Universidade de Brasília

Faculdade de Administração, Contabilidade e Economia
Departamento de Economia
Programa de Pós-Graduação em Economia

WHEN PRODUCTIVITY IS COSTLY: THE RELATION BETWEEN TRANSPORT COSTS AND INFRASTRUCTURE STOCK

Eduardo Dornelas Munhoz

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Banca Examinadora:
Profª. Dra. Marina Delmondes de Carvalho Rossi (Orientadora)
Profª Dra. Geovana Lorena Bertussi (Banca)
Profº Drº José Luiz Rossi Júnior (Banca)

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When productivity is costly: the relation between transport costs and infrastructure stock

Eduardo Dornelas Munhoz

University of Brasília

Marina Delmondes de Carvalho Rossi, PhD

University of Brasília

Abstract

A well-known result is that infrastructure stock leads to higher productivity of production factors such as capital and labor. This means that the marginal cost of those factors may increase after a raise in infrastructure investment. However, it is also known that infrastructure investment induces lower marginal costs to the transportation sector. We propose a two-firm model with transportation as an mandatory and intermediary good bought by the producer of consumer goods. We modify the depreciation function of private transportation capital stock so that it varies inversely with the infrastructure stock. This makes the capital stock higher and, thus, lowers its marginal productivity and marginal cost. As a consequence, the transportation firm has a reduction in its capital expenditures and this allows it to set lower freight rates. Our numeric simulation shows that our specification lowers the optimum investment rate of the economy when compared to the benchmark model used in the literature. Moreover, our results shows that if the Brazilian government invests just about 2% of GDP in transportation infrastructure, the steady state GDP could be 27% higher.

Contents

1 Introduction

Year after year, Brazilians strive to overcome the difficulties imposed by the poor transportation infrastructure that haunts the country. During every rain season, the news channels notice queues of trucks jammed into the mud on unpaved roads. What if we improved the quality of roads in Brazil? It is clear that this would benefit Brazilian economy and it is not hard to find any study that emphasize the relation between infrastructure investment, freight rates and production (BANISTER and BERECHMAN, 2000; BANERJEE, DUFLO and QIAN, 2012). A more subtle and interesting aspect is understanding the transmission channels through which transportation infrastructure impacts economic activity.

We develop a two-firm model assuming the infrastructure stock as reducing the depreciation rate of private capital. As the stock of infrastructure increases, the stock of private capital depreciates less and, therefore, its marginal productivity and marginal cost decreases. We can show that part of the effect that links infrastructure investment, freight rates, and economic product is the reduction of transportation firms' expenditures. In essence, we are adding one more transmission mechanism through which infrastructure affects the economy, namely, the "*cost reduction effect*", This effect plays along with the "*production effect*", which is normally presented by the literature by adding infrastructure stock into the production function. In practice, this mean that the freight truck does not spend as much time in the garage as before because it travels on a less damage highway. Therefore, this truck requires less maintenance cost, has greater availability, and does not be to be replaced as often.

The literature on public goods argues that infrastructure stock increases the productivity of production factors. Consequently, most papers adds the infrastructure stock directly into the representative firm's production function. If we assume that firm is a carrier, this approach does have some practical correspondence, since the quality of infrastructure affects the transit time of transportation (and the number of trips) and, thus, productivity. However, in such papers, the price of some production factors increase as a consequence of this higher marginal productivity, specially the cost of the private capital unit. As we know, the cost of production factors marginally equals the marginal productivity of these components. This means that there is an increase in the expenditures of the transportation firm, which is not in line with empirical findings (for example, Banister and Berechman (2003) argue that as much the quality of infrastructure improves, the production expenditure reduces).

By adding a transportation firm as an intermediary service brought by the firm that produces the consumer good, we can see that our consumer goods firm reacts likewise the firms in current literature - not drive by the production effect, but rather because freight rate is lower and that means a production expenditure decrease.

We do a numerical simulation using a Dynamic Stochastic General Equilibrium (DSGE) framework. The shock is an unexpected raise in infrastructure investment. We use a parametrization that intents to replicate the Brazilian economy. We also consider alternative parametrization scenarios as a robustness exercise. The comparative statics results show that the best infrastructure investment rate to Brazil is about 5% of GDP, since it optimizes the welfare. That rate is far low from others in literature, like Rioja (1999) that sets this rate about 10%. This happens because we have another mechanism, the "cost reduction effect", that boost the benefits of infrastructure stock even after the "production effect" diminishes. Even if the Brazilian government choose not be that benevolent, Brazilian production could raise by 27.10% if the government invest permanently 1.96% of GDP, the rate that Frischtak (2017) calculated that modernizes the Brazilian transport infrastructure stock up to 2060.

We also simulate the impact of price control policies, such as the one made by Brazilian government after 2017's truck drivers strike. We fixed the price to 5%, 10% and 15%-up the steady state and the results show a decrease of 4.96% to 14.60% of production. Because the models we usually find in the literature have just one representative firm (it does not distinguish between carrier and goods producer), such simulations could not be done before.

2 Literature review and discussion

The literature of neoclassic growth models with public goods started with Aschauer (1989), as the author decomposes the US Government investment in infrastructure between 1949 and 1985. He assumed a logarithmized Cobb-Douglas function and finds positive effects of transport, energy and sanitation infrastructure stock in US economic growth. Ascheuer (1989) was the first to empirically state the benefits of infrastructure investments.

Aschauer's work inspired Barro (1991) to developed a theoretical neoclassic model with infrastructure effects. In his work, he assumes one representative consumer and one representative firm with a Cobb-Douglas production function with constant returns. Besides the private capital and labor supply, the representative firm also uses a third element in the production function, a public investment of infrastructure. Barro (1991) model was lately improved by Glomm and Ravikumar (1994) as they included the stock of infrastructure and congestion adjustments.

After Barro (1991) and Glomm and Ravikumar (1994), several papers use this improved model structure to study specific aspects of transport infrastructure and growth. That means the basic model logic is the same to all these papers, but with some features extended: multiple firms (RIOJA, 2004); heterogeneous consumers (GIBSON and RIOJA, 2017a); choice of maintaining or expanding infrastructure stock (GIBSON and RIOJA, 2017b); time to build new infrastructure and the consequential delay of commute time (GALLEN and WINSTON, 2018); and public goods rivalry (RIOJA, 2005) are good examples of such new features aggregated.

Neoclassical growth models with infrastructure displays peculiar effects regards to others neoclassical growth models with public goods. This happens because infrastructure stock is not in the utility function of the representative consumer, nor it is a component of the consumer's budget constraint (GALLEN and WINSTON, 2018). Actually, its effect is indirect and ambiguous: just as the infrastructure stock increases the marginal productivity of inputs (ASCHAUER, 1989; BARRO, 1990; RIOJA, 1999; and others), effect called "*resource benefit*", it also induces sub-optimum economic equilibrium as the increase in infrastructure stock forces depends on a raise in the tax rate. Therefore, the equilibrium of these models usually cannot be derived analytically and the marginal effect of infrastructure stock in product and welfare generally shows a parabolic form.

A microeconomic consequence of higher productivity is higher cost of production factors since it marginally equals its productivity. This means the cost of private capital, also known as interest rates or marginal productivity of capital (MPK), would increase after an expansion of infrastructure stock. We can clearly see that in the literature. Rioja (1999) shows that, after an tax rate increase to finance an infrastructure investment, the economy shrinks; but after this "fiscal effect", the production, interest rates and private capital investments rapid grow above the steady state.

Although this behavior fits well the effects of investment in several infrastructure sectors, the idiosyncrasies of transport sector may induce others behaviors and results. Actually, we think the aggregation of the literature may offset the idiosyncrasies of transport sector. For example, on the one hand if roads are improved to permit faster trips, trucks drivers can make more trips with the same capital stock and labor quantities as before – therefore, the marginal productivity grows. We will call this the "*production effect*". On the other hand, their trucks may brake less and spend less time in garage because infrastructure is better and with less potholes. As consequence, the vehicle is more available and the maintenance costs are lower. Thus, the cost of production factors may reduce as the transport infrastructure improves because the marginal productivity of the capital stock is lower than before. It is like we would not be so interested to buy a new truck to our imaginary transport company because the trucks we already have are getting the job done, since they break less.

The link between transport investment and operation cost reduction is not new to the literature. As cited in Banister and Berechman (2000, p. 7), *"The debate over the links between transport infrastructure investment and economic development is not new. Ever since roads and railways were built, one of the main arguments has been the impact that the infrastructure would have on production costs."* In Lakashimanan (2011), the author comment that the microeconomic analysis emphasizes the cost savings from transport improvements as one way of how transport infrastructure impacts the economy. Cohen and Paul (2004) analyze the impact of infrastructure investment with spatialized data using a cost-saving function model. Other works cite such cost-reducing effect as well (MOUROUGANE and PISU, 2011; KIM, HEWINGS and AMIR, 2016; TATANO and TSUCHIYA, 2007; CRAFTS, 2009; KLINGTHONG, 2017; among others).

Most operational expenditures are mostly linked to costs of capital as you may have notice from our previous examples. Whence, our approach to model the cost reduction behavior relies on new specification of private capital depreciation function to carrier, general equilibrium effects and the fact that we separated the transport firm from the consumer goods firm, so we can better understand the effects.

If the truck breaks less, the depreciation of this stocked capital is lower than before[1]. So transport infrastructure improvements may reduce the depreciation of private capital. But if the private capital depreciates less, transport firms will need a smaller amount of investment to keep the capital stock up. Another way to see it is realize that truck will have greater availability, so the private capital stock will be higher than before. Because the private stock increases (depreciates less), the production grows and the freight decreases. That is what we call "*cost reduction effect*". As we will show, the freight also decreases because the diminishing cost of capital, so the shipping company production expenses also shrinks.

We resume these general equilibrium effects in Figure 1. There, the link between "Infrastructure stock" and "Production of transport" is what we call *production effect* and the links between "Carrier private capital depreciation", "Marginal product of factors" and "Production of transport" are the transmission mechanism that we propose (the *cost reduction effect*). Note that our proposed model does have the "production effect" *and* the "cost reduction effect", but both appear just in transportation firm.

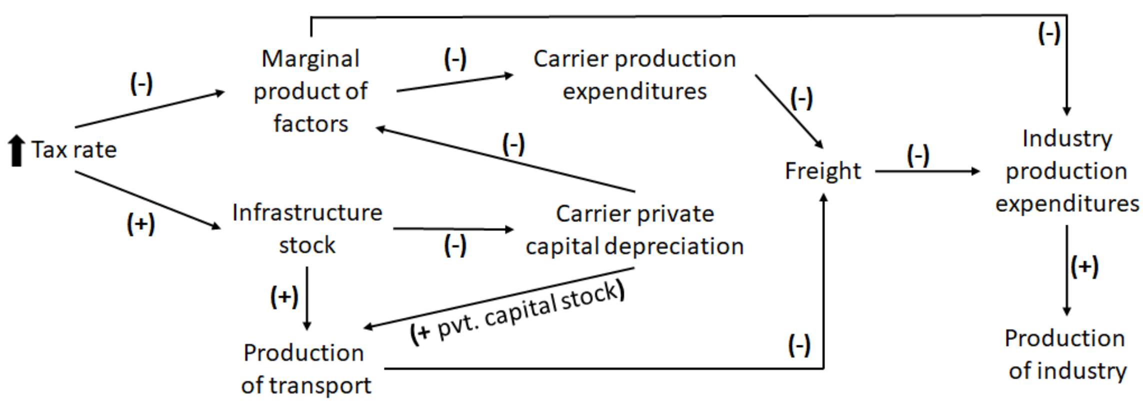


Figure 1: General equilibrium effects of our model.

Those are the effects in the shipping company. However, due to general equilibrium, the consumer goods firm also benefits from freight reduction. Because this firm is obligate to buy units of transport at the same amount of produced goods, its production expenditures decreases and the production raises.

[1]We can think depreciation as maintenance of vehicles and edification as well.

We are not the first ones to think about depreciation as a mechanism of transmitting infrastructure benefits. In Gibson and Rioja (2017b), they make the private capital depreciation, the efficiency of infrastructure and the public maintenance of infrastructure spend as functions of infrastructure maintenance-GDP ratio, a parameter, so their could study the implications of the allocation of public money on new infrastructure (raise in the stock) or in infrastructure maintenance. We differ from them because our depreciation functions depends directly on infrastructure stock and we do not parametrize this. As result, our depreciation is given endogenously and, therefore, reacts to unpredicted shocks and variables movements. We also use a two-firms model and our endogenously depreciation affects only one of these firms.

Finally, the use of depreciation shock in RBC models already exists, although it is not very common and may be never used to analyze infrastructure impact (at least, the we could not find any paper related). Albonico, Kalyvitis and Pappa (2013) developed a DSGE model with endogenously depreciation rate influenced by capital maintenance. The authors find that depreciation rate is procyclical and very volatile, in contrast as the usual business cycle literature adopts. Deli (2016) studies the impact of capital maintenance on endogenous depreciation rate and suggest that it reduces the Solow residuals usually assumed by standard RBC models.

In the next chapter, we explain the model we use and how we try to replicate the theoretical effects that we explained before. If the reader got interested by the good and worth reading papers we briefly cited, we have provided in the Annex I a more comprehensive discussion of the current literature.

3 Model specification

We describe our alternative model below in four blocks: consumers, industry, carrier and government (see Figure 2). The consumer buy products from industry (Q^I) and offers capital and labor to the carrier firm (K^C, L^C) and to the industry firm (K^I, L^I). The carrier sells transport services to industry (pQ^C) and both firms pay taxes (λ). Finally, the government invests in infrastructure (G)

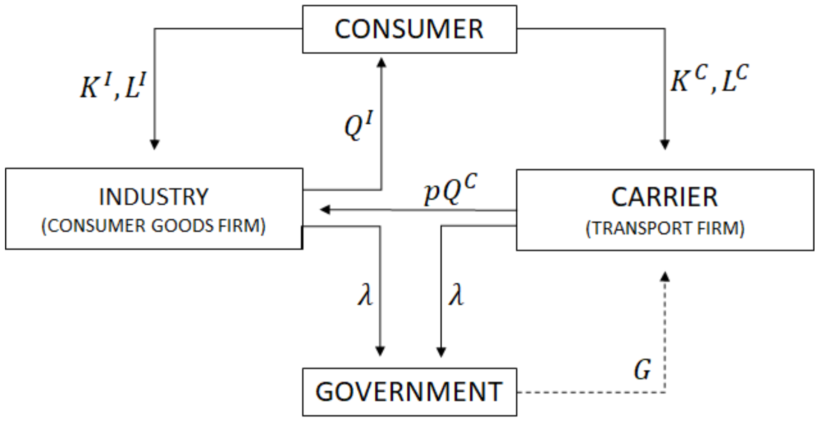


Figure 2: Schematic map of our model.

3.1 Consumers

Consumer utility function is the Constant Relative Risk Aversion (CRRA), a very common function in the literature. The maximisation problem is set as[2]:

$$\max_{K_t^{\mathrm{C}},K_t^{\mathrm{I}},C_t^{\mathrm{I}},L_t^{\mathrm{C}},L_t^{\mathrm{I}}} U_t = \beta \mathrm{E}_t \left[U_{t+1}\right] + (1-\eta)^{-1} \left(C_t^{\mathrm{I}\mu} \left(1 - L_t^{\mathrm{C}} - L_t^{\mathrm{I}}\right)^{1-\mu} \right)^{1-\eta} \tag{1}$$

where the subscript t is the time index; β is the intertemporal discount rate; C_t^{I} is the consume of goods; K_t^{C} is the private capital associated with carrier sector; K_t^{I} is the private capital associated with industry sector; L_t^{C} is the labor supply to carrier; L_t^{I} is the labor supply to industry sector; and μ, η are CRRA parameters. We will use the superscript "I" to all industry related variables and "C" to all carrier related variables.

The consumers are free to choose which firm they want to work and which one they want to invest, but they must buy just consumer goods - that means they do not consume transport services. These are their spending. For their labor, they receive wages; and for their capital, they receive interest rates. These are their income. So, we set the budget constraint as:

$$C_t^{\mathrm{I}} + I_t^{\mathrm{C}} + I_t^{\mathrm{I}} = K_{t-1}^{\mathrm{C}} r_t^{\mathrm{C}} + K_{t-1}^{\mathrm{I}} r_t^{\mathrm{I}} + L_t^{\mathrm{C}} W_t^{\mathrm{C}} + L_t^{\mathrm{I}} W_t^{\mathrm{I}} + \pi_t^{\mathrm{C}} \tag{2}$$

where I_t^{I} is the investment of private capital in industry sector; I_t^{C} is the the investment of private capital in carrier sector; r_t^{C} is the interest rate/marginal production of private capital in carrier sector; r_t^{I} is the interest rate/marginal production of private capital in industry sector; W_t^{C} is the wage/marginal product of labor in carrier sector; W_t^{I} is the wage/marginal product of labor in industry sector.

The law of motion of carrier firm private capital is a bit different than traditional use.

[2]The following consumer problem will use the recursive structure of $E_t(\sum_t \beta^t U_t)$.

We assume the following depreciation function:

$$\delta^{\star} = \delta \frac{K_t^{\mathrm{Cd}\,\psi}}{G_t} \tag{3}$$

where δ is the private capital depreciation and G_t is the transport infrastructure stock. So the law of motion of transport sector private capital is:

$$I_t^{\mathrm{C}} \equiv K_t^{\mathrm{C}} - K_{t-1}^{\mathrm{C}} \left(1 - \delta^{\star}\right) \tag{4}$$

Because transport infrastructure is subject to congestion with usage, we must adjust the actual infrastructure stock using the carrier private capital. The ψ is the congestion parameter. This formulation is the inverse of the term used in Rioja (1999).

Although not sophisticated, our depreciation function ($\delta^{\star}$) have the desired property of transmitting the changes in infrastructure stock: if the infrastructure stock grows at higher rate than carrier private capital, the term $\frac{K_t^{\mathrm{Cd}\,\psi}}{G_t}$ diminishes and, as consequence, the carrier private capital depreciation also decreases. With this function, we establish our new transmission mechanism of effects derived from infrastructure stock.

The law of motion for consumer goods follows:

$$I_t^{\mathrm{I}} \equiv K_t^{\mathrm{I}} - K_{t-1}^{\mathrm{I}} \left(1 - \delta\right) \tag{5}$$

3.2 Firms

In our economy, there are innumerous consumers and two sectors of firms: one that only produces transport services (carrier) and other that only produces homogeneous consumer goods (industry). Because the industry needs to deliver the products to buyers, it must buy, at a freight value p_t, transport services from the carrier. To make it simple, we suppose the consumer goods as numeraire and every unit of consumer goods produced needs one unit of transport service ($Q^C = Q^I$). Because this restriction, the $p_t \in (0, 1)$, whereas, if not, the industry will not be profitable. So the maximization problem of industry is:

$$\max_{K_t^{\mathrm{Id}}, L_t^{\mathrm{Id}}, Q_t^{\mathrm{I}}} \pi_t^{\mathrm{I}} = \left(1 - \lambda_t\right) Q_t^{\mathrm{I}} - r_t^{\mathrm{I}} K_t^{\mathrm{Id}} - L_t^{\mathrm{Id}} W_t^{\mathrm{I}} - p_t Q_t^{\mathrm{I}} \tag{6}$$

s.t. :

$$Q_t^{\mathrm{I}} = K_t^{\mathrm{Id}\,\alpha} L_t^{\mathrm{Id}\,1-\alpha} \tag{7}$$

where π_t^{I} is the profit of industry; λ_t is the tax rate levied by the government; K_t^{Id} is the private capital demanded by the consumer goods sector, L_t^{Id} is the labor demanded by industry and α is the coefficient of production elasticity. The read must see that we used the term $p_t Q_t^{\mathrm{I}}$ instead of $p_t Q_t^{\mathrm{C}}$ in order to emphasize the equality of both quantities.

Again, the transport sector is a bit different from usual. Here, it is important to note that our proposed transport firm does have the *production effect* and the *cost reduction effect*. As we explained latter, the literature was not wrong to assume production effect, but since they just have one firm, they could not separate the carrier and the industry as we do.

Last section where we said that better infrastructure means faster trips. So our carrier production function must aggregates this "*production effect*", as we name before. But shall we take as the literature handles and add the term $\left(G_t K_t^{\mathrm{Cd}-\psi}\right)^{\omega}$? We think this may overestimates the impact in production. Using a software of traffic modelling, we adjust the production effect as the same gain in travel speed. We describe our methodology in the parametrization section. To mirror these ideas, we use a "cycle effect inflator" (τ), so if the infrastructure stock increases, the marginal productivity of labor and private capital slightly grows. Thus, the maximization problem of transport firm is:

$$\max_{K_t^{\mathrm{Cd}}, L_t^{\mathrm{Cd}}, Q_t^{\mathrm{C}}} \pi_t^{\mathrm{C}} = p_t Q_t^{\mathrm{C}} \left(1 - \lambda_t\right) - L_t^{\mathrm{Cd}} W_t^{\mathrm{C}} - r_t^{\mathrm{C}} K_t^{\mathrm{Cd}} \tag{8}$$

s.t. :

$$Q_t^{\mathrm{C}} = \left(1 + \tau \left(G_t K_t^{\mathrm{Cd}-\psi}\right)^{\omega}\right) K_t^{\mathrm{Cd}\,\sigma} L_t^{\mathrm{Cd}\,1-\sigma} \tag{9}$$

where π_t^{C} is the profit of transport firm; λ_t is the tax rate levied by the government; K_t^{Cd} is the private capital demanded by the transport sector, L_t^{Cd} is the labor demanded by carrier and σ is the coefficient of production elasticity.

The term $G_t K_t^{\mathrm{Cd}-\psi}$ is the inverse of the one we used in the depreciation of carrier's private capital. It is inversed because more infrastructure stock improves production.

3.2.1 First order

We do not intent to describe all first order derivations (we left in the Annex II as well). However, it is easy to see how the raise in tax rate reduces the productivity (or the cost of) private capital and labor if we analyze the first order of firms problem:

$$r_t^{\mathrm{I}} = \left(1 - \lambda_t - p_t\right) Q^{\mathrm{I}}_{K_t^I} \tag{10}$$

$$W_t^{\mathrm{I}} = (1 - \lambda_t - p_t)\, Q^{\mathrm{I}}_{L_t^I} \tag{11}$$

$$r_t^{\mathrm{C}} = p_t\,(1 - \lambda_t)\, Q^{\mathrm{C}}_{K_t^C} \tag{12}$$

$$W_t^{\mathrm{C}} = p_t\,(1 - \lambda_t)\, Q^{\mathrm{C}}_{L_t^C} \tag{13}$$

where $Q^{\mathrm{I}}_{K_t^I}, Q^{\mathrm{I}}_{L_t^I}, Q^{\mathrm{C}}_{K_t^C}, Q^{\mathrm{C}}_{L_t^C}$ are the derivations of the production function related to the resource in subscript. If the tax increases, the term $(1 - \lambda_t)$ decreases and so the marginal productivity of the factor.

3.3 Government

We assume a passive government that levies the tax on firms and instantaneously converts into infrastructure stock. To be fair, this is a heritage from Barro (1991). Nonetheless, this conversion is not one-by-one and suffers from govern inefficiencies. With inefficiencies, infrastructure investment rate is very close, however different from tax rate.

The accumulation process of infrastructure stock is:

$$G_t = G_{t-1}\,(1 - \delta^{\mathrm{g}}) + \lambda_t\,(1 - \zeta)\left(Q_t^{\mathrm{I}} + p_t Q_t^{\mathrm{C}}\right) \tag{14}$$

where G_t is the infrastructure stock; δ^{g} is the depreciation rate of infrastructure stock; and ζ is the govern inefficiency rate.

Because tax rate is our shock variable, there is no optimization problem to set the best tax rate to our economy. As in Rioja (1999), there is a autocorrelated process that drives the tax rate decay after a shock, set below:

$$\lambda_t = \epsilon_t^{\lambda} + \lambda^{\mathrm{mean}}\,(1 - \phi) + \phi\lambda_{t-1} \tag{15}$$

where ϵ_t^{λ} is the exogenous shock; λ^{mean} is the mean tax rate. For example, when we do the shock experiment, we set $\epsilon_t^{\lambda} = 1\%$, so λ_t increases in 1%.

3.4 Equilibrium Identities

The following equations must be hold in equilibrium:

$$K_C^{\mathrm{Id}} = K_{t-1}^{\mathrm{I}} \tag{16}$$

$$K_t^{\mathrm{Cd}} = K_{t-1}^{\mathrm{C}} \tag{17}$$

$$L_t^{\mathrm{Id}} = L_t^{\mathrm{I}} \tag{18}$$

$$L_t^{\mathrm{Cd}} = L_t^{\mathrm{C}} \tag{19}$$

$$Q_t^{\mathrm{C}} = Q_t^{\mathrm{I}} \tag{20}$$

$$Y_t = p_t Q_t^{\mathrm{C}} + Q_t^{\mathrm{I}} \tag{21}$$

equations (16) and (17) have different period subscript because K_t^{I} is the capital stock at the end of time t. So, in period t, firms rent the capital from the stock of the last end of period, $t-1$. Equations (18) and (19) are just the equalization of labor - all supplied labor must equal the demanded labor. Equation (20) is our restriction of one consumer good produced must be delivered by one unit of transport service. Finally, (21) is just the product account.

4 Parametrization

This study do not aim to get empirical results, but an intent to replicate Brazilian parameters were made. Almost all of our parameters come from two studies. Rioja (1999) is our benchmark paper and much of its parameters, such as consumer parametrization and output-elasticities of firms, are equal or very close to newest papers. Also, if we use his parametrization, we could better compare our models. We also use the work of Frischtak (2017). His work is the most recent study that calculates depreciation rate and the stock of Brazilian infrastructure. The sad conclusion of his results is that Brazil invested, on average, 0.67% of GDP between 2010 and 2016 and this was not enough to overcome the deprecation.

In 2018, the Inter-American Development Bank made a series of studies of efficiency of public spending. With that work, we parametrize the ζ according to the inefficiency of Brazilian government.

We used the software Highway Development and Management (HDM-4) calibrated accordingly the Brazilian National Department of Transport Infrastructure (DNIT) parameters to estimate the average speed of trucks in one-lane roads at three qualities of pavement: terrible, bad, regular, good and great. We choose one-lane because it is the most common kind of road in Brazil. The average speed increases, on average, 4.56% across these pavement qualities. So we assume that the production would increase in the same amount because the time used in travels would decrease about this value.

Specially for ψ parameter, it was not attributed any value since it is defined by the following calibration equation:

$$K_{\text{ss}}^{\text{C}^{\psi}} = \frac{1}{0.75} \tag{22}$$

This equation mimics the relation appointed by Rioja (1999) that Latin America infrastructure is congested about 20% and 30%, so the adjusted stock of infrastructure related to congestion is 75% of the "raw" stock. If we calibrate ψ using (22), so the term $G_t K_t^{\text{td}^{-\psi}}$ equals $0.75 * G_t$, therefore 75% of the "raw" stock.

Because we could not find any parametrization specifically to transport firms, we estimate the capital output-elasticity of carrier using KLEM database of US transportation sector (road, rail and water transportation). Our parametrization values are listed in Table 1.

Table 1: Standard parametrization setting. Quarterly values.

Parameter	**Value**	**Description**	**Source**
α	0.54	Industry private capital output-elasticity	Rioja (1999)
β	0.9975	Intertemporal discount rate	Rioja (1999)
δ	0.025	Depreciation rate of private capital	Rioja (1999)
δ_g	0.0106	Depreciation rate of public infrastructure	Frischtak (2017)
β	2.33	Utility curvature parameter	Rioja (1999)
λ^{mean}	0.0067	Long-term tax rate	Frischtak (2017)
μ	0.35	Consumption share	Rioja (1999)
ω	0.1	Public infrastructure output-elasticity	Rioja (1999)
ϕ	0.9	Persistence of lagged shock	Rioja (1999)
ψ	0.091075	Congestion parameter	*
σ	0.64	Carrier private capital output-elasticity	Authors
τ	0.0456	Increased cycles coefficient	Authors
ζ	0.039	Public investment inefficiency coefficient	IADB (2018)

We also simulates many scenarios that are described in Table 2.

Is his study, Frischtak (2017) proposes a infrastructure investment target to Brazil, 1.96% (2.92 times higher than the standard parametrization) of the product. The "Frischtak's setting" were test as a way to observe the [positive] distortions of this highest infrastructure investment in Brazilian economy. However, Frischtak scenario is not a good way to analyze marginal increases in the tax rate, so we also present a "1.01 times higher", that is, 1% increase in standard tax rate.

Moreover, we also test for higher depreciation scenarios (1% absolute raise in the private depreciation rate we used and 1% absolute increase in the depreciation rate - that is 40% higher), higher depreciation of infrastructure stock (from 0.0106 to 0.0107 - 1% increase - and 0.0207 - 94% higher).

Table 2: Alternative scenarios

Scenario	Changes
Standard	No changes
Higher capital depreciation rate (HCD)	$\delta = 0.02525$
Higher infrastructure depreciation rate (HID)	$\delta_g = 0.0107$
Frischtak	$\lambda^{mean} = 0.0196$
+1% Tax rate	$\lambda^{mean} = 0.006767$
HCD +1% absolute increase	$\delta = 0.035$
HID +1% absolute increase	$\delta_g = 0.0206$

5 Results

We have been arguing that there are another transition mechanism of infrastructure benefits, the cost reduction effect, that plays along with the production effect. We expect that you are convinced by our theoretical arguments, but now we want you to be convinced by numerical results. To do so, we do two exercises: first, a comparative statics; second, a shock experiment. The first intents to describe the long run effects of a persistent raise in the tax rate and transport infrastructure investment by the government. The second is important to explain how the variables react to an unexpected raise in the tax rate (and, thus, in the investment rate).

5.1 Comparative statics

The comparative statics results are shown below and are presented as deviations from standard scenario. We add three more variables, EXP_t^C, EXP_t^I and Production effect, that represent, respectively, the carrier production expenditures; the industry production expenditures; and the term $\left(1 + \tau\left(G_t K_t^{\mathrm{Cd}-\psi}\right)^{\omega}\right)$. The cost reduction effect is represented by $\delta^{\star}$.

Table 3: Comparative statics results. Selected variables (complete table at Annex III)

Variable	+1% Tax rate	Frischtak	HID	HCD	HID+1%	HCD+1%
p	-0,54%	-40,42%	0,55%	0,67%	48,99%	27,79%
r_t^{C}	-1,18%	-64,70%	1,21%	2,09%	169,65%	118,91%
r_t^{I}	0,00%	0,00%	0,00%	0,91%	0,00%	36,36%
G_t	1,41%	271,81%	-1,41%	-1,51%	-18,55%	-34,35%
U	0,30%	18,09%	-0,32%	-0,92%	-45,34%	-48,62%
Y	0,41%	27.10%	-0.43%	-1.51%	-48,79%	-51,24%

Let us recall Figure 2. We raise the tax rate in 1% permanently and, as expected, the new infrastructure stock is higher than before. The depreciation of carrier capital decreases and, as consequence of lower depreciation of capital, the stock of carrier private capital increase. It is interesting to see the variation of carrier private capital stock is higher than the industry's because of that.

It is also interesting that the carrier investment drops compared to standard setting, because now, since the capital depreciates less, is not necessary to invest the same amount as before – recall our argument in Section 2. This is clear if you see that the difference between the variations of the depreciation of carrier capital (-1.30%) and its stock (+1.19%) is almost the same as the variation of carrier investment (-0.12%). Because capital is cheaper to carrier, it also substitute some labor factor by private capital.

The changes in the resource allocation of the carriers makes its production expenses unaltered and, actually, the profit raises because the decrease in prices are lower than the growth of production, tax rate and transport cycles. It can't be said about industry production expenses, that raise compared to the standard setting. Since the industry has no profit (it equals zero), the increase in production expenses follows the increase of production and tax. Consequently, the increase in the production induce higher labor and capital usage and, therefore, higher welfare.

By one side, our experiment with Frischtak investment rate suggestion also presents the same movements, but in large scale. By the other side, the results of HID show opposite but very similar to the +1% tax rate. This result is intuitive, since the infrastructure stock variation is almost the same as in tax rate increase and it affects the same structures (production and carrier capital depreciation). HID with +1% absolute increase show the same pattern, but magnified.

Finally, the HCD is the "worst" scenario in terms of product and welfare. This happens because it's the only scenario that prejudices both firms at the same time. Both firms must pay more to use capital factor and, to the carrier, the variation is higher because the infrastructure stock declines, consequence of lower collection of taxes, so it loses both production and cost reduction effect. Again, HCD with +1% absolute increase also shows the same movements, in a large scale, and the production decrease is still higher than the HID with +1% absolute increase.

But how much of this results are induced by the cost reduction effect? To answer this, we made a "benchmark model" with just production effects. This means equation (4) has the same design as equation (5). On the one hand, if the infrastructure investment grows, cost reduction effect induces higher product and lower freight, so the "benchmark model" product increases less and the freight reduces less. On the other hand, the benchmark model shows greater production in higher depreciation scenarios because it depends less of infrastructure stock.

Table 4 show a comparison of the steady states on both models to selected scenarios. We can see that, for example, the product in the "+1% Tax rate" scenario is 0.63% higher in cost reduction model than in benchmark model.

Table 4: Variation of cost reduction model results in relation to the benchmark model results.Selected variables (complete table at Annex III)

Variable	**+1% Tax rate**	**Frischtak**	**HID**	**HCD**
p	-0,53%	-40,08%	0,54%	0,52%
r_t^{C}	-1,18%	-64,70%	1,21%	1,17%
r_t^{I}	0,00%	0,00%	0,00%	0,00%
G_t	0,42%	29,81%	-0,42%	-0,41%
U	0,31%	19,50%	-0,32%	-0,31%
Y	0,42%	29,81%	-0,42%	-0,41%

Now, let compare the results with the literature. As said before, we were inspired by Rioja (1999) and his work is the most comparable to ours. With "most comparable" we mean that there are no structures that difference his work from ours beyond the two-firm economy, fright prices, depreciation function and that he uses a tax rate of 6.24%.

According to Rioja (1999) simulations, +1% absolute increase in the tax rate (from 6.24% to 7.24%) increase the product in 2.53%. If we increase our tax rate in +1% (from 0.67% to 1.67%), the product raises about 24% [3]. Why our model show such a high increase? The main factor here is that our standard tax rate is far below the one used in Rioja (1999) and the both models shows marginal decrease of benefits of infrastructure. If more infrastructure means less marginal benefits, so Rioja's model presents less benefits since it has higher tax rate.

One may recall this marginal decreasing benefit is due to the raise of tax that is onerous to the economy. That means a very tax rate can make the economy worse, since the effects of higher infrastructure stock does not offsets the bad effects of tax levied. So both model, ours or Rioja's, present a parabolic curve of marginal gain in welfare (and production) relate to the tax rate. The interesting fact here is that, in our model, the peak of this parabola occurs with a tax rate of about 5% and in Rioja's this peak happens when the tax rate is around 10%.

What does it tell us? When we add the "cost reduction effect", we need less investment in infrastructure to achieve the optimum investment rate because there is another transmission of benefit. In practice, the "overimprovement" of some road would be undesirable since the cost reduction and the average speed of trips are so marginally low that does not compen-

[3] We did not showed this result in the tables before because it is very close to the Frischtak scenario's results

sate the raise of tax. These are good news to Latin America and others underdevelopment countries.

5.2 Short-term effects of the tax increase

In the comparative statics we assume a persistent shock in tax rate so that we could analyze the long run effects of higher infrastructure capital. Our model produces a complex movement, as there is the carrier service price that drives the effects to industry firm. All figures with the movements we are going to describe are listed in Annex IV.

Let us start with the impact of tax shock (and, raise of infrastructure investment). First, it drop the factor payments (wages and interest) to both firms and also the production of industry and, thus, the transport production. By the carrier side, it sees two effects of cost reduction that reduces its production expenses: first, the drop in cost of production factors, as said before; second, since infrastructure stock increases, the fall in its private capital depreciation rate that affects the cost of capital.

Now, the carrier must set the transport price. Remember that industry has zero profit, so all its expenses ($r^I K^I + w^I L^I + pQ^I$) must equal all they liquid revenue ($(1-\lambda)\, Q r^I$). If some cost of production factor (r^I, w^I or p) raises, so it must adjust the use of inputs through the new level of production. For example, if r^I increases, Kr^I decreases and others production factors (as Lr^I) may decrease as well, so that liquid revenue and production expenses equalize again. Likewise, If the freight (p) grows, the industry will be forced to give up some capital stock and labor and, as consequence, production and transport demand:

$$(1-\lambda)Q^I < r^I K^I + w^I L^I + \uparrow pQ^I \rightarrow (1-\lambda) \downarrow Q^I = r^I \downarrow K^I + w^I \downarrow L^I + \uparrow p \downarrow Q^I$$

Carrier, however, does not want a lower industry production, because it will mean a lower transport demand. It has a alternative: it can give up some profit (since carrier has positive profit) and reduce freight. And that is what happens. Such movements show how transport input differs from a usual factors: it does not contributes to production (is not explicitly in Q_{nt}), but it use is obligatory. So, if freight increases, the only way to react is giving up production. The firm does not have substitution effect to this kind of shock.

Carrier investment show an interesting movement. It raises after the tax shock. Actually, we understand that as a result of the drop in cost of capital (-1.91%) higher than the wages (-0.82%). Because the capital is more value than labor to the carrier, since output-elasticity is higher to capital, it substitute labor (-1.09%) by capital stock (0.53%). The difference between the variations are due the loss of production. Of course, it decreases as the lower depreciation makes capital stock more "available".

Future is bright to industry. The freight prices continue to drop as the carrier capital depreciation forces down the cost of capital. Although wages and interest rates of industry increase after the tax shock (kind of "resource benefit" applies here), the industry expenses do not, so production has room to expand. And, as production (of both firms) expand, the government collects more taxes and the infrastructure raises – and the "wheel of fortune" spins again.

There is no much to compare our model with Rioja (1999), since the last does not have two firms or freight prices. However, one can see that the industry movements are very close to the firm that Rioja assumes. That mean our industry *does* benefits from infrastructure such as in Rioja (1999), however in diferent ways.

5.3 Price control policy simulation

After a serial of political scandals, the giant state-owner Petrobras oil company was almost complete dismantled. As a reaction, the board of the company approved a bunch of reforms in October 2016 such as periodic review of fuel prices in the light of changes in the international oil market, exchange rates and others relevant factors. As investors view the new price setting as a way to rebuilt the company finances, the truck drivers would not have the same enthusiasm. Many factors drive the exchange rate to appreciate (in US dollars terms) in early 2018 and, as consequence to the new Petrobras price setting, the diesel price rose in local currency. As the self-employed truck drivers saw the cost shock, they claimed an action by the government.

After a few days without any answer, the drivers started a huge strike that literally stopped the country. Even the transportation firms could not operate, since the strikers blocked and threatened any trucker who dare to work. As the strikes took several days, jets barely could be refueled in some of the main airports, the poultry production suffered extensive lost as the birds could not be feed, supermarkets lacked of food and many gas stations, as they fuel stocks were not restored, rose the fuel prices. After all, the impacts were estimated in billions of dollars to Brazilian economy. After days of intense strike, the government finally answered the trucker's claimed and, among other benefits, forced down the diesel price and introduced a minimum shipping price table.

Is expected that the impacts of transportation minimum price could be tested in DSGE models with infrastructure. However, as the literature usually presents these models, the use of fixed prices cannot be tested since there are not a specialized transportation firm or transportation price transmission. Alternative specifications, as ours presented here, enables such tests.

Therefore, the model was rerun, but modified with fixed prices for transportation service. However, since the model would present more equations than variables, we drop the quantity control variable of carrier firm, but we left the production constraint:

$$Q_t = \left(1 + \tau \left(G_t K_t^{\mathrm{Cd}-\psi}\right)^{\omega}\right) K_t^{\mathrm{Cd}\sigma} L_t^{\mathrm{Cd}1-\sigma} \tag{23}$$

If we apply the same calibrated parameters in that "modified" model, the same results of "original" model will be produced. So, we are clear to go on. The price was fixed to 5%, 10% and 15% bigger than standard steady state as shown before. The Table 5 shows the results as deviations from the long-run values with free transportation prices structure.

Table 5: Results from price control policy simulation (complete table at Annex III).

Variable	**+5% above SS**	**+10% above SS**	**+15% above SS**
r_t^{C}	3,98%	8,29%	12,96%
r_t^{I}	0,00%	0,00%	0,00%
G_t	-4,96%	-9,82%	-14,60%
U	1,40%	2,93%	4,61%
Y	-4,96%	-9,82%	-14,60%

The results are intuitively. The product drops almost the same rate of freight increase. Because the cost of transport is higher, the industry must drop its production. As reaction, the carrier firm also decreases its production as well. However, since the firms are not using as much inputs as before, the consumers rent fall. It is interesting to note that infrastructure stock also drops, so the depreciation of carrier private capital increases and the cost of capital to the carrier react the same way. The final result, as expected, is lower welfare to consumers and a very higher profit to carrier firms.

6 Conclusion

Current literature normally assumes the effect of a raise in infrastructure stock just in the production function of a representative firm, generating what we called "*production effects*". However, this makes the production expenditures to raise with higher infrastructure stock. If we assume a consumer goods firm, this result may makes sense (as we showed), but if we analyse a transport firm, the result does not have practical meaning. We think the problem here is because use just one representative firm and that hides the effects in transport sector and how they are transmited to consumer goods sector.

We proposed a two-firm model that just the transport firm has depreciation as a function of infrastructure stock along with modified production effect structure. This depreciation structure is a new transmission effect mechanics that complements the production effect. The very point of adding new transmission mechanics is to make sure that costs of production factors to transport firm, specially private capital costs, does not grow up after a raise in infrastructure stock. This fact correlates with the usual result that more infrastructure investment drops transport costs.

Our Brazilian-parametrized model is populated by consumers, carrier firms and industry firms that produce consumer goods and must buy transport services. The numeric simulations show that a small raise in infrastructure investment, provided by a raise in tax rate, induces several general equilibrium effects, as lower freight values and higher economic product. Specially, the optimal investment rate that maximizes the welfare in our model is lower than the findings of the literature, because our new transmission mechanics.

We also used our model to analyze the effects of freight value control policy implemented by Brazilian government after the truck drivers' strike in 2018. Our model points out that consumers may lost welfare and the carriers firms and truck drivers raised their profits.

We think that improving the depreciation functional form, adding other sectors of infrastructure and including public debt emission to finance the infrastructure investment are interesting ways to improve our work.

7 References

[1] BANISTER, D.; BERECHMAN, J. *Transport investment and economic development.* Taylor & Francis, 2003.

[2] DELI, Y. D. Endogenous capital depreciation and technology shocks. *Journal of International Money and Finance*, v. 69, p. 318–338, Dec. 2016.

[3] MOUROUGANE, A.; PISU, M. Promotinginfrastructure development in brazil. *OECD EconomicsDepartment Working Papers*, v. 898, p. 1–35, 2011.

[4] ALBONICO, A.; KALYVITIS, S.; PAPPA, E. Capital maintenance and depreciation over the business cycle. *Journal of Economic Dynamics and Control*, v. 39, p. 273–286, Feb. 2014.

[5] KIM, E.; HEWINGS, G. J.; AMIR, H. Economic evaluation of transportation projects: An application of financial computable general equilibrium model. *Research in Transportation Economics*, v. 61, p. 44–55, Mar. 2017.

[6] BANERJEE, A.; DUFLO, E.; QIAN, N. On the road: Access to transportation infrastructure and economic growth in china. NBER Working Papers 17897, National Bureau of Economic Research, Inc, 2012.

[7] BARRO, R. J. Government spending in a simple model of endogeneous growth. *Journal of Political Economy*, v. 98, n. 5, Part 2, p. S103–S125, Oct. 1990.

[8] CRAFTS, N. Transport infrastructure investment: implications for growth and productivity. *Oxford Review of Economic Policy*, v. 25, n. 3, p. 327–343, 10 2009.

[9] GALLEN, T.; WINSTON, C. Transportation capital and its effects on the u.s. economy: A general equilibrium approach. *University of Purdue*, p. 1–45, Aug. 2018.

[10] GIBSON, J.; RIOJA, F. Public infrastructure maintenance and the distribution of wealth. *Economic Inquiry*, v. 55, n. 1, p. 175–186, 2017b.

[11] GIBSON, J.; RIOJA, F. The welfare effects of infrastructure investment in a heterogenous agents economy. *Georgia State University*, p. 1–48, may 2017a.

[12] KLINGTHONG, P. *Essays on fiscal policy, public infrastructure, and public r&d and heterogeneous labor*. may 2017. Tese (Doutorado em Física) - Washington State University, may 2017.

[13] LAKSHMANAN, T. The broader economic consequences of transport infrastructure investments. *Journal of Transport Geography*, v. 19, n. 1, p. 1–12, 2011.

[14] MACHICADO, C. G. Macroeconomic and welfare effects of public infrastructure investment in five latin american countries. Development Research Working Paper Series 14/2007, La Paz, 2007.

[15] RIOJA, F. Productiveness and welfare implications of public infrastructure: a dynamic two-sector general equilibrium analysis. *Journal of Development Economics*, v. 58, n. 2, p. 387–404, 1999.

[16] RIOJA, F. Growth, welfare, and public infrastructure: A general equilibrium analysis of latin american economies. *Journal of Economic Development*, v. 26, n. 2, p. 119–130, 2001.

[17] RIOJA, F. Infrastructure and sectoral output along the road to development. *International Economic Journal*, v. 18, n. 1, p. 49–64, 2004.

[18] RIOJA, F. Roads versus schooling: Growth effects of government choices. *The B.E. Journal of Macroeconomics*, v. 5, n. 1, p. 1–24, 2005.

[19] SPENCE, N.; ROVOLIS, A. Duality theory and cost function analysis in a regional context: the impact of public infrastructure capital in the greek regions. *The Annals of Regional Science*, v. 36, n. 1, p. 55–78, 2002.

[20] TATANO, H.; TSUCHIYA, S. A framework for economic loss estimation due to seismic transportation network disruption: a spatial computable general equilibrium approach. *Natural Hazards: Journal of the International Society for the Prevention and Mitigation of Natural Hazards*, v. 44, n. 2, p. 253–265, February 2008.

[21] FRISCHTAK, C. R.; MOURÃO, J. O estoque de capital de infraestrutura no brasil: uma abordagem setorial. In: NEGRI, J. A. D.; ARAÚJO, B. C.; BACELETTE, R. (Eds.) *Desafios da Nação*. Instituto de Pesquisa Econômica Aplicada (IPEA), 2018. v. 1, Cap. 3, p. 1–24. Auxiliary paper.

[22] ASCHAUER, D. Is public expenditure productive? *Journal of Monetary Economics*, v. 23, n. 2, p. 177–200, 1989.

[23] GLOMM, G.; RAVIKUMAR, B. Public investment in infrastructure in a simple growth model. *Journal of Economic Dynamics and Control*, v. 18, n. 6, p. 1173–1187, 1994.

[24] IZQUIERDO, A.; PESSINO, C.; VULETIN, G. J. *Melhores gastos para melhores vidas: como a américa latina e o caribe podem fazer mais com menos.* Inter-American Development Bank, 1992.

Annex I - Extended literature review

Before we start to develop a new approach to modelling growth with infrastructure problems, we must review the current literature. This annex aims to glance the most important papers in economic growth guided by stock of infrastructure, mainly DSGE modelling. Although this work focus in transport infrastructure and as this specific topic is not well covered by literature, a more broadly meaning of the term "infrastructure" will be used: the term will denote not just refer to transportation infrastructure, but also energy, sanitation, telecommunication and so on.

The economic benefits related to maintaining a infrastructure stock is relatively new since the main seminal paper was Aschauer (1989). In his empirical study, the author decomposes the US Government investment in infrastructure between 1949 and 1985. Assuming a logarithmized Cobb-Douglas' production function, the author finds positive effects of transport, energy and sanitation infrastructures stocks in US economic growth. Moreover, Aschauer attributes the decrease of American productivity between 1971-1985 to the reduction in non-military infrastructure stock (which englobes transport, energy and sanitation infrastructure).

Accordingly to Machicado (2007), empirical studies normally regress steady state equations or "growth accounting" identities. Doing that, its shows the importance of infrastructure in aggregate production, but can't capture the feedbacks effects that infrastructure generates in the economy. In order to do so, economic equilibrium models are preferable.

Inspired by Aschauer (1989), Barro (1990) developed the benchmark neoclassic model and, by that, writes the seminal paper of theoretical infrastructure economic modelling. In his work, he assumes one representative consumer and one representative firm with a Cobb-Douglas production function with constant returns. Besides the private capital and labor supply, the representative firm also uses a third element in the production function, a public investment of infrastructure that could be represented by the follow term:

$$f(G, K, L) = G^{\psi} K^{\alpha} L^{(}1 - \alpha)$$

where "G" is the public infrastructure provided; "K" is the private capital; and "L" is the labor supply. In such fashion, it is possible a decrease in the marginal production of private capital if the public infrastructure does not follow de increase in that input. Consequently, public inputs will not be a perfect substitute of private inputs (capital) – an example would be the public services that could not be charged efficiently if were provided by private agents.

In addition to the assumption of a third variable in the Cobb-Douglas production function, Barro also assumes that public services are funding by a revenue tax. The government is responsible to setting the rate and levying taxes. Since it is assumed that the government uses all the levying taxes contemporaneously (i.e., there is no government bound), there is no discussion of the best temporal use of government bounds.

The Barro's model was lately improved by Glomm and Ravikumar (1994), in a study that included the stock of infrastructure and congestion – so far, it was used just the flow of infrastructure. Posterior works in economic equilibrium with infrastructure normally use these both studies as base model, but modify the returns of the production function (1) to be constant returns in private inputs (i.e., $0 < \alpha < 1$) and decreasing returns in public infrastructure (i.e., $0 < \psi < 1$). With this setting, one can observe that the aggregate production function, $f(G, K, L)$ shows a increasing returns of scale.

It is important to highlight that neoclassical growth models with infrastructure displays peculiar effects regards to others neoclassical growth models with government spending. This happens because infrastructure stock is not in the utility function of the representative consumer, nor it is a component of the consumer's budget constraint (GALLEN and WINSTON, 2018). Actually, its effect is indirect and ambiguous: just as the infrastructure stock increases the productivity of marginal inputs ASCHAUER, 1989; BARRO, 1990; RIOJA, 1999; and others), effect normally called "resource benefit", it also induces sub-optimum economic equilibrium as the increase in infrastructure stock forces a raise in the tax rate. Therefore, the equilibrium of these models usually cannot be derived analytically and the marginal effect of infrastructure stock in product and wellfare generally shows a parabolic form.

As an extension of Barro's and Glomm and Ravikumar's work, Rioja (1999) is credited as one of the first empirical apply of DSGE models with infrastructure. In his paper, he finds the optimal welfare occurs near 4% of aggregate production allocated for maintaining infrastructure and a greater stock of infrastructure seems to have a positive effect in private investment. Similar results are found at Rioja (2001). Using Rioja (2001) as a base model and with data from five Latin-American countries, Machiado (2007) finds evidence that the macroeconomics and welfare effects provided by infrastructure are usually related with the infrastructure-aggregate production ratio. Also, the author advocate that, as the crowding out effects are present in such modelling, minor quantities of public investment in infrastructure (about 4% of aggregate production, as he comment) are preferable rather than bigger allocations (about 10% of aggregate production, as he examples) as it allows private investment.

Modifications on the quantity of agents were made in Rioja (2004), Gibson and Rioja (2017a, 2017b). A very common hypothesis in economic equilibrium modelling with infrastructure is the existence of one representative firm, so that infrastructure effects all productive sectors of the economy. In the first work, Rioja (2004) breaks this assumption when it models three production firms, each one for agricultural sector; manufactures; and services. Infrastructure stock affects the production function of all three firms in the same manner, similar to the benchmark model. The model is calibrated using Latin-American countries data for the

1960s and 1990s, as the author wishes to compare de effects of infrastructure between these periods. The results indicates that infrastructure investments in the 1960s had a greater impact than in 1990s, due to the fact that these countries were less developed and, therefore, needed more infrastructure stock. Moreover, in the 1960s, the manufacture sector were the most benefited by infrastructure investments.

The second paper, Gibson and Rioja (2017a), also aimed to break a commonly assumption that consumers are homogeneous, so that one can aggregate them in a single utility function (representative consumer). In these models, a set of heterogeneous consumers were assumed in order to allow the analysis of the effects of infrastructure in the social welfare. Gibson and Rioja (2017a) modify the benchmark model as to create a theorical model that allows welfare and social inequality effects analysis accordingly to the public infrastructure stock. The results shows that increases in public infrastructure investment provokes a gradual and long-term increase in welfare. In respect to inequality, the authors find that it depends on the level of taxation: if the tax is levied on consume and labor, then the income inequality diminishes; oppositely, if the tax is levied on the interest rates, the income inequality increases.

The third paper, Gibson and Rioja (2017b), focus in the effects of the trade-off between maintaining an existing infrastructure or expand. In that study, the government aims two objectives: maintain the quality of the infrastructure, which requires investment due to the depreciation by usage; or expand the exist infrastructure. Both objectives impacts the firms, since its affects the production function and the depreciation of the private capital. A modified version of this setting will be used in this work lately. The results shows that a policy focused on quality over expansion diminishes the income inequality, while a more equilibrated policy between quality and expansion favors the aggregate production.

Gallen and Winston (2018) changes the dynamic transition of the economic model using the time do build new infrastructure and the delay of commute time due these works. Consequently, the transition of the effects are given by: increase in the productivity of private firms' input; the decrease of transit time for commuters to shop or work; the raise of taxes on consumer spending. The authors find that public infrastructure investment have a positive greater effect in aggregate production than in welfare, since the agents consume more and, therefore, work more and spent more time in transit to go shopping. Another interesting result is listed: the delays on the build infrastructures are actually efficient, since it allows the private capital to adjust to the new infrastructure stock before the construction time ends.

Economic models with public infrastructure also permits evaluate the best allocation of public resources in a set of investment sectors that rivalry the government funding. As an example, Rioja (2005), studies an economy with education and infrastructure sections that rivalry the public resources. He argues that both investment are welfare-improvement, but education investment have a slightly greater impact than infrastructure investment. However, this substitution relation is beneficial to the economy until a threshold is reached.

Just a few papers bring alternatives formulations of the benchmark model on. Rovolis and Spence (2002) specifies alternative ways to Aschauer's work. As the authors argue, besides the Aschauer's results were discussed by many others studies after, "The production function analysis dominance of the infrastructure debate, however, is not uncontested" (ROVOLIS and SPENCE, 2002, page 57). So, the authors assume the same production function of Achauer's, but they analyze the dual problem, namely, the firms' problem of cost minimization. By that setting, the infrastructure investment reduces the cost of the firms (aggregately), since the infrastructure investment increases the marginal production of the inputs. In addition, the authors find a substitution relation between public infrastructure and intermediate inputs and a complementary relation between public infrastructure and private capital.

The comment of Rovolis and Spence (2002) is relevant, because production inputs and functional form o production function in neoclassical growth models with infrastructure are not well debated. One may see two attention points that these models arise: the raise of marginal cost of inputs and the concept of production function itself.

Annex II - Model structure documentation

Consumer

Optimisation problem

$$\max_{K_t^{\mathrm{C}},K_t^{\mathrm{I}},C_t^{\mathrm{I}},L_t^{\mathrm{C}},L_t^{\mathrm{I}},I_t^{\mathrm{C}},I_t^{\mathrm{I}}} U_t = \beta \mathrm{E}_t\left[U_{t+1}\right] + \left(1-\eta\right)^{-1}\left({C_t^{\mathrm{I}}}^{\mu}\left(1-L_t^{\mathrm{C}}-L_t^{\mathrm{I}}\right)^{1-\mu}\right)^{1-\eta} \tag{24}$$

s.t. :

$$C_t^{\mathrm{I}} + I_t^{\mathrm{C}} + I_t^{\mathrm{I}} = \pi_t^{\mathrm{C}} + \pi_t^{\mathrm{I}} + K_{t-1}^{\mathrm{C}} r_t^{\mathrm{C}} + K_{t-1}^{\mathrm{I}} r_t^{\mathrm{I}} + L_t^{\mathrm{C}} W_t^{\mathrm{C}} + L_t^{\mathrm{I}} W_t^{\mathrm{I}} \quad \left(\lambda_t^{\mathrm{CONSUMER}^1}\right) \tag{25}$$

$$I_t^{\mathrm{C}} = K_t^{\mathrm{C}} - K_{t-1}^{\mathrm{C}}\left(1-\delta {G_t}^{-1} {K_t^{\mathrm{Cd}}}^{\psi}\right) \quad \left(\lambda_t^{\mathrm{CONSUMER}^2}\right) \tag{26}$$

$$I_t^{\mathrm{I}} = K_t^{\mathrm{I}} - K_{t-1}^{\mathrm{I}}\left(1-\delta\right) \quad \left(\lambda_t^{\mathrm{CONSUMER}^3}\right) \tag{27}$$

First order conditions

$$\lambda_t^{\mathrm{CONSUMER}^2} + \beta\left(\mathrm{E}_t\left[\lambda_{t+1}^{\mathrm{CONSUMER}^1} r_{t+1}^{\mathrm{C}}\right] + \mathrm{E}_t\left[\lambda_{t+1}^{\mathrm{CONSUMER}^2}\left(-1+\delta {G_{t+1}}^{-1} {K_{t+1}^{\mathrm{Cd}}}^{\psi}\right)\right]\right) = 0 \quad \left(K_t^{\mathrm{C}}\right) \tag{28}$$

$$\lambda_t^{\mathrm{CONSUMER}^3} + \beta\left(\left(-1+\delta\right)\mathrm{E}_t\left[\lambda_{t+1}^{\mathrm{CONSUMER}^3}\right] + \mathrm{E}_t\left[\lambda_{t+1}^{\mathrm{CONSUMER}^1} r_{t+1}^{\mathrm{I}}\right]\right) = 0 \quad \left(K_t^{\mathrm{I}}\right) \tag{29}$$

$$-\lambda_t^{\mathrm{CONSUMER}^1} + \mu {C_t^{\mathrm{I}}}^{-1+\mu}\left(1-L_t^{\mathrm{C}}-L_t^{\mathrm{I}}\right)^{1-\mu}\left({C_t^{\mathrm{I}}}^{\mu}\left(1-L_t^{\mathrm{C}}-L_t^{\mathrm{I}}\right)^{1-\mu}\right)^{-\eta} = 0 \quad \left(C_t^{\mathrm{I}}\right) \tag{30}$$

$$\lambda_t^{\mathrm{CONSUMER}^1} W_t^{\mathrm{C}} + \left(-1+\mu\right){C_t^{\mathrm{I}}}^{\mu}\left(1-L_t^{\mathrm{C}}-L_t^{\mathrm{I}}\right)^{-\mu}\left({C_t^{\mathrm{I}}}^{\mu}\left(1-L_t^{\mathrm{C}}-L_t^{\mathrm{I}}\right)^{1-\mu}\right)^{-\eta} = 0 \quad \left(L_t^{\mathrm{C}}\right) \tag{31}$$

$$\lambda_t^{\mathrm{CONSUMER}^1} W_t^{\mathrm{I}} + \left(-1+\mu\right){C_t^{\mathrm{I}}}^{\mu}\left(1-L_t^{\mathrm{C}}-L_t^{\mathrm{I}}\right)^{-\mu}\left({C_t^{\mathrm{I}}}^{\mu}\left(1-L_t^{\mathrm{C}}-L_t^{\mathrm{I}}\right)^{1-\mu}\right)^{-\eta} = 0 \quad \left(L_t^{\mathrm{I}}\right) \tag{32}$$

$$-\lambda_t^{\mathrm{CONSUMER}^1} - \lambda_t^{\mathrm{CONSUMER}^2} = 0 \quad \left(I_t^{\mathrm{C}}\right) \tag{33}$$

$$-\lambda_t^{\mathrm{CONSUMER}^1} - \lambda_t^{\mathrm{CONSUMER}^3} = 0 \quad \left(I_t^{\mathrm{I}}\right) \tag{34}$$

Industry Firm

Optimisation problem

$$\max_{K_t^{\mathrm{Id}}, L_t^{\mathrm{Id}}, Q_t^{\mathrm{I}}} \pi_t^{\mathrm{I}} = -r_t^{\mathrm{I}} K_t^{\mathrm{Id}} - L_t^{\mathrm{Id}} W_t^{\mathrm{I}} + Q_t^{\mathrm{I}} \left(1 - \lambda_t\right) - p_t Q_t^{\mathrm{I}} \tag{35}$$

s.t. :

$$Q_t^{\mathrm{I}} = {K_t^{\mathrm{Id}}}^{\alpha} {L_t^{\mathrm{Id}}}^{1-\alpha} \tag{36}$$

First order conditions

$$-r_t^{\mathrm{I}} + \alpha \left(1 - \lambda_t - p_t\right) {K_t^{\mathrm{Id}}}^{-1+\alpha} {L_t^{\mathrm{Id}}}^{1-\alpha} = 0 \quad \left(K_t^{\mathrm{Id}}\right) \tag{37}$$

$$-W_t^{\mathrm{I}} + \left(1 - \alpha\right) \left(1 - \lambda_t - p_t\right) {K_t^{\mathrm{Id}}}^{\alpha} {L_t^{\mathrm{Id}}}^{-\alpha} = 0 \quad \left(L_t^{\mathrm{Id}}\right) \tag{38}$$

Carrier firm

Optimisation problem

$$\max_{K_t^{\mathrm{Cd}}, L_t^{\mathrm{Cd}}, Q_t^{\mathrm{C}}} \pi_t^{\mathrm{C}} = -L_t^{\mathrm{Cd}} W_t^{\mathrm{C}} - r_t^{\mathrm{C}} K_t^{\mathrm{Cd}} + p_t Q_t^{\mathrm{C}} \left(1 - \lambda_t\right) \tag{39}$$

s.t. :

$$Q_t^{\mathrm{C}} = \left(1 + \tau \left(G_t {K_t^{\mathrm{Cd}}}^{-\psi}\right)^{\omega}\right) {K_t^{\mathrm{Cd}}}^{\sigma} {L_t^{\mathrm{Cd}}}^{1-\sigma} \tag{40}$$

First order conditions

$$-r_t^{\mathrm{C}} + p_t \left(1 - \lambda_t\right) \left(\sigma \left(1 + \tau \left(G_t {K_t^{\mathrm{Cd}}}^{-\psi}\right)^{\omega}\right) {K_t^{\mathrm{Cd}}}^{-1+\sigma} {L_t^{\mathrm{Cd}}}^{1-\sigma} - \omega \psi \tau G_t {K_t^{\mathrm{Cd}}}^{-1-\psi+\sigma} {L_t^{\mathrm{Cd}}}^{1-\sigma} \left(G_t {K_t^{\mathrm{Cd}}}^{-\psi}\right)^{-1+\omega}\right) = 0 \quad \left(K_t^{\mathrm{Cd}}\right) \tag{41}$$

$$-W_t^{\mathrm{C}} + p_t \left(1 - \sigma\right) \left(1 - \lambda_t\right) \left(1 + \tau \left(G_t {K_t^{\mathrm{Cd}}}^{-\psi}\right)^{\omega}\right) {K_t^{\mathrm{Cd}}}^{\sigma} {L_t^{\mathrm{Cd}}}^{-\sigma} = 0 \quad \left(L_t^{\mathrm{Cd}}\right) \tag{42}$$

Equilibrium

Identities

$$K_t^{\mathrm{Id}} = K_{t-1}^{\mathrm{I}} \tag{43}$$

$$K_t^{\mathrm{Cd}} = K_{t-1}^{\mathrm{C}} \tag{44}$$

$$L_t^{\mathrm{Id}} = L_t^{\mathrm{I}} \tag{45}$$

$$L_t^{\mathrm{Cd}} = L_t^{\mathrm{C}} \tag{46}$$

$$Y_t = Q_t^{\mathrm{I}} + p_t Q_t^{\mathrm{C}} \tag{47}$$

$$G_t = G_{t-1}\left(1 - \delta^{\mathrm{g}}\right) + \lambda_t \left(1 - \zeta\right)\left(Q_t^{\mathrm{I}} + p_t Q_t^{\mathrm{C}}\right) \tag{48}$$

$$Q_t^{\mathrm{C}} = Q_t^{\mathrm{I}} \tag{49}$$

Exogenous

Identities

$$\lambda_t = \epsilon_t^{\lambda} + \lambda^{\mathrm{mean}}\left(1 - \phi\right) + \phi \lambda_{t-1} \tag{50}$$

Equilibrium relationships (after reduction)

$$-r_t^{\mathrm{C}} + p_t\left(1 - \lambda_t\right)\left(\sigma\left(1 + \tau\left(G_t {K_{t-1}^{\mathrm{C}}}^{-\psi}\right)^{\omega}\right) {K_{t-1}^{\mathrm{C}}}^{-1+\sigma} {L_t^{\mathrm{C}}}^{1-\sigma} - \omega\psi\tau G_t {K_{t-1}^{\mathrm{C}}}^{-1-\psi+\sigma} {L_t^{\mathrm{C}}}^{1-\sigma}\left(G_t {K_{t-1}^{\mathrm{C}}}^{-\psi}\right)^{-1+\omega}\right) = 0 \tag{51}$$

$$-r_t^{\mathrm{I}} + \alpha\left(1 - \lambda_t - p_t\right) {K_{t-1}^{\mathrm{I}}}^{-1+\alpha} {L_t^{\mathrm{I}}}^{1-\alpha} = 0 \tag{52}$$

$$-Q_t^{\mathrm{I}} + {K_{t-1}^{\mathrm{I}}}^{\alpha} {L_t^{\mathrm{I}}}^{1-\alpha} = 0 \tag{53}$$

$$Q_t^{\mathrm{I}} - Q_t^{\mathrm{C}} = 0 \tag{54}$$

$$-Q_t^{\mathrm{C}} + \left(1 + \tau\left(G_t {K_{t-1}^{\mathrm{C}}}^{-\psi}\right)^{\omega}\right) {K_{t-1}^{\mathrm{C}}}^{\sigma} {L_t^{\mathrm{C}}}^{1-\sigma} = 0 \tag{55}$$

$$-W_t^{\mathrm{C}} + p_t\left(1 - \sigma\right)\left(1 - \lambda_t\right)\left(1 + \tau\left(G_t {K_{t-1}^{\mathrm{C}}}^{-\psi}\right)^{\omega}\right) {K_{t-1}^{\mathrm{C}}}^{\sigma} {L_t^{\mathrm{C}}}^{-\sigma} = 0 \tag{56}$$

$$-W_t^{\mathrm{I}} + \left(1 - \alpha\right)\left(1 - \lambda_t - p_t\right) {K_{t-1}^{\mathrm{I}}}^{\alpha} {L_t^{\mathrm{I}}}^{-\alpha} = 0 \tag{57}$$

$$\beta(\mu \mathrm{E}_t\left[r_{t+1}^{\mathrm{C}} {C_{t+1}^{\mathrm{I}}}^{-1+\mu}\left(1 - L_{t+1}^{\mathrm{C}} - L_{t+1}^{\mathrm{I}}\right)^{1-\mu}\left({C_{t+1}^{\mathrm{I}}}^{\mu}\left(1 - L_{t+1}^{\mathrm{C}} - L_{t+1}^{\mathrm{I}}\right)^{1-\mu}\right)^{-\eta}\right] \tag{58}$$

$$
\begin{aligned}
&-\mu \mathrm{E}_t \left[\left(-1 + \delta {G_{t+1}}^{-1} {K_t^{\mathrm{C}}}^{\psi} \right) {C_{t+1}^{\mathrm{I}}}^{-1+\mu} \left(1 - L_{t+1}^{\mathrm{C}} - L_{t+1}^{\mathrm{I}}\right)^{1-\mu} \left({C_{t+1}^{\mathrm{I}}}^{\mu} \left(1 - L_{t+1}^{\mathrm{C}} - L_{t+1}^{\mathrm{I}}\right)^{1-\mu} \right)^{-\eta} \right]) \\
&-\mu {C_t^{\mathrm{I}}}^{-1+\mu} \left(1 - L_t^{\mathrm{C}} - L_t^{\mathrm{I}}\right)^{1-\mu} \left({C_t^{\mathrm{I}}}^{\mu} \left(1 - L_t^{\mathrm{C}} - L_t^{\mathrm{I}}\right)^{1-\mu} \right)^{-\eta} = 0
\end{aligned}
$$

$$
\begin{aligned}
&\beta (\mu \mathrm{E}_t \left[r_{t+1}^{\mathrm{I}} {C_{t+1}^{\mathrm{I}}}^{-1+\mu} \left(1 - L_{t+1}^{\mathrm{C}} - L_{t+1}^{\mathrm{I}}\right)^{1-\mu} \left({C_{t+1}^{\mathrm{I}}}^{\mu} \left(1 - L_{t+1}^{\mathrm{C}} - L_{t+1}^{\mathrm{I}}\right)^{1-\mu} \right)^{-\eta} \right] \\
&-\mu \left(-1 + \delta\right) \mathrm{E}_t \left[{C_{t+1}^{\mathrm{I}}}^{-1+\mu} \left(1 - L_{t+1}^{\mathrm{C}} - L_{t+1}^{\mathrm{I}}\right)^{1-\mu} \left({C_{t+1}^{\mathrm{I}}}^{\mu} \left(1 - L_{t+1}^{\mathrm{C}} - L_{t+1}^{\mathrm{I}}\right)^{1-\mu} \right)^{-\eta} \right]) \\
&-\mu {C_t^{\mathrm{I}}}^{-1+\mu} \left(1 - L_t^{\mathrm{C}} - L_t^{\mathrm{I}}\right)^{1-\mu} \left({C_t^{\mathrm{I}}}^{\mu} \left(1 - L_t^{\mathrm{C}} - L_t^{\mathrm{I}}\right)^{1-\mu} \right)^{-\eta} = 0
\end{aligned}
\tag{59}
$$

$$
\begin{aligned}
&\left(-1 + \mu\right) {C_t^{\mathrm{I}}}^{\mu} \left(1 - L_t^{\mathrm{C}} - L_t^{\mathrm{I}}\right)^{-\mu} \left({C_t^{\mathrm{I}}}^{\mu} \left(1 - L_t^{\mathrm{C}} - L_t^{\mathrm{I}}\right)^{1-\mu} \right)^{-\eta} \\
&+\mu W_t^{\mathrm{C}} {C_t^{\mathrm{I}}}^{-1+\mu} \left(1 - L_t^{\mathrm{C}} - L_t^{\mathrm{I}}\right)^{1-\mu} \left({C_t^{\mathrm{I}}}^{\mu} \left(1 - L_t^{\mathrm{C}} - L_t^{\mathrm{I}}\right)^{1-\mu} \right)^{-\eta} = 0
\end{aligned}
\tag{60}
$$

$$
\begin{aligned}
&\left(-1 + \mu\right) {C_t^{\mathrm{I}}}^{\mu} \left(1 - L_t^{\mathrm{C}} - L_t^{\mathrm{I}}\right)^{-\mu} \left({C_t^{\mathrm{I}}}^{\mu} \left(1 - L_t^{\mathrm{C}} - L_t^{\mathrm{I}}\right)^{1-\mu} \right)^{-\eta} \\
&+\mu W_t^{\mathrm{I}} {C_t^{\mathrm{I}}}^{-1+\mu} \left(1 - L_t^{\mathrm{C}} - L_t^{\mathrm{I}}\right)^{1-\mu} \left({C_t^{\mathrm{I}}}^{\mu} \left(1 - L_t^{\mathrm{C}} - L_t^{\mathrm{I}}\right)^{1-\mu} \right)^{-\eta} = 0
\end{aligned}
\tag{61}
$$

$$
-G_t + G_{t-1} \left(1 - \delta^{\mathrm{g}}\right) + \lambda_t \left(1 - \zeta\right) \left(Q_t^{\mathrm{I}} + p_t Q_t^{\mathrm{C}} \right) = 0 \tag{62}
$$

$$
-I_t^{\mathrm{C}} + K_t^{\mathrm{C}} - K_{t-1}^{\mathrm{C}} \left(1 - \delta {G_t}^{-1} {K_{t-1}^{\mathrm{C}}}^{\psi} \right) = 0 \tag{63}
$$

$$
-I_t^{\mathrm{I}} + K_t^{\mathrm{I}} - K_{t-1}^{\mathrm{I}} \left(1 - \delta\right) = 0 \tag{64}
$$

$$
Q_t^{\mathrm{I}} - Y_t + p_t Q_t^{\mathrm{C}} = 0 \tag{65}
$$

$$U_t - \beta \mathrm{E}_t\left[U_{t+1}\right] - (1-\eta)^{-1}\left(C_t^{\mathrm{I}\mu}\left(1 - L_t^{\mathrm{C}} - L_t^{\mathrm{I}}\right)^{1-\mu}\right)^{1-\eta} = 0 \tag{66}$$

$$\epsilon_t^{\lambda} - \lambda_t + \lambda^{\mathrm{mean}}\left(1-\phi\right) + \phi\lambda_{t-1} = 0 \tag{67}$$

$$\pi_t^{\mathrm{C}} + K_{t-1}^{\mathrm{C}} r_t^{\mathrm{C}} + L_t^{\mathrm{C}} W_t^{\mathrm{C}} - p_t Q_t^{\mathrm{C}}\left(1-\lambda_t\right) = 0 \tag{68}$$

$$\pi_t^{\mathrm{I}} + K_{t-1}^{\mathrm{I}} r_t^{\mathrm{I}} + p_t Q_t^{\mathrm{I}} + L_t^{\mathrm{I}} W_t^{\mathrm{I}} - Q_t^{\mathrm{I}}\left(1-\lambda_t\right) = 0 \tag{69}$$

$$\pi_t^{\mathrm{C}} + \pi_t^{\mathrm{I}} - y_t^{\mathrm{s}} + C_t^{\mathrm{I}} + I_t^{\mathrm{C}} + I_t^{\mathrm{I}} + \lambda_t\left(Q_t^{\mathrm{I}} + p_t Q_t^{\mathrm{C}}\right) + p_t Q_t^{\mathrm{C}} = 0 \tag{70}$$

$$\pi_t^{\mathrm{C}} + \pi_t^{\mathrm{I}} - C_t^{\mathrm{I}} - I_t^{\mathrm{C}} - I_t^{\mathrm{I}} + K_{t-1}^{\mathrm{C}} r_t^{\mathrm{C}} + K_{t-1}^{\mathrm{I}} r_t^{\mathrm{I}} + L_t^{\mathrm{C}} W_t^{\mathrm{C}} + L_t^{\mathrm{I}} W_t^{\mathrm{I}} = 0 \tag{71}$$

Steady state relationships (after reduction)

$$-\delta_{\mathrm{ss}}^{\mathrm{C}} + \delta {G_{\mathrm{ss}}}^{-1} {K_{\mathrm{ss}}^{\mathrm{C}}}^{\psi} = 0 \tag{72}$$

$$-r_{\mathrm{ss}}^{\mathrm{C}} + p_{\mathrm{ss}}\left(1-\lambda_{\mathrm{ss}}\right)\left(\sigma\left(1+\tau\left(G_{\mathrm{ss}} {K_{\mathrm{ss}}^{\mathrm{C}}}^{-\psi}\right)^{\omega}\right) {K_{\mathrm{ss}}^{\mathrm{C}}}^{-1+\sigma} {L_{\mathrm{ss}}^{\mathrm{C}}}^{1-\sigma} - \omega\psi\tau G_{\mathrm{ss}} {K_{\mathrm{ss}}^{\mathrm{C}}}^{-1-\psi+\sigma} {L_{\mathrm{ss}}^{\mathrm{C}}}^{1-\sigma}\left(G_{\mathrm{ss}} {K_{\mathrm{ss}}^{\mathrm{C}}}^{-\psi}\right)^{-1+\omega}\right) = 0 \tag{73}$$

$$-r_{\mathrm{ss}}^{\mathrm{I}} + \alpha\left(1-\lambda_{\mathrm{ss}}-p_{\mathrm{ss}}\right) {K_{\mathrm{ss}}^{\mathrm{I}}}^{-1+\alpha} {L_{\mathrm{ss}}^{\mathrm{I}}}^{1-\alpha} = 0 \tag{74}$$

$$-Q_{\mathrm{ss}}^{\mathrm{I}} + {K_{\mathrm{ss}}^{\mathrm{I}}}^{\alpha} {L_{\mathrm{ss}}^{\mathrm{I}}}^{1-\alpha} = 0 \tag{75}$$

$$Q_{\mathrm{ss}}^{\mathrm{I}} - Q_{\mathrm{ss}}^{\mathrm{C}} = 0 \tag{76}$$

$$-Q_{\mathrm{ss}}^{\mathrm{C}} + \left(1+\tau\left(G_{\mathrm{ss}} {K_{\mathrm{ss}}^{\mathrm{C}}}^{-\psi}\right)^{\omega}\right) {K_{\mathrm{ss}}^{\mathrm{C}}}^{\sigma} {L_{\mathrm{ss}}^{\mathrm{C}}}^{1-\sigma} = 0 \tag{77}$$

$$-W_{\mathrm{ss}}^{\mathrm{C}} + p_{\mathrm{ss}}\left(1-\sigma\right)\left(1-\lambda_{\mathrm{ss}}\right)\left(1+\tau\left(G_{\mathrm{ss}} {K_{\mathrm{ss}}^{\mathrm{C}}}^{-\psi}\right)^{\omega}\right) {K_{\mathrm{ss}}^{\mathrm{C}}}^{\sigma} {L_{\mathrm{ss}}^{\mathrm{C}}}^{-\sigma} = 0 \tag{78}$$

$$-W_{\mathrm{ss}}^{\mathrm{I}} + \left(1-\alpha\right)\left(1-\lambda_{\mathrm{ss}}-p_{\mathrm{ss}}\right) {K_{\mathrm{ss}}^{\mathrm{I}}}^{\alpha} {L_{\mathrm{ss}}^{\mathrm{I}}}^{-\alpha} = 0 \tag{79}$$

$$\begin{gathered}\beta(\mu r_{\mathrm{ss}}^{\mathrm{C}} {C_{\mathrm{ss}}^{\mathrm{I}}}^{-1+\mu}\left(1-L_{\mathrm{ss}}^{\mathrm{C}}-L_{\mathrm{ss}}^{\mathrm{I}}\right)^{1-\mu}\left({C_{\mathrm{ss}}^{\mathrm{I}}}^{\mu}\left(1-L_{\mathrm{ss}}^{\mathrm{C}}-L_{\mathrm{ss}}^{\mathrm{I}}\right)^{1-\mu}\right)^{-\eta} \\ -\mu(-1+\delta {G_{\mathrm{ss}}}^{-1} {K_{\mathrm{ss}}^{\mathrm{C}}}^{\psi}) {C_{\mathrm{ss}}^{\mathrm{I}}}^{-1+\mu}(1-L_{\mathrm{ss}}^{\mathrm{C}}-L_{\mathrm{ss}}^{\mathrm{I}})^{1-\mu}({C_{\mathrm{ss}}^{\mathrm{I}}}^{\mu}\left(1-L_{\mathrm{ss}}^{\mathrm{C}}-L_{\mathrm{ss}}^{\mathrm{I}}\right)^{1-\mu})^{-\eta}) \\ -\mu {C_{\mathrm{ss}}^{\mathrm{I}}}^{-1+\mu}\left(1-L_{\mathrm{ss}}^{\mathrm{C}}-L_{\mathrm{ss}}^{\mathrm{I}}\right)^{1-\mu}\left({C_{\mathrm{ss}}^{\mathrm{I}}}^{\mu}\left(1-L_{\mathrm{ss}}^{\mathrm{C}}-L_{\mathrm{ss}}^{\mathrm{I}}\right)^{1-\mu}\right)^{-\eta} = 0\end{gathered} \tag{80}$$

$$\begin{aligned}&\beta(\mu r^{\mathrm{I}}_{\mathrm{ss}} C^{\mathrm{I}\;-1+\mu}_{\mathrm{ss}} \left(1 - L^{\mathrm{C}}_{\mathrm{ss}} - L^{\mathrm{I}}_{\mathrm{ss}}\right)^{1-\mu} \left(C^{\mathrm{I}\;\mu}_{\mathrm{ss}} \left(1 - L^{\mathrm{C}}_{\mathrm{ss}} - L^{\mathrm{I}}_{\mathrm{ss}}\right)^{1-\mu}\right)^{-\eta} \\ &-\mu\left(-1+\delta\right) C^{\mathrm{I}\;-1+\mu}_{\mathrm{ss}} \left(1 - L^{\mathrm{C}}_{\mathrm{ss}} - L^{\mathrm{I}}_{\mathrm{ss}}\right)^{1-\mu} \left(C^{\mathrm{I}\;\mu}_{\mathrm{ss}} \left(1 - L^{\mathrm{C}}_{\mathrm{ss}} - L^{\mathrm{I}}_{\mathrm{ss}}\right)^{1-\mu}\right)^{-\eta}) \\ &-\mu C^{\mathrm{I}\;-1+\mu}_{\mathrm{ss}} \left(1 - L^{\mathrm{C}}_{\mathrm{ss}} - L^{\mathrm{I}}_{\mathrm{ss}}\right)^{1-\mu} \left(C^{\mathrm{I}\;\mu}_{\mathrm{ss}} \left(1 - L^{\mathrm{C}}_{\mathrm{ss}} - L^{\mathrm{I}}_{\mathrm{ss}}\right)^{1-\mu}\right)^{-\eta} = 0\end{aligned} \tag{81}$$

$$\begin{aligned}&\left(-1+\mu\right) C^{\mathrm{I}\;\mu}_{\mathrm{ss}} \left(1 - L^{\mathrm{C}}_{\mathrm{ss}} - L^{\mathrm{I}}_{\mathrm{ss}}\right)^{-\mu} \left(C^{\mathrm{I}\;\mu}_{\mathrm{ss}} \left(1 - L^{\mathrm{C}}_{\mathrm{ss}} - L^{\mathrm{I}}_{\mathrm{ss}}\right)^{1-\mu}\right)^{-\eta} \\ &+\mu W^{\mathrm{C}}_{\mathrm{ss}} C^{\mathrm{I}\;-1+\mu}_{\mathrm{ss}} \left(1 - L^{\mathrm{C}}_{\mathrm{ss}} - L^{\mathrm{I}}_{\mathrm{ss}}\right)^{1-\mu} \left(C^{\mathrm{I}\;\mu}_{\mathrm{ss}} \left(1 - L^{\mathrm{C}}_{\mathrm{ss}} - L^{\mathrm{I}}_{\mathrm{ss}}\right)^{1-\mu}\right)^{-\eta} = 0\end{aligned} \tag{82}$$

$$\begin{aligned}&\left(-1+\mu\right) C^{\mathrm{I}\;\mu}_{\mathrm{ss}} \left(1 - L^{\mathrm{C}}_{\mathrm{ss}} - L^{\mathrm{I}}_{\mathrm{ss}}\right)^{-\mu} \left(C^{\mathrm{I}\;\mu}_{\mathrm{ss}} \left(1 - L^{\mathrm{C}}_{\mathrm{ss}} - L^{\mathrm{I}}_{\mathrm{ss}}\right)^{1-\mu}\right)^{-\eta} \\ &+\mu W^{\mathrm{I}}_{\mathrm{ss}} C^{\mathrm{I}\;-1+\mu}_{\mathrm{ss}} \left(1 - L^{\mathrm{C}}_{\mathrm{ss}} - L^{\mathrm{I}}_{\mathrm{ss}}\right)^{1-\mu} \left(C^{\mathrm{I}\;\mu}_{\mathrm{ss}} \left(1 - L^{\mathrm{C}}_{\mathrm{ss}} - L^{\mathrm{I}}_{\mathrm{ss}}\right)^{1-\mu}\right)^{-\eta} = 0\end{aligned} \tag{83}$$

$$-G_{\mathrm{ss}} + G_{\mathrm{ss}}\left(1-\delta^{\mathrm{g}}\right) + \lambda_{\mathrm{ss}}\left(1-\zeta\right)\left(Q^{\mathrm{I}}_{\mathrm{ss}} + p_{\mathrm{ss}} Q^{\mathrm{C}}_{\mathrm{ss}}\right) = 0 \tag{84}$$

$$-I^{\mathrm{C}}_{\mathrm{ss}} + K^{\mathrm{C}}_{\mathrm{ss}} - K^{\mathrm{C}}_{\mathrm{ss}}\left(1 - \delta G_{\mathrm{ss}}^{-1} K^{\mathrm{C}\;\psi}_{\mathrm{ss}}\right) = 0 \tag{85}$$

$$-I^{\mathrm{I}}_{\mathrm{ss}} + K^{\mathrm{I}}_{\mathrm{ss}} - K^{\mathrm{I}}_{\mathrm{ss}}\left(1-\delta\right) = 0 \tag{86}$$

$$Q^{\mathrm{I}}_{\mathrm{ss}} - Y_{\mathrm{ss}} + p_{\mathrm{ss}} Q^{\mathrm{C}}_{\mathrm{ss}} = 0 \tag{87}$$

$$U_{\mathrm{ss}} - \beta U_{\mathrm{ss}} - \left(1-\eta\right)^{-1}\left(C^{\mathrm{I}\;\mu}_{\mathrm{ss}} \left(1 - L^{\mathrm{C}}_{\mathrm{ss}} - L^{\mathrm{I}}_{\mathrm{ss}}\right)^{1-\mu}\right)^{1-\eta} = 0 \tag{88}$$

$$-\lambda_{\mathrm{ss}} + \lambda^{\mathrm{mean}}\left(1-\phi\right) + \phi\lambda_{\mathrm{ss}} = 0 \tag{89}$$

$$\pi^{\mathrm{C}}_{\mathrm{ss}} + r^{\mathrm{C}}_{\mathrm{ss}} K^{\mathrm{C}}_{\mathrm{ss}} + L^{\mathrm{C}}_{\mathrm{ss}} W^{\mathrm{C}}_{\mathrm{ss}} - p_{\mathrm{ss}} Q^{\mathrm{C}}_{\mathrm{ss}}\left(1-\lambda_{\mathrm{ss}}\right) = 0 \tag{90}$$

$$\pi^{\mathrm{I}}_{\mathrm{ss}} + p_{\mathrm{ss}} Q^{\mathrm{I}}_{\mathrm{ss}} + r^{\mathrm{I}}_{\mathrm{ss}} K^{\mathrm{I}}_{\mathrm{ss}} + L^{\mathrm{I}}_{\mathrm{ss}} W^{\mathrm{I}}_{\mathrm{ss}} - Q^{\mathrm{I}}_{\mathrm{ss}}\left(1-\lambda_{\mathrm{ss}}\right) = 0 \tag{91}$$

$$\pi^{\mathrm{C}}_{\mathrm{ss}} + \pi^{\mathrm{I}}_{\mathrm{ss}} - y^{\mathrm{s}}_{\mathrm{ss}} + C^{\mathrm{I}}_{\mathrm{ss}} + I^{\mathrm{C}}_{\mathrm{ss}} + I^{\mathrm{I}}_{\mathrm{ss}} + \lambda_{\mathrm{ss}}\left(Q^{\mathrm{I}}_{\mathrm{ss}} + p_{\mathrm{ss}} Q^{\mathrm{C}}_{\mathrm{ss}}\right) + p_{\mathrm{ss}} Q^{\mathrm{C}}_{\mathrm{ss}} = 0 \tag{92}$$

$$\pi^{\mathrm{C}}_{\mathrm{ss}} + \pi^{\mathrm{I}}_{\mathrm{ss}} - C^{\mathrm{I}}_{\mathrm{ss}} - I^{\mathrm{C}}_{\mathrm{ss}} - I^{\mathrm{I}}_{\mathrm{ss}} + r^{\mathrm{C}}_{\mathrm{ss}} K^{\mathrm{C}}_{\mathrm{ss}} + r^{\mathrm{I}}_{\mathrm{ss}} K^{\mathrm{I}}_{\mathrm{ss}} + L^{\mathrm{C}}_{\mathrm{ss}} W^{\mathrm{C}}_{\mathrm{ss}} + L^{\mathrm{I}}_{\mathrm{ss}} W^{\mathrm{I}}_{\mathrm{ss}} = 0 \tag{93}$$

Calibrating equations

$$-1.33333 + K_{\mathrm{ss}}^{\mathrm{C}\psi} = 0 \tag{94}$$

Annex III - Comparative statics complete tables

Table 6: Comparative statics results

Variable	+1% Tax rate	Frischtak	HIC	HDC	HIC+1%	HDC+1%
$\delta^{\star}$	-1,30%	-71,18%	1,33%	2,30%	186,66%	130,83%
λ	1,00%	192,54%	0,00%	0,00%	0,00%	0,00%
p	-0,54%	-40,42%	0,55%	0,67%	48,99%	27,79%
π_t^{C}	0,12%	-6,07%	-0,13%	-1,14%	-26,25%	-37,40%
r_t^{C}	-1,18%	-64,70%	1,21%	2,09%	169,65%	118,91%
r_t^{I}	0,00%	0,00%	0,00%	0,91%	0,00%	36,36%
Production effect	0,01%	0,64%	-0,01%	-0,01%	-38,41%	-44,93%
C_t^{I}	0,62%	50,10%	-0,66%	-1,87%	-38,41%	-44,93%
EXP_t^C	0,00%	-16,54%	-0,02%	-1,03%	-0,48%	-0,23%
EXP_t^I	0,54%	40,09%	-0,57%	-1,68%	-49,36%	-51,91%
G_t	1,41%	271,81%	-1,41%	-1,51%	-18,55%	-34,35%
I_t^{C}	-0,12%	-31,87%	0,10%	-0,83%	-45,34%	-48,62%
I_t^{I}	0,83%	70,12%	-0,86%	-1,95%	-68,31%	-44,93%
K_t^{C}	1,19%	136,42%	-1,22%	-3,05%	-13,42%	-30,77%
K_t^{I}	0,83%	70,12%	-0,86%	-2,92%	-59,82%	-55,18%
L_t^{C}	-0,61%	-43,71%	0,62%	0,82%	-69,80%	-70,01%
L_t^{I}	0,22%	14,74%	-0,22%	-0,21%	-59,82%	-67,99%
Q_t^{I}	0,55%	41,93%	-0,57%	-1,68%	59,05%	34,64%
Q_t^{C}	0,55%	41,93%	-0,57%	-1,68%	-21,53%	-10,48%
U	0,30%	18,09%	-0,32%	-0,92%	-45,34%	-48,62%
W_t^{C}	0,61%	48,27%	-0,64%	-1,83%	-45,34%	-48,62%
W_t^{I}	0,61%	48,27%	-0,64%	-1,83%	38,61%	42,30%
Y	0,41%	27.10%	-0.43%	-1.51%	-48,79%	-51,24%

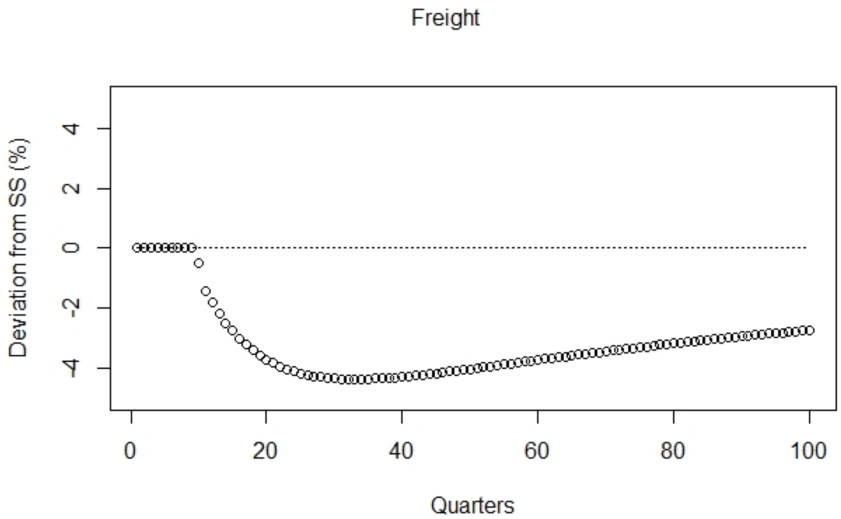
Table 7: Variation of cost reduction model results in relation to the benchmark model results.

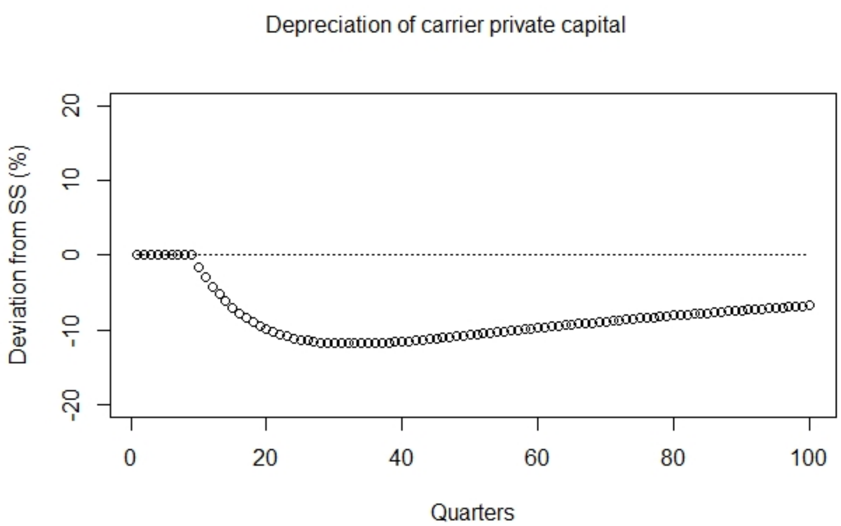
Variable	+1% Tax rate	Frischtak	HIC	HDC
p	-0,53%	-40,08%	0,54%	0,52%
π_t^{C}	0,05%	-11,71%	-0,03%	-0,05%
r_t^{C}	-1,18%	-64,70%	1,21%	1,17%
r_t^{I}	0,00%	0,00%	0,00%	0,00%
Production effect	0,00%	0,10%	0,00%	0,00%
C_t^{I}	0,64%	55,79%	-0,65%	-0,63%
EXP_t^C	0,02%	-13,27%	-0,02%	-0,02%
EXP_t^I	0,55%	44,75%	-0,56%	-0,54%
G_t	0,42%	29,81%	-0,42%	-0,41%
I_t^{C}	-0,10%	-29,21%	0,10%	0,09%
I_t^{I}	0,84%	76,49%	-0,86%	-0,83%
K_t^{C}	1,21%	145,67%	-1,22%	-1,18%
K_t^{I}	0,84%	76,49%	-0,86%	-0,83%
L_t^{C}	-0,61%	-43,64%	0,62%	0,60%
L_t^{I}	0,21%	14,68%	-0,22%	-0,21%
Q_t^{I}	0,55%	44,75%	-0,56%	-0,54%
Q_t^{C}	0,55%	44,75%	-0,56%	-0,54%
U	-0,31%	-19,50%	0,32%	0,31%
W_t^{C}	0,63%	53,90%	-0,64%	-0,62%
W_t^{I}	0,63%	53,90%	-0,64%	-0,62%
Y	0,42%	29,81%	-0,42%	-0,41%

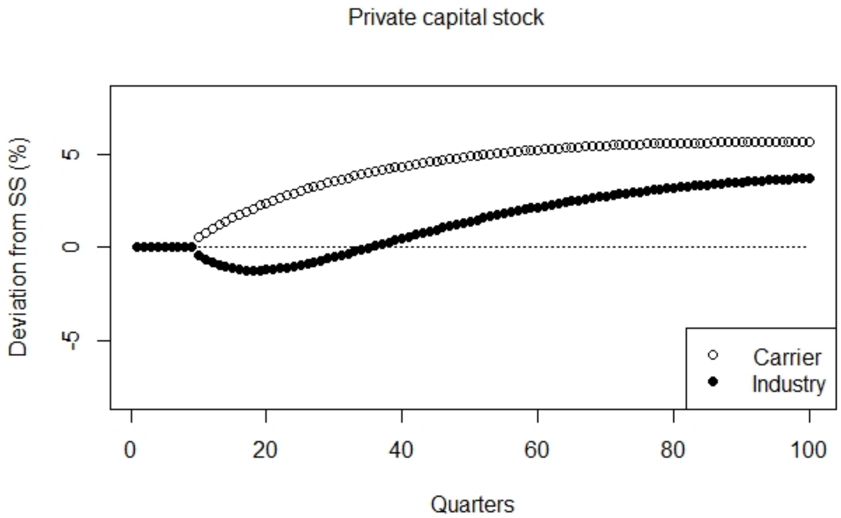
Table 8: Results from price control policy experiment.

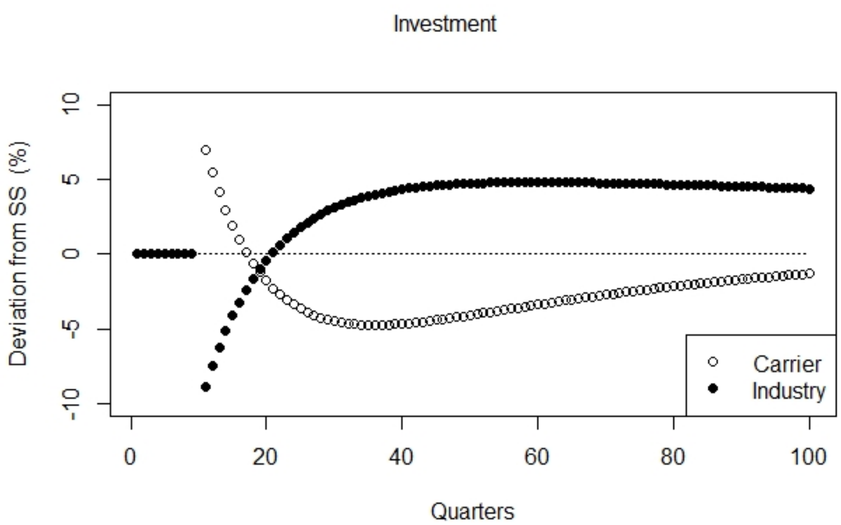
Variable	+5% above SS	+10% above SS	+15% above SS
$\delta^\star$	4,38%	9,12%	14,26%
λ	0,00%	0,00%	0,00%
π_t^{C}	11260.02%	21004,66%	29332,31%
r_t^{C}	3,98%	8,29%	12,96%
r_t^{I}	0,00%	0,00%	0,00%
Production effect	-0,02%	-0,04%	-0,06%
C_t^{I}	-4,80%	-9,54%	-14,23%
EXP_t^C	-5,82%	-11,40%	-16,76%
EXP_t^I	-6,17%	-12,09%	-17,78%
G_t	-4,96%	-9,82%	-14,60%
I_t^{C}	-5,46%	-10,72%	-15,80%
I_t^{I}	-8,71%	-16,85%	-24,46%
K_t^{C}	-9,42%	-18,18%	-26,31%
K_t^{I}	-8,71%	-16,85%	-24,46%
L_t^{C}	-0,03%	-0,01%	0,05%
L_t^{I}	-3,10%	-6,16%	-9,20%
Q_t	-6,17%	-12,09%	-17,78%
U	1,40%	2,93%	4,61%
W_t^{C}	-5,79%	-11,39%	-16,80%
W_t^{I}	-5,79%	-11,39%	-16,80%
Y	-4,96%	-9,82%	-14,60%

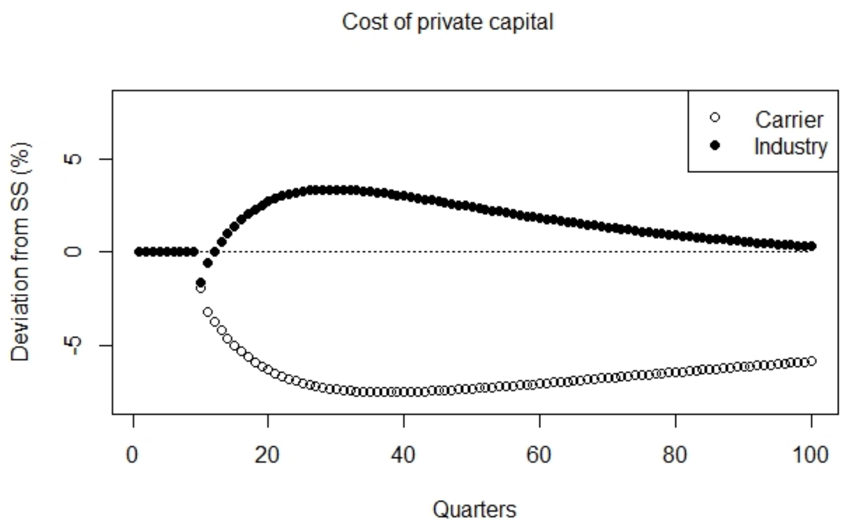
Annex IV - Shock simulation figures

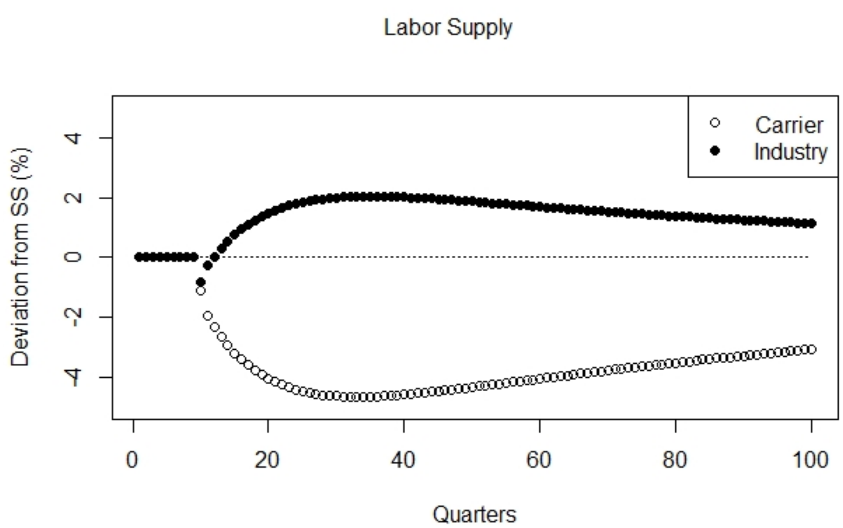




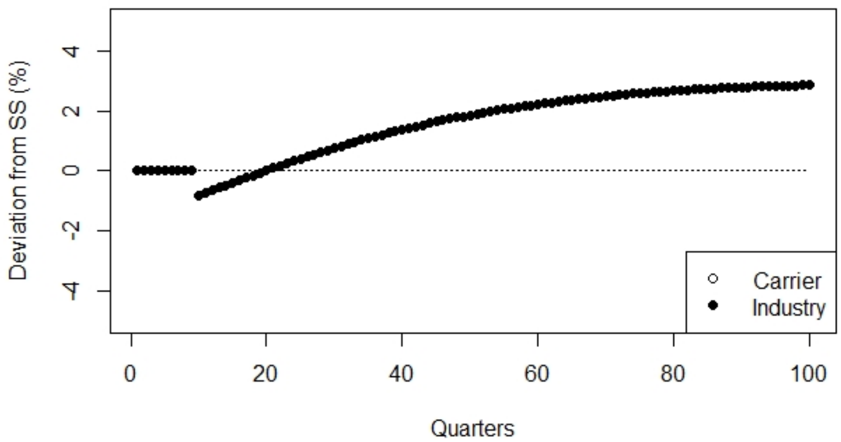

Private capital stock
Deviation from SS (%)
5
0
-5
0
20
40
60
80
100
Quarters
Carrier
Industry


Investment
Deviation from SS (%)
10
5
0
-5
-10
0
20
40
60
80
100
Quarters
Carrier
Industry

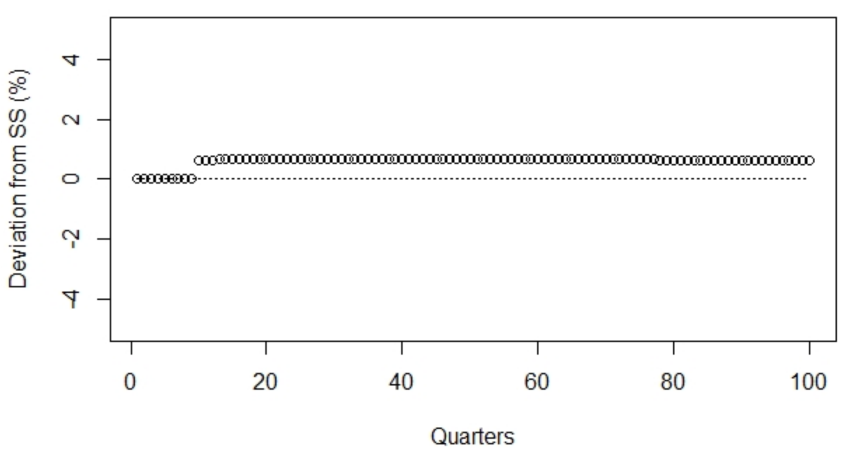

Cost of private capital
Carrier
Industry
Deviation from SS (%)
5
0
-5
0
20
40
60
80
100
Quarters

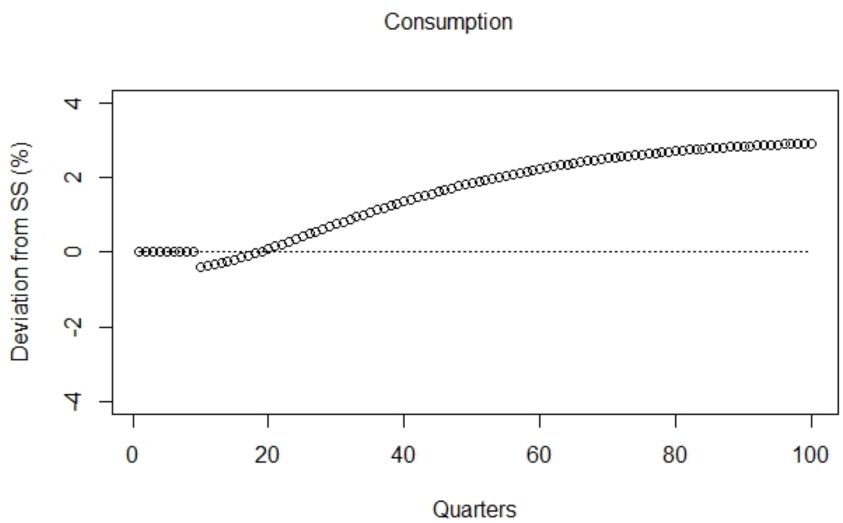

Labor Supply
Carrier
Industry
Deviation from SS (%)
4
2
0
-2
-4
0
20
40
60
80
100
Quarters

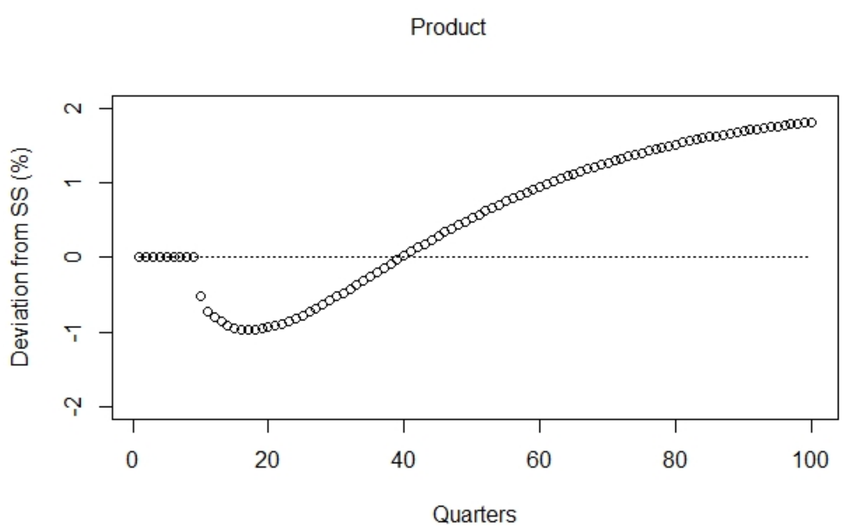
Wages



Welfare




Consumption
Deviation from SS (%)
4
2
0
-2
-4
0
20
40
60
80
100
Quarters


Product
Deviation from SS (%)
2
1
0
-1
-2
0
20
40
60
80
100
Quarters