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Probabilistic Analysis Applied to the Risk Assessment of Dams

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ABSTRACT

Conventional engineering primarily uses a deterministic analysis of the parameters to obtain results. Therefore, an evaluation was used as a unique solution; however, the parameters used in the analysis present considerable variability due to the nature of the materials. That disregard of a variable of the materials has been wrong in the safety assessment, resultant in which: solutions economically more expensive or higher risk to the enterprise. Thereby, the work elaborates a probabilistic analysis treatment of factor of safety of dams for a more realistic perception. In order to evaluate the influence of the variability of the measurements applied to dams design, the case study of a hypothetical composed dam. Through this analysis, it was possible to conclude that the probabilities of the risk assessment process may be more significant than the acceptable risk, even if the deterministic safety factor is within the limits allowed by the rule, which may prescribe an unfeasibility of the enterprise, in addition to demonstrating the most substantial amount of safety of the dam.

Keywords: risk, dam security, probabilistic analysis, variability.

1 INTRODUCTION

Historically, conventional engineering defines the security of buildings by their Factor of safety (FS), which is a deterministic approach. This operation mode starts to be doubtful once most of the geotechnical materials features a variability behavioral. To overlook this behavior incurs in unacceptable risks (Assis et al, 2018).

Therefore, the concept of the probabilistic analysis was introduced to design and treat the data of engineering problems. This view considers the parameters as variable values that occurs in a range, according to probabilistic distributions (Araujo & Sayão, 2018).

The parameters are determined by tests that are realized at field or laboratory. The tests results did not reproduce enough data to determine a representative sample of the properties.

On smaller buildings, it is possible to use a deterministic approach without big losses due to the little risk of the enterprise. However, the small amount of data is not an obstacle to apply probabilistic methods, thence, for large impact buildings, it is necessary to use a probabilistic method for design and treating of data.

1.1 *Risk*

Definition of risk can be applied by different means; the most kwon is related to insecurity. However, risk can be also related with benefits (Almeida, 2001).

The equation below represents a mathematic definition of risk:

$$R= P(A).C \tag{1}$$

where R is risk in ($), P(A) is probability of rupture (%), C is the damage caused by a break of construction in ($).

Therefore, the risk can be evaluated with a monetary value. This value corresponds to the value of the consequence of the rupture of the construction multiplied by the probability of rupture of the same. This index is able to determine the viability of a construction. For example, big constructions can have small probabilities of rupture, but the consequences can be countless, turning the enterprise inviable and vice versa (Agência Nacional de Águas, 2011).

1.2 *Security of Dams*

The absolute safety of dams cannot be guaranteed; this fact is already known among engineers, but it is not widely known and accepted by the population in general (Almeida, 2001). For a long time, catastrophes were attributed to natural causes, whereas that construction was made as rigorous and correct as technology allowed.

The *Malpasset France* accident in 1959 killed 421 people and changed the view of dam safety. Dam disruption probably occurred due to the disregard of a geological fault that could have been predicted if the variability of the material foundation had been considered (Carmo, 2013).

From this incident, there were discussions that consider three fundamental points to evaluate the safety of dams (Almeida, 2001):

- Evaluation of Operational technique that consisted in observing the design norms;
- Monitoring that allows to evaluate the conditions of the dam, as well as the feedback of the process of operational technical evaluation;
- Emergency plan and risk management.

The emergency and risk management plan is a recent practice, which needs to be better applied in Brazil (Medeiros & Pinto, 2014). They used to believe that by adopting the best operating techniques and by monitoring the dam, the probability of rupture is negligible.

2 JUSTIFICATIVE

A recent event, which was the disruption of the Fundão tailings dam that occurred in November 2016, showed that only the deterministic approach is not enough to guarantee the safety of a dam (SAMARCO, 2016).

Another catastrophe affected Brazil this year: the rupture of the Feijão dam, located in Brumadinho, shown in Figure 1. This case also demonstrates that the deterministic method is not enough to verify the safety of dams once that dam was designed with the deterministic approach, and considered safe according to the standard factors. However, it presented signs that it could be at risk (*Estadão*, 2019).

FIGURE 1 Feijão Dam in Brumadinho before and after de rupture (*Google Earth*, 2019).

In both situations, the importance of the emergency action plan was proved; both of dams presented this safety item. However, the effectiveness of the plans were questionable when considered the number of deaths and the environmental catastrophe. If there were a more efficient emergency plan, combined with people training, fewer fatalities would have occurred. As so, if mitigating measures for environmental impact existed, the consequences could be smaller. (Vale, 2019).

According to studies, one in every 100 dams fails, and the annual probability of dam fail is 0.045, which means that in each 22 years there is a dam failure (Foster & Spannagle, 1998).

For the specific subject of dams, there is a diagram (Figure 2) that relates the probability of rupture to the number of fatalities. The diagram also categorizes the risks into: intolerable; insignificant and ALARP, which would be the tolerable limit zone in which works must be constantly observed and where the probability of rupture has to be low enough so that its benefits are better than the hazard involved in the failure (FEMA, 2015).

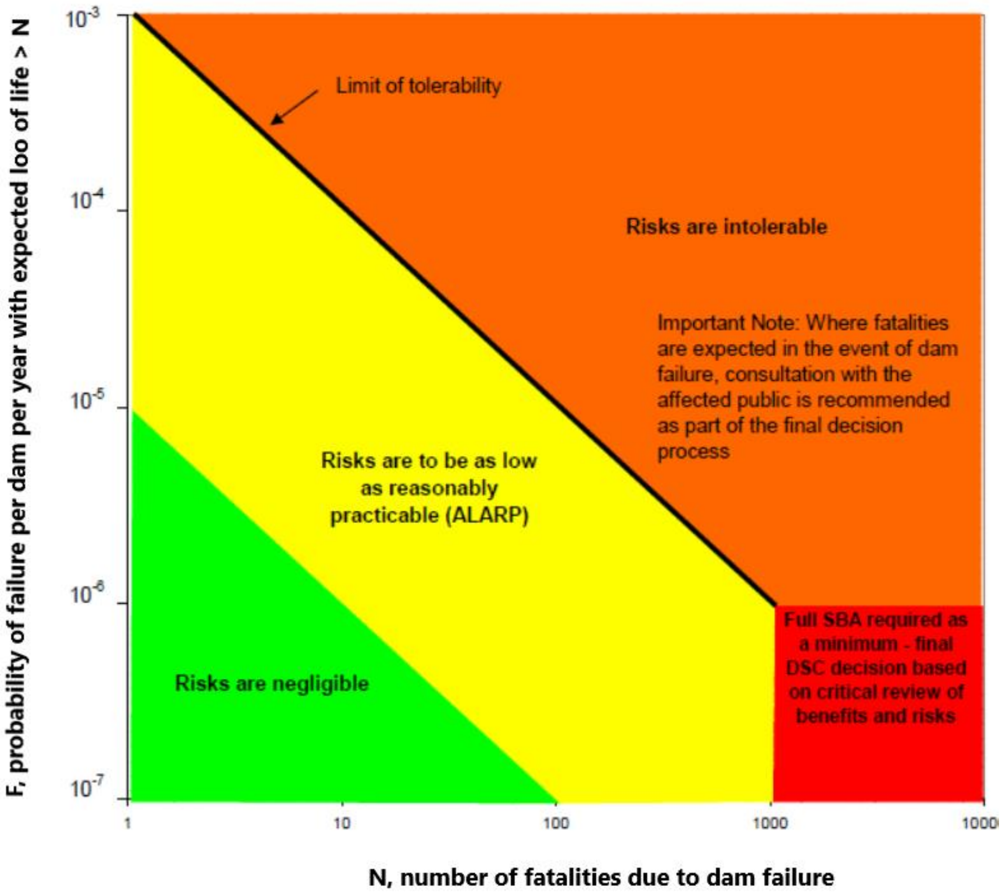

FIGURE 2 Diagram that classifies the risks of the dams (DSC, 2006)

It is possible to analyze the dams in a precise way in relation to the risks; however, to do this methodology, a probabilistic analysis is more appropriate to dimension the probability of rupture. This aspect is essential to give more reliability to the dam.

3 METHODOLOGY

The result of the probabilistic method is a probability distribution of a performance factor, in which a certain value of critical performance determines the probability of rupture, as for example the factor of safety smaller than one (1) defined the probability of rupture of a dam . As shown in Figure 3, the factor of safety of the deterministic method is not sufficient to determine the probability of rupture, and a build with a lower factor of safety may have a lower probability of rupture, therefore, to be safer than that the deterministic methodology indicate (Assis et al, 2018).

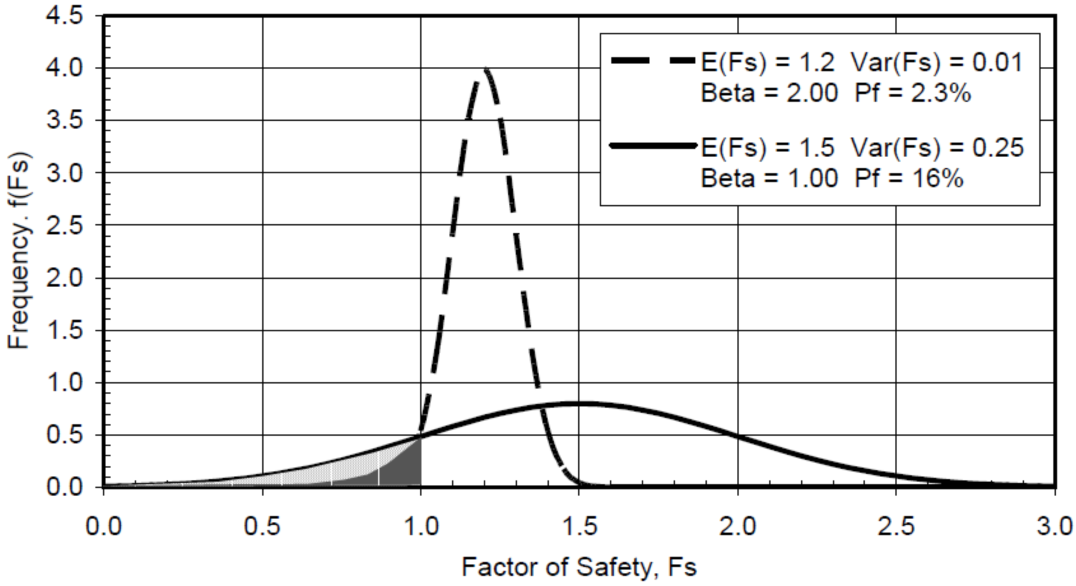

FIGURA 3 Example of calculation of probability of failure according to the performance indicator Factor of Safety (Assis et al, 2018).

The probabilistic methodology requires that input data to be probabilistic distributions. The use of probabilistic distributions in the geotechnical properties adapts perfectly to the characteristics of the parameters due to the great variability of the materials. This variability of parameters can be represented by the statistical concept of standard deviation, however, to measure the standard deviation a reasonably large sampling is required that are obtained by tests (Duncan, 2000).

Most of constructions do not have enough tests to define the statistical data; the average can be considered with only one data, however, the variance needs more data. To use the probabilistic methods, the coefficient of variation of parameters is a property information that can be determined by bibliography, and can be used to determine the statistical data of variance, presented in the following equation (Ang & Tang, 1975):

$$COV = \left(\frac{\sigma}{\mu}\right)100\% \tag{2}$$

where COV is the coefficient of variation in (%), σ is the standard deviation in (%), and μ is the mean.

This paper will evaluate the safety of dams using classical probabilistic methods to determine the variables most relevant to the performance factor, statistical data of interest and the probabilistic distribution most appropriate to the problem. The statistical methods used are presented below.

3.1 *First Order Second Moment Method (FOSM)*

This method consists of truncating the first-order derivative of the Taylor series using the derivatives of the function, which can solve numerically; with the results of the truncation of the series, it is possible to determine the statistical parameters essential for the determination of the probabilistic distribution of the performance factor. The standard deviation of the function is determined by Equation and mean is a parameter known for the method (Baecher & Cristian, 2003):

$$\sigma_F^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial F}{\partial x_i}\frac{\partial F}{\partial x_j}\rho_{x_i x_j}\sigma_{x_i x_j} \tag{3}$$

where F is the function of the performance factor, n is the number of variables, σ is the standard deviation, and x is the independent variable.

The most interesting result of this method is the determination of a sensibility for each variable involved in the problem, because the product of the derivative is a function that relates to a variable with its standard deviation, determining the interference of each variable in the solution function (Assis et al, 2018).

3.2 *Point Estimate Method (PEM)*

This method created by Rosenblueth (1975), consists in applying points of estimation of input parameters, considering the mean and standard deviation of each. Thus, is not necessary to know the complete distribution of each variable (Rosenblueth, 1975).

The main difference between FOSM and PEM is that, in the first method, the variation is the same for all input parameters, while the second one is the standard deviation for each input variable in the estimation points (Assis et al, 2018).

From the points of the parameters applied in the function, we can calculate the mean and the standard deviation of the performance factor by means of the following Equations, respectively (Rosenblueth, 1975):

$$\bar{Y} = \frac{1}{2^n} \sum_{i=1}^{2n} y_i \tag{4}$$

$$\sigma_y^2 = \frac{1}{2^n} \sum_{i=1}^{2n} (y_i - \bar{Y})^2 \tag{5}$$

where n is the number of variables, yi is the result of the performance factor for each point.

The statistical data calculated by Equations 4 and 5 to determine the probability distribution of the performance factor and, by means of the critical value of the function, the probability of rupture is calculated.

3.3 *Monte Carlo Method (MMC)*

This method is the most accurate of the classic methods. The procedure is to search for random values of the independent variable through successive attempts and to apply in the function. Performing this random procedure countless times results in a histogram of frequencies of the results found. From the histogram, it is possible to determine directly the mean, the standard deviation or probability of occurrence of the event. (Ang & Tang, 1984).

4 CASE STUDIED

The main failure modes of the dam are overflow; piping and structural failure of the material used both in the body and in the foundation of the dam (*Agência Nacional de Águas*, 2011). The structural failure will be the type of fault used in this work and defined by the Factor of safety (FS), either by the deterministic method when probabilistic only changes the type of approach.

The deterministic method calculates a FS value, considering that the properties are constant, whereas the probabilistic methods take into account the variability of the properties and finds a probabilistic distribution of the FS, in which values smaller than one is the probability of failure (pf).

Using a rock fill dam with vertical clay core in operation, values consistent with the bibliography are applied, both for mean values and for the COV and the statistical data of the materials presented in Table 1.

Table 1 Parameters used

	Foundation	Rockfill	Random	Clay	Sand
Unit Weight (kN/m³)	28	22	22	22	17
COV	3	3	3	3	3
Standard Deviation	1	0,66	0,66	0,66	0,51
Cohesion (kPa)	100	5	10	10	1
COV	40	40	50	40	40
Standard Deviation	40	2	5	4	0,4
Friction Angle	50	44	35	28	35
COV	10	10	15	10	10
Standard Deviation	5	4,4	5,25	2,8	3,5

Deterministic evaluation of security was conducted in permanent state using the Software Geostudio, which considers limit equilibrium method. Figure 4 shows the place where the litter security factor (FS) was found.

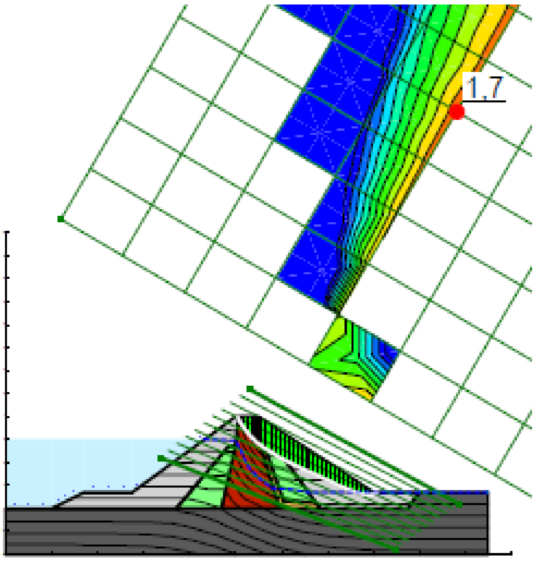

FIGURE 4. Fator of Safety – Deterministic method

The calculations of probabilistic methods applied to the same dam that was used in the deterministic method are presented next.

4.1 *FOSM Analysis*

The method used was the double FOSM, in which considered two changes of the input parameter. An increase and a decrease of 10% in the average of the following input parameters and applied specific weight, angle of friction and cohesion of the materials.

This variation determinate the influence of each input parameter in function. Table 2 presents the results obtained, where it is evident that the most important property in the slope stability of this dam is the rock fill friction angle with 97% and friction angle of Random with 2% of the weight in the factor of safety.

The most important result of probabilistic methods is the probability of rupture, that in the case of this dam was $8{,}5.10^{-4}$.

Table 2 Results of FOSM double

	Mean	Standard Deviation	Delta X	X+	X-	FS+	FS-	Derivate	Derivate²StandadrDeviation²	Sensitivity (%)
Foundation										
Unit weight (kN/m³)	28	1	2,8	30,8	25,2	1,669	1,669	0,000	0,000	0,000
Cohesion (kPa)	100	40	10	110	90	1,669	1,669	0,000	0,000	0,000
Friction Angle (º)	50	5	5	55	45	1,669	1,669	0,000	0,000	0,000
Rock fill										
Unit weight (kN/m³)	22	0,66	2,2	24,2	19,8	1,678	1,661	0,013	0,000	0,159
Cohesion (kPa)	5	2	0,5	5,5	4,5	1,674	1,665	0,002	0,000	0,045
Friction Angle (º)	44	4,4	4,4	48,4	39,6	1,863	1,443	0,048	0,044	97,110
Radom										
Unit weight (kN/m³)	22	0,66	2,2	24,2	19,8	1,669	1,669	0,000	0,000	0,000
Cohesion (kPa)	10	5	1	11	9	1,67	1,669	0,000	0,000	0,001
Friction Angle (º)	35	5,25	3,5	38,5	31,5	1,703	1,64	0,006	0,001	2,185
Clay										
Unit weight (kN/m³)	22	0,66	2,2	24,2	19,8	1,66	1,669	-0,007	0,000	0,045
Cohesion (kPa)	10	4	1	11	9	1,671	1,667	0,001	0,000	0,009
Friction Angle (º)	28	2,8	2,8	30,8	25,2	1,684	1,658	0,005	0,000	0,372
Sand										
Unit weight (kN/m³)	17	0,51	1,7	18,7	15,3	1,667	1,671	-0,004	0,000	0,009
Cohesion (kPa)	1	0,4	0,1	1,1	0,9	1,669	1,669	0,000	0,000	0,000
Friction Angle (º)	35	3,5	3,5	38,5	31,5	1,676	1,665	0,002	0,000	0,067
FS mean	**1,669**		**Standard Deviation FS**		**0,21310**			**Probability of Failure**		**0,00085**

4.2 *PEM Analysis*

From the FOSM, it was possible to conclude that the parameter with the greatest influence on the factor of safety is the friction angle of rock fill, random and clay, also unit of weight of rock fill. This method was restricted to evaluating only these properties in the result of the factor of safety.

PEM apply an increase and a decrease of the standard deviation of the parameter in the variable, that is, the points of estimation. Table 3 shows the input parameters, while Table 4 presents the calculations for each scenario.

Table 3 Input parameter of PEM

Parameters		**Means**	**Standard Deviation**	**P+**	**P-**	**FS mean**
Rock fill	Unit Weight(kN/m³)	22	0,66	22,66	21,34	
	Friction Angle (º)	44	4,4	48,4	39,6	
Random	Friction Angle (º)	35	5,25	40,25	29,75	
Clay	Friction Angle (º)	28	2,8	30,8	25,2	1,671

Table 4 Results of PEM

Scenario					**FSi**	**FSi-FS mean**	**FSi²-FS mean²**
C1	Rock fill	Unit Weight (kN/m³)	+	22,66	1,916	0,245	0,060
		Friction Angle (º)	+	48,4			
	Random	Friction Angle (º)	+	40,25			
	Clay	Friction Angle (º)	+	30,8			
C2	Rock fill	Unit Weight (kN/m³)	-	21,34	1,444	-0,227	0,052
		Friction Angle (º)	-	39,6			
	Random	Friction Angle (º)	-	29,75			
	Clay	Friction Angle (º)	-	25,2			
C3	Rock fill	Unit Weight (kN/m³)	-	21,34	1,926	0,255	0,065
		Friction Angle (º)	+	48,4			
	Random	Friction Angle (º)	+	40,25			
	Clay	Friction Angle (º)	+	30,8			
C4	Rock fill	Unit Weight (kN/m³)	+	22,66	1,446	-0,225	0,051
		Friction Angle (º)	-	39,6			
	Random	Friction Angle (º)	+	40,25			
	Clay	Friction Angle (º)	+	30,8			
C5	Rock fill	Unit Weight (kN/m³)	+	22,66	1,841	0,17	0,029
		Friction Angle (º)	+	48,4			
	Random	Friction Angle (º)	-	29,75			
	Clay	Friction Angle (º)	+	30,8			
C6	Rock fill	Unit Weight (kN/m³)	+	22,66	1,928	0,257	0,066
		Friction Angle (º)	+	48,4			
	Random	Friction Angle (º)	+	40,25			
	Clay	Friction Angle (º)	-	25,2			
C7	Rock fill	Unit Weight (kN/m³)	-	21,34	1,452	-0,219	0,048
		Friction Angle (º)	-	39,6			
	Random	Friction Angle (º)	+	40,25			

	Clay	Friction Angle (º)	+	30,8			
C8	Rock fill	Unit Weight (kN/m³)	+	22,66	1,446	-0,225	0,051
		Friction Angle (º)	-	39,6			
	Random	Friction Angle (º)	-	29,75			
	Clay	Friction Angle (º)	+	30,8			
C9	Rock fill	Unit Weight (kN/m³)	+	22,66	1,802	0,131	0,017
		Friction Angle (º)	+	48,4			
	Random	Friction Angle (º)	-	29,75			
	Clay	Friction Angle (º)	-	25,2			
C10	Rock fill	Unit Weight (kN/m³)	+	22,66	1,445	-0,226	0,051
		Friction Angle (º)	-	39,6			
	Random	Friction Angle (º)	+	40,25			
	Clay	Friction Angle (º)	-	25,2			
C11	Rock fill	Unit Weight (kN/m³)	-	21,34	1,831	0,16	0,026
		Friction Angle (º)	+	48,4			
	Random	Friction Angle (º)	-	29,75			
	Clay	Friction Angle (º)	+	30,8			
C12	Rock fill	Unit Weight (kN/m³)	-	21,34	1,489	-0,182	0,033
		Friction Angle (º)	-	39,6			
	Random	Friction Angle (º)	-	29,75			
	Clay	Friction Angle (º)	+	30,8			
C13	Rock fill	Unit Weight (kN/m³)	+	22,66	1,447	-0,224	0,050
		Friction Angle (º)	-	39,6			
	Random	Friction Angle (º)	-	29,75			
	Clay	Friction Angle (º)	-	25,2			
C14	Rock fill	Unit Weight (kN/m³)	-	21,34	1,797	0,126	0,016
		Friction Angle (º)	+	48,4			
	Random	Friction Angle (º)	-	29,75			
	Clay	Friction Angle (º)	-	25,2			
C15	Rock fill	Unit Weight (kN/m³)	-	21,34	1,45	-0,221	0,049
		Friction Angle (º)	-	39,6			
	Random	Friction Angle (º)	+	40,25			
	Clay	Friction Angle (º)	-	25,2			
C16	Rockfill	Unit Weight (kN/m³)	-	21,34	1,893	0,222	0,049
		Friction Angle (º)	+	48,4			
	Random	Friction Angle (º)	+	40,25			
	Clay	Friction Angle (º)	-	25,2			
				FS Standard Deviation			0,2109
				Probability of Failure			0,0007

From these results, it was possible to calculate the probability of failure that is $7.\,10^{-4}$, as well as in the FOSM method in the magnitude scale, demonstrating the convergence of probabilistic methods for the problem in question.

4.3 *Monte Carlo Analysis*

The Software used in this work, *GeoStudio*, uses Monte Carlo Method for determines probability of failure simulations of 100, 1000, 10,000 and 100,000 interactions were performed. The results in Table 5.

The choice of quantity of interactions is related to the amount of parameters involved in the problem. In this case, the only variation was the friction angle rockfill because it was the parameter with 97% of weight in the Performance Factor, so was used just that to optimize the computer time. The results converge at 10,000 simulations. Therefore, the FOSM and PEM demonstrated the problem has this magnitude scale, so the simulation stops at 100.000 trials.

Table 5 Results of Monte Carlo

100 trials		**1000 trials**		**10.000 trials**		**100.000 trials**	
Mean FS	1,701	Mean FS	1,681	Mean FS	1,680	Mean FS	1,681
Reliability index	2,968	Reliability index	2,772	Reliability index	2,772	Reliability index	2,778
FS<1 (%)	0,000	FS<1 (%)	0,200	FS<1 (%)	0,220	FS<1 (%)	0,160
Standard Deviation	0,236	Standard Deviation	0,246	Standard Deviation	0,245	Standard Deviation	0,245
Smaller FS	1,154	Smaller FS	0,855	Smaller FS	0,779	Smaller FS	0,770
Greatest FS	2,303	Greatest FS	2,780	Greatest FS	3,160	Greatest FS	3,643
Number of trials	100,000	Number of trials	1000,000	Number of trials	10.000	Number of trials	100.000
Pf	0,0015	Pf	0,0028	Pf	0,0028	Pf	0,0027

Figure 5 shows histograms of Monte Carlo Method, as they increase the quantity of simulations, the results were converging.

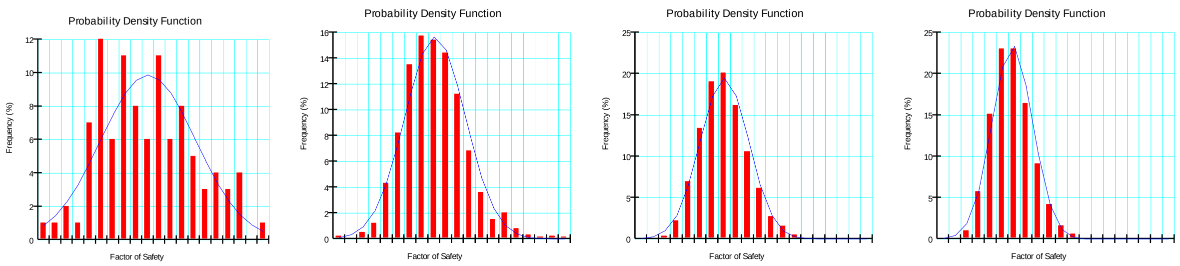

Figure 5 Histogram of the factor of safety 100, 1,000, 10,000, 100,000 respectively

Considering that the greatest convergence occurred in 100.000 attempts, the probability of failure of this dam was $2{,}7.10^{-3}$.

5 CONCLUSION

From this paper, it was concluded that the probabilistic calculation is a procedure that must be performed to evaluate the safety of a dam. Although the input parameters seem challenging to acquire, the literature already presents a database that can help to use the probabilistic methods, which are essential information for risk management.

In the studied case, it was possible to analyze that even though the dam possesses a factor of safety equal to 1.7 in the permanent regime, it presents a probability of rupture in a range 7.10^{-4} to$2{,}7.10^{-3}$, that depending the consequence can be in the acceptable zone and even unacceptable zone, as shown in Figure 2.

Therefore, the risk evaluation makes it possible to monetize the eventual damages caused by a failure. This process can classify a dam in infeasible or prepares the owner of the construction for a possible incident.

This study shows that even the dam with adequate factor of safety to the rule may have the probability of failure unacceptable, for example, the case of the recently broken tailings dams, which were in accordance with the norm, according to the average factor of safety, however, has led to enormous catastrophes for society.

Finally, this work aims to encourage the evaluation of risks of significant works using the probabilistic method, once it is a viable procedure and can add more reliability to the safety of the dam.

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