Lambda + \Lambda\Lambda)$	$\langle r'^2_{\Lambda\Lambda}\rangle^{1/2}$	$\langle r'^2_{\alpha-\Lambda\Lambda}\rangle^{1/2}$	$\langle r'^2_{\Lambda\Lambda}\rangle^{1/2}$ Filikhin	$\langle r'^2_{\alpha-\Lambda\Lambda}\rangle^{1/2}$ and Gal
Gibson+NSC97b	3.51	1.95		
Gibson+NSC97e	3.41	1.91		
Gibson+ND	2.75	1.67		
Gibson+ESC00	2.47	1.58		
Isle+NSC97b	3.91	2.20		
Isle+NSC97e	3.83	2.18	3.93	2.29
Isle+ND	3.25	2.07	3.36	2.11
Isle+ESC00	2.94	2.03	3.09	2.04

Table III. Contributions (in %) of the partial waves (0, 0), (2, 2), (4, 4) to the ${}^{6}_{\Lambda\Lambda}$He ground-state energy and to the wave function $\langle\Psi|\Psi\rangle$ ($N = 30$ approximation).

Potentials $(\alpha\Lambda + \Lambda\Lambda)$	Energy (0,0)+(2,2)+(4,4)	Wave function (0,0)+(2,2)+(4,4)
Gibson+NSC97b	86.03+12.68+1.29	97.12+2.74+0.13
Gibson+NSC97e	87.47+11.44+1.09	97.40+2.48+0.11
Gibson+ND	95.09+4.65+0.26	98.88+1.09+0.03
Gibson+ESC00	97.11+2.78+0.12	99.30+0.69+0.02
Isle+NSC97b	86.22+11.70+2.08	98.63+1.27+0.10
Isle+NSC97e	86.74+11.30+1.96	98.72+1.19+0.10
Isle+ND	90.67+8.21+1.12	99.19+0.74+0.07
Isle+ESC00	92.61+6.64+0.75	99.34+0.61+0.05

${}^{10}_{\Lambda\Lambda}$Be

In order to improve convergence of the HO basis in four-body calculations we included the Jastrow-type correlations functions as shown in Eqs. (10)–(13). In all calculations we obtained $c_a = c_b = 0$, $c'_a = c'_b = -1$, $\rho'_1 = 2.5$ fm as best parameters, leaving changes only in ρ'_2. We limited $N = 2(n_a + n_b + n_c)$ to 12 which proved to be enough from the standpoint of convergence in a former calculation of ours [3] with the purely attractive $\Lambda\Lambda$ Dalitz potential [11, 12]. We show in Table IV our results for the ground-state energy of ${}^{10}_{\Lambda\Lambda}$Be in the $N = 12$ approximation (dimension 336) considering the combinations (0, 0, 0), (2, 2, 0), (2, 0, 2) and (0, 2, 2) to (l_a, l_b, l_c) and using the $\alpha\alpha$ potential of Chien and Brown [14]. The values of ϵ and ρ'_2 were fixed in the $N = 8$ approximation (dimension 140) and extended to the $N = 12$ approximation. We consider both Gibson I ($\rho_{\rm GI}$) [16] and Isle [15] $\alpha\Lambda$ potentials. We notice again that the combination Isle+NSC97e gives a result close to the experimental ground-state energy [24, 25], as happened with ${}^{6}_{\Lambda\Lambda}$He. We also show the Faddeev-Yakubovsky results of Filikhin and Gal [5]. It is clear that our results present more binding than theirs. As $\alpha\alpha$ interaction they use the older Ali-Bodmer a_0 potential [26] but this cannot account for the discrepancy. They expect an uncertainty of 0.5 MeV in their figures due to limiting the calculations to s-waves. In order to have a precise idea about the convergence quality of our calculations we plot in Fig. 1 the behaviour of the ground-state energy against N for all combinations of

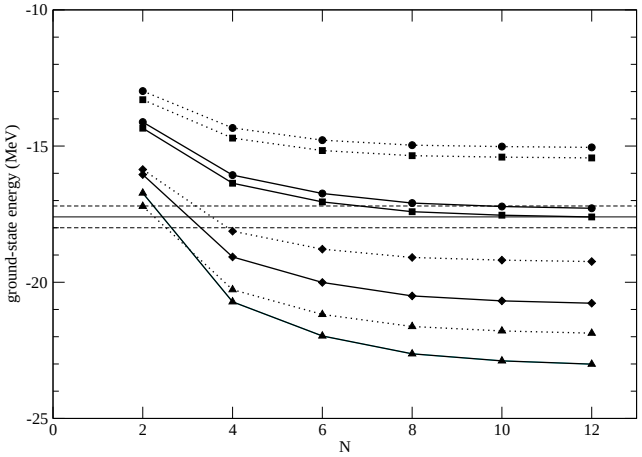

Figure 1. Ground-state energy of ${}^{10}_{\Lambda\Lambda}$Be as a function of N, illustrating the convergence of the method. Full lines refer to the Isle $\alpha\Lambda$ potential and dashed lines to the Gibson potential. The $\Lambda\Lambda$ potentials are represented by circles (NSC97b), squares (NSC97e), diamonds (ND) and triangles (ESC00). It is also shown the experimental value and respective error limits through the horizontal lines.

Gibson and Isle $\alpha\Lambda$ potentials with NSC97b, NSC97e, ND and ESC00 $\Lambda\Lambda$ potentials. It is also shown the experimental value and the error limits. The results obtained with the combination of the Isle potential with NSC97b and NSC97e potentials are inside the experimental error limits. We see that the convergence is satisfactory. However we should mention that the simulations to the Nijmegen $\Lambda\Lambda$ potentials due to Filikhin and Gal are supposed to be applied to the ${}^1S_0$ interaction. Since in Table IV the energies were calculated with bases involving $l_b \geq 0$ we are assuming that those interactions are still valid for higher relative angular momenta between the Λ-particles. Other results for the ground-state energy, obtained also with simulations of the Nijmegen D model, combined with the G matrix formalism, are –17.6 MeV [19], –17.15 MeV [20], –17.0 MeV [21], –15.05 MeV [22]; and –16.7 MeV (RPA) [23].

We show in Table V the rms distances between the α-particles, between the Λ-particles and between the $\alpha\alpha$ and $\Lambda\Lambda$ centers-of-mass. We notice that $\langle r'^2_{\Lambda\Lambda}\rangle^{1/2}$ is very sensitive to the $\Lambda\Lambda$ potential: the deeper is its well, the shorter is $\langle r'^2_{\Lambda\Lambda}\rangle^{1/2}$. Other results for $\langle r'^2_{\alpha\alpha}\rangle^{1/2}$ are 3.40 fm [20] and 3.44 fm [22], for $\langle r'^2_{\Lambda\Lambda}\rangle^{1/2}$ are 2.81 fm [19], 3.02 fm [20] and 2.8 fm [21], and for $\langle r'^2_{\alpha\alpha-\Lambda\Lambda}\rangle^{1/2}$ are 1.67 fm [19] and 1.90 fm [20].

In Table VI we present the corresponding contributions of the several combinations of (l_a, l_b, l_c) to the ground-state energy in the $N = 12$ approximation. We notice that the (0, 0, 0) restriction is completely unsatisfactory in our approach, although it can be perfectly acceptable in the Faddeev-Yakubovsky formalism of Filikhin and Gal. We also see from Table VI that the $l_b \neq 0$ contributions are not negligible, what reinforces the necessity of detailed studies of the $\Lambda\Lambda$ interaction beyond the ${}^1S_0$ channel.

Table IV. 0^+ ground-state energy of ${}^{10}_{\Lambda\Lambda}$Be in the $N = 12$ approximation, with the combinations of (l_a, l_b, l_c) as (0, 0, 0)+(2, 2, 0)+(2, 0, 2)+(0, 2, 2) and basis dimension 336. The $\alpha\alpha$ potential is due to Chien and Brown [14]. Also shown are the values of the parameters ϵ (in MeV) and ρ'_2 (in fm), the expectation values of the kinetic and potential energies, and the results due to Filikhin and Gal [5]. Energies are in MeV.

Table IV

Potentials ($\alpha\Lambda + \Lambda\Lambda$)	ϵ	ρ'_2	g.s. energy	kinetic energy	potential energy	g.s. energy Filikhin and Gal
Gibson+NSC97b	11.843	0.8085	–15.048	28.807	–43.855	
Gibson+NSC97e	12.015	0.7952	–15.435	29.361	–44.796	
Gibson+ND	13.879	0.7221	–19.240	34.894	–54.134	
Gibson+ESC00	15.168	0.6987	–21.870	38.648	–60.518	
Isle+NSC97b	12.642	0.7993	–17.283	24.491	–41.774	–15.2
Isle+NSC97e	12.774	0.7866	–17.603	24.921	–42.524	–15.4
Isle+ND	14.064	0.7144	–20.771	29.494	–50.265	–17.7
Isle+ESC00	15.047	0.6898	–23.007	32.866	–55.873	–19.4
Experimental			–17.6±0.4			

Table V. rms values of the distances between the α-particles, between the Λ-particles and between both centers-of-mass (in fm) for $^{10}_{\Lambda\Lambda}$Be in the $N = 12$ approximation, with combinations of (l_a, l_b, l_c) as (0, 0, 0)+(2, 2, 0)+(2, 0, 2)+(0, 2, 2) and basis dimension 336. The $\alpha\alpha$ potential is due to Chien and Brown [14]. In the last three columns are the results of Filikhin and Gal [5].

Potentials $(\alpha\Lambda + \Lambda\Lambda)$	$\langle r'^2_{\alpha\alpha}\rangle^{1/2}$	$\langle r'^2_{\Lambda\Lambda}\rangle^{1/2}$	$\langle r'^2_{\alpha\alpha-\Lambda\Lambda}\rangle^{1/2}$	$\langle r'^2_{\alpha\alpha}\rangle^{1/2}$ Filikhin	$\langle r'^2_{\Lambda\Lambda}\rangle^{1/2}$ and	$\langle r'^2_{\alpha\alpha-\Lambda\Lambda}\rangle^{1/2}$ Gal
Gibson+NSC97b	3.13	3.10	1.62			
Gibson+NSC97e	3.12	3.05	1.61			
Gibson+ND	3.02	2.63	1.54			
Gibson+ESC00	2.97	2.42	1.51			
Isle+NSC97b	3.25	3.26	1.72			
Isle+NSC97e	3.24	3.21	1.71	3.5	4.2	2.4
Isle+ND	3.20	2.81	1.64	3.4	3.9	2.3
Isle+ESC00	3.18	2.59	1.60	3.3	3.7	2.2

Table VI – Contributions (in %) to the ground-state energy of $^{10}_{\Lambda\Lambda}$Be in the $N = 12$ approximation of the each one of the partial waves (0, 0, 0), (2, 0, 2), (2, 2, 0) and (0, 2, 2).

Potentials $(\alpha\Lambda + \Lambda\Lambda)$	(0,0,0)	(2,0,2)	(2,2,0)	(0,2,2)
Gibson+NSC97b	85.16	6.67	7.10	1.07
Gibson+NSC97e	85.66	6.69	6.67	0.98
Gibson+ND	89.20	6.58	3.77	0.45
Gibson+ESC00	90.72	6.32	2.66	0.30
Isle+NSC97b	93.14	2.51	3.12	1.23
Isle+NSC97e	93.21	2.56	3.05	1.18
Isle+ND	93.67	3.11	2.48	0.74
Isle+ESC00	93.81	3.48	2.15	0.56

In Fig. 2 we plot $B_{\Lambda\Lambda}$, the $\Lambda\Lambda$ separation energies for $^{6}_{\Lambda\Lambda}$He (which are just the extrapolated ground-state energies in Table I with sign changed, $-E_{\rm g.s.}$) as a function of the corresponding energies for $^{10}_{\Lambda\Lambda}$Be (which in this case are $-E_{\rm g.s.} + 0.09$ MeV – see Table IV), each point calculated with the same set of $\alpha\Lambda$ and $\Lambda\Lambda$ potentials. We notice that there is a correlation between those points through straight lines, what was already observed by Filikhin and Gal [5], Bodmer *et al.* [27] and Wang *et al.* [28]. The full line fits the points calculated with the Isle $\alpha\Lambda$ potential and the dashed line fits points calculated with the Gibson $\alpha\Lambda$ potential. We also indicate through horizontal and vertical lines the experimental values and errors of $B_{\Lambda\Lambda}$ for $^{6}_{\Lambda\Lambda}$He and $^{10}_{\Lambda\Lambda}$Be, respectively. We note that the full line crosses the rectangle defined by the dotted lines that represent the limits of the experimental errors and that the point that corresponds to the combination Isle+NSC97e of $\alpha\Lambda$ and $\Lambda\Lambda$ potentials is just inside the rectangle. This could indicate, as far as the extrapolated ground-state energies of $^{6}_{\Lambda\Lambda}$He in Table I are reliable, and the Faddeev calculations of Filikhin, Gal and Suslov [18] seem to confirm that since they reproduce our old result [4] obtained with the Bando $\alpha\Lambda$ potential, and the application of $\Lambda\Lambda$ potentials from Nijmegen model simulations beyond the $^1S_0$ channel is acceptable, that a unified description of both $^{6}_{\Lambda\Lambda}$He and $^{10}_{\Lambda\Lambda}$Be hypernuclei is possible, as represented through the combination Isle+NSC97e. However, for a consistent description, the Isle $\alpha\Lambda$ potential should reproduce in our

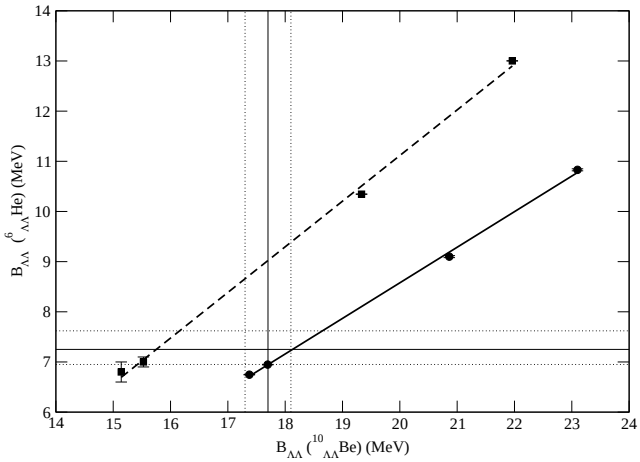

Figure 2. $\Lambda\Lambda$ separation energy $B_{\Lambda\Lambda}$ (in MeV) for $^{6}_{\Lambda\Lambda}$He as a function of the corresponding separation energy (in MeV) for $^{10}_{\Lambda\Lambda}$Be. Each point is calculated with the same combination of $\alpha\Lambda$ and $\Lambda\Lambda$ potentials in both hypernuclei. The full straight line fits points calculated with the Isle $\alpha\Lambda$ potential, while the dashed straight line is related to the Gibson $\alpha\Lambda$ potential. Experimental values and respective errors are also represented by the horizontal and vertical lines.

model the ground-state energies of the single hypernuclei $^{5}_{\Lambda}$He and $^{9}_{\Lambda}$Be. The results are respectively -3.095 MeV and -8.307 MeV [4], that should be compared with the experimental values -3.12 ± 0.02 MeV and -6.62 ± 0.04 MeV. It is clear that while the Isle potential reproduces the ground-state energy of $^{5}_{\Lambda}$He quite well, it overbinds $^{9}_{\Lambda}$Be by about 1.7 MeV. For this reason we cannot state that the combination Isle+NSC97e gives a consistent description of the single and double hypernuclei $^{5}_{\Lambda}$He, $^{9}_{\Lambda}$Be, $^{6}_{\Lambda\Lambda}$He and $^{10}_{\Lambda\Lambda}$Be. On the other hand, the Gibson $\alpha\Lambda$ potential gives -3.086 MeV and -6.839 MeV [4] for the ground-state energies of $^{5}_{\Lambda}$He and $^{9}_{\Lambda}$Be, respectively, being more reasonable than the Isle potential in this respect. However the dashed line in Fig. 2, which represent results for $^{6}_{\Lambda\Lambda}$He and $^{10}_{\Lambda\Lambda}$Be, is far from the rectangle it should cross. The conclusion is that a consistent description is not possible, agreeing with Filikhin and Gal [5] and Yamamoto *et al.* [19] and contrary to the conclusion of Albertus *et al.* [23].

Acknowledgments

We would like to thank the Laboratório de Cálculo Científico of the Institute of Physics of University of Brasília and CENAPAD MG/CO for providing the machines where calculations were performed.

References

[1] H. Takahashi *et al.*, Phys. Rev. Lett. **87**, 212502 (2001).

[2] D. J. Prowse, Phys. Rev. Lett. **17**, 782 (1966).

[3] O. Portilho, J. Phys. G: Nucl. Part. Phys. **28**, 2409 (2002); there is a mistyping in the r.h.s. of its Eq. (28): n that appears inside the parentheses and square-roots shall be replaced by n'.

[4] O. Portilho and S. A. Coon, J. Phys. G: Nucl. Part. Phys. **17**, 1375 (1991).

[5] I. N. Filikhin and A. Gal, Phys. Rev. C **65**, 041001(R) (2002); I. N. Filikhin and A. Gal, Nucl. Phys. A **707**, 491 (2002).

[6] Th. A. Rijken, V. G. J. Stoks and Y. Yamamoto, Phys. Rev. C **59**, 21 (1999).

[7] V. G. J. Stoks and Th. A. Rijken, Phys. Rev. C **59**, 3009 (1999).

[8] Th. A. Rijken, Nucl. Phys. A **691**, 322c (2001).

[9] M. M. Nagels, T. A. Rijken and J. J. de Swart, Phys. Rev. D **12**, 744 (1975).

[10] M. M. Nagels, T. A. Rijken and J. J. de Swart, Phys. Rev. D **15**, 2547 (1975).

[11] R. H. Dalitz and G. Rajasekaran, Nucl. Phys. **50**, 450 (1964).

[12] K. Ikeda, H. Bando and T. Motoba, Prog. Theor. Phys. (Suppl.) **81**, 147 (1985).

[13] O. Portilho, J. Phys. G: Nucl. Part. Phys. **25**, 961 (1999).

[14] W. S. Chien and R. E. Brown, Phys. Rev. C **10**, 1767 (1974).

[15] Y. Kurihara, Y. Akaishi and H. Tanaka, Phys. Rev. C **31**, 971 (1985).

[16] C. Daskaloyannis, M. Grypeos and H. Nassena, Phys. Rev. C **26**, 702 (1982).

[17] L. M. Delves, Adv. Nucl. Phys., edited by M. Baranger and E. Vogt (Plenum, New York, 1972), vol. 5, p. 1.

[18] I. N. Filikhin, A. Gal and V. M. Suslov, nucl-th/0303028.

[19] Y. Yamamoto, T. Motoba, H. Himeno, K. Ikeda and S. Nagata, Prog. Theor. Phys. (Suppl.) **117**, 361 (1994).

[20] E. Hiyama, M. Kamimura, T. Motoba, T. Yamada and Y. Yamamoto, Prog. Theor. Phys. **97**, 881 (1997).

[21] T. Yamada and C. Nakamoto, Phys. Rev. C **62**, 034319 (2000).

[22] E. Hiyama, M. Kamimura, T. Motoba, T. Yamada and Y. Yamamoto, Phys. Rev. C **66**, 024007 (2002).

[23] C. Albertus, J. E. Amaro and J. Nieves, Phys. Rev. Lett. **89**, 032501 (2002).

[24] M. Danysz *et al.*, Phys. Rev. Lett. **11**, 29 (1963); M. Danysz *et al.*, Nucl. Phys. **49**, 121 (1963).

[25] J. Pniewski and D. Zieminska, in *Proceedings of the International Conference on Kaon-Nuclear Interaction and Hypernuclei, Zvenigorod, 1977*, edited by P. A. Cherenkov *et al.* (Nauka, Moskow, 1979), p. 33.

[26] S. Ali and A. R. Bodmer, Nucl. Phys. **80**, 99 (1966).

[27] A. R. Bodmer, Q. N. Usmani and J. Carlson, Nucl. Phys. A **422**, 510 (1984).

[28] X. C. Wang, H. Takaki and H. Bando, Prog. Theor. Phys. **76**, 865 (1986).