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TESE DE DOUTORADO Nº 21

Estatísticas de Lei de Potência Aplicadas no Estudo de Terremotos

Área de concentração: Geofísica Aplicada

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Orientador: George Sand Leão Araújo de França

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Instituto de Geociências

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Autora

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Tese apresentada ao Instituto de Geociências da Universidade de Brasília, para obtenção do título de Doutor em Geociências Aplicadas, na área de Geofísica Aplicada.

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Brasília, 05 de dezembro de 2014.

RELATÓRIO DE DEFESA DE TESE

DOUTORADO

Universidade de Brasília - UnB
Decanato de Pesquisa e Pós-Graduação - DPP
Secretaria de Administração Acadêmica - SAA

1 - Identificação do Aluno

Nome	Matrícula
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Curso
Geociências Aplicadas

Área de Concentração	Código	Departamento
Geofísica Aplicada	2615	IGD

2 - Sessão de Defesa de Tese

Título

"ESTATÍSTICAS DE LEI DE POTÊNCIAS APLICADAS NO ESTUDO DE TERREMOTOS"

3 - Comissão Examinadora

Nome	Função	Assinatura
GEORGE SAND LEAO ARAUJO DE FRANCA (Doutor)	Membro Interno vinculado ao programa (Presidente) Instituto de Geociências	[assinatura]
GIULIANO SANT'ANNA MAROTTA (Doutor)	Membro Interno vinculado ao programa Instituto de Geociências	[assinatura]
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DORY HÉLIO AIRES DE LIMA ANSELMO (Doutor)	Membro Externo não vinculado ao programa Universidade Federal do Rio Grande do Norte	[assinatura]
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ROBERTA MARY VIDOTTI (Doutor)	Membro Interno vinculado ao programa (Suplente) Instituto de Geociências	

4 - Resultado

A Comissão Examinadora, em 05/12/2014 após exame da **Defesa de Tese** e argüição do candidato, decidiu:

- [ ] Pela aprovação da **Tese**
- [x] Pela aprovação da **Tese**, com revisão de forma, indicando o prazo de até 30 dias para apresentação definitiva do trabalho revisado.
- [ ] Pela reprovação da **Tese**
- [ ] Pela reformulação da **Tese**, indicando o prazo de ____________ para nova versão.

Preencher somente em caso de revisão de forma:

- [ ] O aluno apresentou a revisão de forma e a **Tese** foi aprovada.
- [ ] O aluno apresentou a revisão de forma e a **Tese** foi reprovada.
- [ ] O aluno não apresentou a revisão de forma.

Autenticação
Presidente da Comissão Examinadora

05/12/2014 — Data
[assinatura] — Assinatura/Carimbo

Autenticação
Coordenador do Curso

___/___/___ — Data
____________________ — Assinatura/Carimbo

Ciente
Aluno

05/12/14 — Data
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Decisão:

- [ ] Homologar

___/___/___ — Data
____________________ — Assinatura do Decano

Dedicatória

Àquele que estabeleceu e firmou todas as coisas, mas nos permitiu perscrutá-las. A Ele, que me deu vida, me capacitou e não me deixou desistir. Que merece muito mais do que sou capaz de oferecer, mas me escolheu e me agraciou para que eu chegasse até aqui.

"Bendize, ó minha alma, ao SENHOR!
SENHOR, Deus meu, como tu és magnificente:
sobrevestido de glória e majestade, coberto de luz como de um manto.
Tu estendes o céu como uma cortina, pões nas águas o vigamento da tua morada,
tomas as nuvens por teu carro e voas nas asas do vento.
Fazes a teus anjos ventos e a teus ministros, labaredas de fogo.
Lançaste os fundamentos da terra, para que ela não vacile em tempo nenhum. [...]
A glória do SENHOR seja para sempre! Exulte o SENHOR por suas obras!
Com só olhar para a terra, ele a faz tremer; toca as montanhas, e elas fumegam.
Cantarei ao SENHOR enquanto eu viver;
cantarei louvores ao meu Deus durante a minha vida.
Seja-lhe agradável a minha meditação; eu me alegrarei no SENHOR."
Sl 104: 1-5, 31-34

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Ainda, há algo que não posso deixar de registrar. Por parte de pai, descendo de agricultores suíços que vieram para o Brasil cheios de esperanças, mas logo descobriram que acreditaram em promessas falsas. Por parte de mãe, descendo de africanos trazidos como escravos e, por isso, enfrentaram obstáculos ainda maiores. Mas nada disso os impediu de lutarem e abrirem caminho para que as gerações futuras avançassem. Sem o exemplo deles, sem a persistência e a história construída por cada um ao longo de tantos anos, lutando contra as circunstâncias adversas e buscando um legado e um futuro para as suas famílias, eu jamais teria as oportunidades que tive e assim jamais conquistaria o que conquistei até este momento.

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Resumo

Após o trabalho pioneiro de Ian Main (1995), estatísticas de lei de potência começaram a ser usadas no estudo de eventos sísmicos. Em especial, a generalização da abordagem clássica de Boltzmann-Gibbs desenvolvida por Tsallis (1998) se mostrou amplamente aplicável. A partir dessa abordagem, modelos para análise de distribuição de energia em Sismologia começaram a ser desenvolvidos e aplicados em diferentes regiões e com diferentes enfoques, sempre apresentando resultados satisfatórios. Entretanto, pouco se avançou na tentativa de associar os parâmetros do ajuste a aspectos geofísicos dos fenômenos e regiões estudadas. Usando o modelo desenvolvido por Sotolongo-Costa e Posadas (2004) e revisado por Silva et al. (2006) esse trabalho buscou um melhor entendimento da aplicabilidade dessa metodologia e ampliação dos significados que podem ser extraídos desse tipo de análise. De fato, foi possível encontrar uma relação entre o parâmetro não extensivo (q) e o modelo de aspereza de Lay e Kanamori (1981), especialmente ao se considerar as zonas de subducção com acoplamento mais intenso e mais suave, indicando a influência de fatores como distribuição de esforços e fragmentação. Ainda, encontrou-se relação entre q e sismos intraplaca em áreas do território brasileiro, com diferentes embasamentos e características tectônicas. Na Margem Passiva, os valores de q foram bem mais elevados. Verificou-se ainda que o uso de diferentes tipos de magnitude na análise impactou os resultados de forma significativa. Estes indicam que a magnitude de superfície influencia mais os valores de q no sentido de se correlacionarem às zonas de subducção, refletindo um efeito predominante da fragmentação em níveis menos profundos.

Palavras-chave: Não extensividade, zonas de subducção, sismos intraplaca, tipos de magnitude

Abstract

After the pioneering work of Ian Main (1995), law power statistics are being used in earthquakes studies. In particular, the classic approach generalization Boltzmann-Gibbs, developed by Tsallis (1998), has showed itself highly applicable. Using this technique, analysis models for earthquakes energy distributions were developed and applied in different regions and with different perspectives, always presenting satisfying results. However, little progress was achieved in trying to associate parameters to adjust the geophysical aspects of phenomena and regions studied. Using the model developed by Sotolongo-Costa e Posadas (2004) e revised by Silva et al. (2006), this work aimed a better understanding of this method, expanding the information that can be obtained by this kind of analysis. Indeed, it was possible to find a relation between the nonextensive parameter (q) and Lay and Kanamori (1981) asperity model, mainly when considered the subduction zones with stronger and weaker coupling, indicating the influence of factors such stress distribution and fragmentation. Also, it was found a relation between q and intraplate quakes in Brazilian areas with different basements and tectonic characteristics. At the Passive Margin, the nonextensive parameter was higher. At least, it's verified that using different kinds of magnitudes impacts significantly in the results. They indicate that when we use surface magnitude the q-values are more correlated with the subduction zones classification, reflecting a predominant effect of fragmentation in less deeper levels.

Key words: Nonextensivity, subduction zones, intraplate quakes, magnitudes types.

Sumário

Capítulo 1 - Introdução

O estudo de terremotos e falhas geológicas é extremamente complexo pois envolve diversas variáveis como: deformação, ruptura, energia liberada, feições do terreno, heterogeneidade na interface sismogênica da placa, entre outros (Kawamura et al. 2012), seja em escala regional ou planetária (Sarlis, 2011; Sarlis e Christopoulos, 2012), ou mesmo em outros planetas. Diferentes ferramentas têm sido usadas a fim de alcançar um melhor entendimento desses fenômenos, muitas delas desenvolvidas empiricamente, como a Lei de Omori (Omori, 1894) que descreve a distribuição temporal de pós-abalos, a Lei de Gutenberg-Richter (Gutenberg e Richter, 1944) que estabelece uma relação entre frequência e magnitude, ou ainda a Lei de Båth (Båth, 1965) delineando uma diferença constante nas magnitudes do sismo principal e do maior pós-abalo gerado por ele.

Recentemente, estatísticas de lei de potência passaram a ser aplicadas também a estudos de terremotos e falhas geológicas. Apesar do sucesso da Mecânica Estatística clássica de Boltzmann-Gibbs na descrição termodinâmica de sistemas, possíveis limitações advindas de propriedades diferenciadas de alguns sistemas incentivaram pesquisadores a desenvolver novos modelos generalizando a definição de entropia adotada por Boltzmann-Gibbs. Do ponto de vista matemático, as estatísticas generalizadas modificam o peso de Boltzmann, trocando o comportamento exponencial por uma lei de potência na função entrópica e na distribuição de probabilidades.

Neste trabalho, nos baseamos na estatística de Tsallis (Tsallis, 1988), que é válida para sistemas em estados estacionários ou meta-estáveis, conforme o modelo desenvolvido para Sismologia por Sotolongo-Costa e Posadas (2004) e posteriormente revisado por Silva et al. (2006). Continua sendo uma abordagem empírica, mas parte-se do pressuposto que ela pode trazer informações adicionais à compreensão e descrição do comportamento e características dos sistemas estudados.

Apesar de este modelo ter apresentado bons ajustes em diversos trabalhos, não foi apresentada explicação física para o significado dos parâmetros calculados. Assim, alguns questionamentos permanecem: há significado físico no parâmetro não extensivo? Há correlação entre q e grandezas geofísicas? Quais grandezas impactam o valor de q de forma mais significativa? O valor de q varia com diferentes características tectônicas ou é apenas um ajuste matemático?

A fim de se avaliar tais questões, o modelo precisaria ser aplicado em diferentes regiões, com uma base de dados confiável e, considerando-se as características de cada uma e os mecanismos geradores de sismicidade, relacioná-los com os valores de q ajustados. A partir das primeiras análises, novas perguntas surgiram: o ajuste por este modelo aplica-se tanto em situações de contato de placas tectônicas como também intraplacas? O uso de diferentes escalas de magnitude, que consideram diferentes aspectos das ondas geradas e do ambiente de propagação delas, tem influência no valor desse parâmetro?

As áreas escolhidas foram primeiramente a zona do Círculo de Fogo do Pacífico, considerando-se a intensa atividade sísmica e vasta cobertura de estações. Para aplicação do modelo em sismicidade intraplaca, foram selecionadas quatro regiões no território brasileiro.

Apresentação da tese

Essa tese é dividida em seis capítulos cuja divisão se encontra a seguir:

O capítulo 1 introduz os temas abordados, motivação e trabalhos realizados, abrangendo justificativas e objetivos.

O capítulo 2 descreve a abordagem da estatística de Tsallis.

A partir do capítulo 3 até o capítulo 6 apresentam-se os artigos fruto desta pesquisa:

- ❖ No capítulo 3 é apresentado o primeiro artigo com abordagem não extensiva no qual a aluna se envolveu, intitulado "Nonextensive triplet in geological faults system", trata da aplicação da não extensividade de tripleto em dados do sistema de falhas de San Andreas na Califórnia, indicando que a atividade sísmica na região apresenta estrutura hierárquica em pequenas escalas. Foi publicado na "*Europhysics Letters*" em maio de 2013.
- ❖ O capítulo 4 traz o artigo chamado "Nonextensivity at the Circum-Pacific Subduction Zones – Preliminary Studies" no qual se discute a relação entre o parâmetro não extensivo q com o modelo de aspereza desenvolvido por Lay e Kanamori (1981) e apresentando correlação entre o valor de q e as zonas de subducção estabelecidas neste modelo. Foi submetido à publicação Physica A em setembro de 2014.
- ❖ O capítulo 5 contém o artigo "Analysis of Four Brazilian Seismic Areas Using a Nonextensive Approach" submetido a "*Europhysics Letters*" em novembro de 2014. Nesse trabalho fez-se o ajuste não extensivo considerando regiões sísmicas intraplaca no território brasileiro e percebe-se que, em regiões de contraste geológico o valor de q ajustado é mais elevado indicando que nesses locais há mais fragmentação, o que por sua vez impacta no comportamento não extensivo.

- O último artigo fruto deste doutoramento é apresentado no capítulo 6 e ainda está em ajustes para posterior submissão. Entitula-se "Nonextensivity at the Circum-Pacific Subduction Zones – The Influence of Magnitudes Types", sendo uma continuidade do artigo apresentado no capítulo 3 no qual se fez a suposição de que considerar os eventos independentemente do tipo de magnitude não traria grande impacto nos resultados. Verifica-se que o uso de diferentes tipos tem impacto no valor de q, mas a relação entre as zonas de subducção permanece, sendo mais evidente para a magnitude M_S e M_B.

O último capítulo sintetiza as principais conclusões deste trabalho.

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Capítulo 2 - Metodologia

A Mecânica Estatística é o ramo da Física em que se parte da dinâmica microscópica de um sistema físico a fim de avaliar probabilisticamente como ele se comporta macroscopicamente no limite termodinâmico, ou seja, são generalizações a fim de simplificar a análise de sistemas mais complexos. Apesar do sucesso da Mecânica Estatística de Boltzmann-Gibbs na descrição termodinâmica de sistemas possíveis limitações advindas de propriedades (p.e. interação de longo alcance, geometrias fractais) incentivaram pesquisadores a desenvolver novos modelos generalizando a definição de entropia adotada por Boltzmann-Gibbs (equação 2.1).

$$S_{BG}(\{p_i\}) = -k \sum_{i-1}^{W} p_i \ln p_i \qquad (2.1)$$

onde k é a constante de Boltzmann e p_i define uma distribuição de probabilidade.

Do ponto de vista matemático, as estatísticas generalizadas modificam o peso de Boltzmann, trocando o comportamento exponencial por uma lei de potência na função entrópica e na distribuição de probabilidades.

Recentemente, estatísticas de lei de potência passaram a ser aplicadas também a estudos de terremotos e falhas geológicas. A primeira aproximação entre a estatística de lei de potências e a Sismologia surgiu no trabalho pioneiro de Ian Main (1995). A partir da análise da distribuição cumulativa de frequência de magnitude dos sismos, o autor classificou diferentes regiões sísmicas como subcríticas, críticas ou supercríticas de acordo com a heterogeneidade e velocidade de deriva da placa tectônica.

Já a partir do ano 2000, vários trabalhos surgiram aplicando Mecânica Estatística no estudo de terremotos. Em 2012, um volume inteiro da Acta Geophysica (vol. 60 - "*Statistical Mechanics in Earth Physics and Natural Hazards*") se dedicou ao tema, considerando a Mecânica Estatística como uma "ferramenta metodológica para descrever fenômenos com distribuição fractal ou multi-fractal de seus elementos e nos quais interações ou intermitências de longo alcance são importantes, como é o caso dos sistemas terrestres" (prefácio, Telesca e Vallianatos).

Neste trabalho, nos baseamos na estatística de Tsallis (Tsallis, 1988), inicialmente chamada também de estatística não extensiva, que é válida para sistemas em estados estacionários ou meta-estáveis. Ela fornece uma descrição estatística e uma termodinâmica convincente para vários cenários físicos, dentre os quais destacamos: comportamento de estrelas politrópicas (Plastino e Plastino 1993, Silva e Alcaniz 2004), turbulência em plasma eletrônicos (Boghosian 1996), o problema do neutrino solar (Kaniadakis et al. 1996), ou de uma maneira geral, sistemas que apresentam interações de longo alcance, efeitos de memória microscópica efetiva, ou comportamento fractal (Tsallis 1995a, 1995b), e pode ser aplicada em sistemas em estado de não equilíbrio e comportamento complexo, além de sistemas naturais cujos elementos têm distribuição fractal ou multi-fractal (Vallianatos, 2009). Nessa abordagem a entropia é definida como:

$$S_q(\{p_i\}) = k\frac{1-\sum_{i-1}^{W} p_i^q}{q-1} \qquad (q \in \Re ; S_1 = S_{BG}) \qquad (2.2)$$

onde o parâmetro q é chamado de parâmetro não-extensivo; quando q é igual a 1 a entropia de Tsallis se iguala a entropia de Boltzmann-Gibbs.

A equação 2.2 pode ainda ser reescrita como:

$$S = -k_B \int p^q(\sigma) \ln_q p(\sigma)\, d\sigma \qquad (2.3)$$

onde k_B é a constante de Boltzmann, q é o parâmetro de não-extensividade, p(σ) é a probabilidade de encontrar um fragmento de superfície σ. Destaca-se que quando q=1, a equação se iguala à definição de Boltzmann-Gibbs (BG).

Mas, diferentemente da entropia de BG, S_q é dita *não aditiva*. Ou seja, considerando dois subsistemas independentes A e B:

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B) \qquad (2.4)$$

Dado que S_q é não-negativo, segue-se que:

$S_q(A+B) \geq S_q(A) + S_q(B)$, se q<1 (caso chamado super-aditivo); (2.4a)

$S_q(A+B) \leq S_q(A) + S_q(B)$, se q>1 (sub-aditivo). (2.4b)

O formalism de Tsallis é considerado adequado para descrição de terremotos pois o violento processo de fragmentação é muito provavelmente um fenômeno não-extensivo, que leva a um acelerado aumento da energia do Sistema com interações de longo alcance entre as partes do objeto que sofre a fragmentação (Tsallis, 2012). Sotolongo-Costa (2012) argumenta ainda que a não-extensividade tem ligação com a interação transcorrente entre as placas.

O modelo usado nos trabalhos desta tese foi desenvolvido a partir da estatística de Tsallis por Sotolongo-Costa e Posadas (2004) e posteriormente revisado por Silva et al. (2006), considerando que a energia liberada por cada terremoto é proporcional à distribuição de tamanho dos fragmentos entre as placas tectônicas. A ideia é que no contato entre as placas as superfícies são irregulares e há constante formação e consumo de fragmentos, o que exige um formalismo diferenciado, considerando a distribuição de tamanhos dos fragmentos. A distribuição de energia definida em Silva et al. (2006) usa a escala $\varepsilon \sim r^3$, isto é, a distribuição de energia gerada reflete a distribuição volumétrica dos fragmentos entre as placas. O modelo se assemelha à Lei de Gutenberg-Richter modificada e é dada por:

$$\log(N_{>m}) = \log N + \left(\frac{2-q}{1-q}\right)\log\left[1-\left(\frac{1-q}{2-q}\right)\left(\frac{10^{2m}}{a^{2/3}}\right)\right] \qquad (2.5)$$

onde $N_{>m}$ é o número de eventos com magnitude maior que m, N é o total de tremores e a é a constante de proporcionalidade entre o volume dos fragmentos e a energia liberada.

Telesca (2010b, 2012), Telesca e Chen (2010), Telesca (2011), Telesca et al. (2012) e Valverde-Esparza et al (2012) usaram este mesmo modelo para analisar a sismicidade na Itália, Taiwan, Sul da Califórnia, Marrocos e México, respectivamente. Ainda, Papadimitriou et al (2008) o utilizou para avaliar emissões eletromagnéticas pré-sismos; Telesca (2010a) estudou sequências sísmicas; Vallianatos et al. (2011) o usou em análises de escala laboratorial; Vallianatos et al. (2013), aplicou-o em sismicidade vulcânica.

Apesar de em todos os casos o modelo ter ajustado adequadamente o comportamento dos dados, não foi apresentada explicação física para o significado dos parâmetros calculados. Apenas Sotolongo-Costa e Posadas (2002) consideraram que q é uma medida quantitativa da escala de interações espaciais: q~1 indicando interação de curto alcance e; a medida que o valor de q aumenta, o estado físico se torna cada vez mais instável; assim, altos valores de q significariam que os planos da falha não estão em equilíbrio e mais tremores são esperados. Ainda, uma relação entre o parâmetro não extensivo q, conforme este modelo, e o parâmetro b da lei de Gutemberg-Richter foi estabelecida por Vallianatos (2009) e Sarlis et al. (2010):

Lei de Gutemberg-Richter $\qquad \text{Log } N = a - b\,M \qquad (2.6)$

onde *N* é o número de ocorrências de tremores de cada magnitude, *M* a magnitude,
a e *b* parâmetros.

De Sarlis et al. (2010): $\qquad b = 2\,(2 - q)\,/\,(q - 1) \qquad (2.7)$

Essa relação é especialmente relevante, pois o parâmetro b já foi relacionado com diferentes características (Kulhanek, 2005) como:

- ❖ Esforço alto e baixo gera séries de tremores com valores baixos e altos de b;
- ❖ Grande heterogeneidade do meio corresponde a valores mais elevados de b;
- ❖ Elevação do gradiente térmico em teste de laboratório elevou o valor de b de 1,2 para 2,7;
- ❖ Pós-abalos apresentam valores de b elevados e pré-abalos apresentam valores baixos;
- ❖ Eventos de falhas de empurrão estão associados com valores de b mais baixos dos que os de falhas normais, o que indica que b tem relação com o mecanismo focal.

Considerando esses fatores e as condições de aplicabilidade da estatística não extensiva, iniciou-se esse trabalho com aprofundamento dos estudos de Vilar et al. (2007). A partir da mesma área de estudo (a falha de San Andreas), aplicou-se novos conceitos ligados à não extensividade: o q-tripleto, explanado em Tsallis (2006), que é uma subdivisão do valor de q em um conjunto de três valores (q_{sen}, q_{rel}, q_{stat}) que representam respectivamente a sensibilidade às condições iniciais, relaxação (capacidade de retornar ao estado de descanso) e estado estacionário. Nessa abordagem se confirmou o comportamento do sistema como sendo consistente com um estado de não equilíbrio e sugerindo correlações de longo prazo.

Posteriormente, relacionou-se o modelo de Silva et al. (2006) com a abordagem empírica de Lay e Kanamori (1981), recentemente revisitada por Uyeda (2013) que definiu e descreveu zonas de subducção ao longo do Círculo de Fogo do Pacífico. Cada zona foi descrita de acordo com diversas características e agrupadas em uma classificação geral, definindo um modelo de asperezas, grandeza definida nos anos 70 por Byerlee (1970) e Scholz e Engelder (1976). Encontra-se uma definição mais precisa em Johnson e Nadeau (2002) que postula: “A falha é considerada heterogênea no sentido de que contém certas regiões, as quais nós chamamos asperezas, que não estão em movimento. Essas asperezas são pequenas áreas da falha que são muito mais resistentes que as redondezas e capazes de resistir ao esforço tectônico até que um limiar seja atingido e a ruptura ocorra”.

Outra questão levantada é o comportamento do modelo não extensivo em regiões com sismicidade intraplaca, como o território brasileiro. Explicar os mecanismos relacionados aos sismos intraplaca ainda é considerado um desafio, mas dois fatores são considerados proeminentes: zonas de fraqueza e concentração de esforços. No trabalho de Silva et al. (2006), o modelo não extensivo foi aplicado em duas regiões intraplaca: a falha de Samambaia e a falha de Nova Madri. Já neste trabalho, o modelo não extensivo foi aplicado nas zonas sísmicas mais ativas no Brasil: província Borborema, faixa Brasília, lineamento Transbrasiliano e margem passiva do Atlântico.

Por fim, verificou-se se o valor de q é impactado pelo uso de diferentes escalas de magnitude numa mesma região. Cada escala foi desenvolvida enfatizando-se aspectos diferentes da natureza da região do sismo e das ondas geradas. Elas foram desenvolvidas a fim de serem coerentes entre si, mas sabe-se que, tendo em vista que representam diferentes propriedades, não há uma calibração perfeita (Kanamori, 1983). Assim, o conjunto de dados na região do Círculo de Fogo foi subdividido de acordo com esses diferentes tipos de magnitudes a fim de se verificar se há impacto e o quão significativo é no comportamento não extensivo do sistema.

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Capítulo 3

Nonextensive triplet in geological faults system

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Nonextensive triplet in a geological faults system

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Abstract – The San Andreas fault (SAF) in the USA is one of the most investigated self-organizing systems in Nature. In this paper, we studied some geophysical properties of the SAF system in order to analyze the behavior of earthquakes in the context of Tsallis's q-Triplet. To this end, we considered 134573 earthquake events in the magnitude interval $2 \leq m < 8$, taken from the Southern Earthquake Data Center (SCEDC, 1932–2012). The values obtained ("q-Triplet" $\equiv \{q_{stat}, q_{sen}, q_{rel}\}$) reveal that the q_{stat}-Gaussian behavior of the aforementioned data exhibit long-range temporal correlations. Moreover, q_{sen} exhibits quasi-monofractal behavior with a Hurst exponent of 0.87.

Introduction. – Earthquakes are among the most complex spatiotemporal phenomena investigated in the context of self-organized criticality (SOC), introduced in ref. [1]. In this regard, let us consider the so-called fault systems, a complex phenomenon related to the deformation and sudden rupture of some parts of the Earth's crust driven by convective motion in the mantle. One of the first examples of self-organizing systems in Nature [2] is the San Andreas fault (SAF) in California. The SAF, one of the world's longest and most active geological faults, is ∼1200 km long, ∼15 km deep, and about 20 million years old. It forms the boundary between the North American and Pacific plates and is classified as a right lateral strike-slip fault, although its movement also involves comparable amounts of reverse slip [3]. From the geophysical standpoint, a considerable number of investigations have been conducted in order to better understand the complexity of this system (see, *e.g.*, [4] and references therein). In contrast to the complexity of earthquakes, empirical laws are extremely simple, *e.g.*, the Gutenberg-Richter law, which gives the number of earthquakes with a magnitude $M > m$ [5], and the Omori law for temporal distribution of aftershocks [6].

Several studies have demonstrated that seismicity exhibits an out-of-equilibrium behavior that is being investigated by different authors, *e.g.*, studies based on wavelet-based multifractal analysis [7] and nonextensive statistical mechanics [8–10], among others. In the present study, we consider a nonextensive formalism, which is a generalization of Boltzmann-Gibbs statistical mechanics (B-G statistics) for out-of-thermal equilibrium systems and is described by the *entropic parameter* q. The celebrated Boltzmann-Gibbs (B-G) statistics is recovered at $q = 1$ [11–13]. This parameter measures the degree of nonextensivity in the stochastic process.

Tsallis statistics is based on the q-exponential and q-logarithm, two central functions defined by

$$\exp_q(f) = [1 + (1-q)f]^{1/1-q}, \qquad (1)$$

and

$$\ln_q(f) = \frac{f^{1-q} - 1}{1-q}, \qquad (2)$$

which produces entropy S_q [13], associated with q-statistics,

$$S_q = k\frac{1 - \int [PDF(x)]^q \mathrm{d}x}{q-1} \quad (q \in \mathbb{R}), \qquad (3)$$

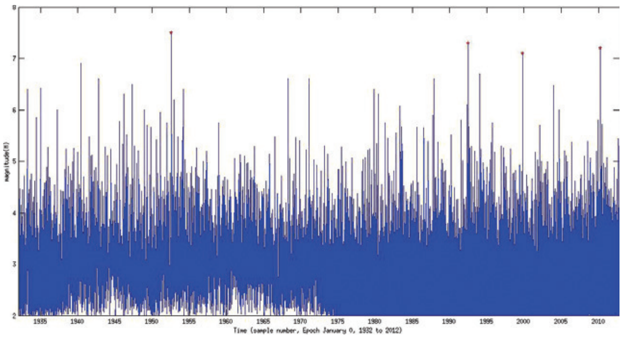

Fig. 1: (Colour on-line) Time series for the magnitude of earthquakes along the SAF. The peaks denote the maximum magnitudes.

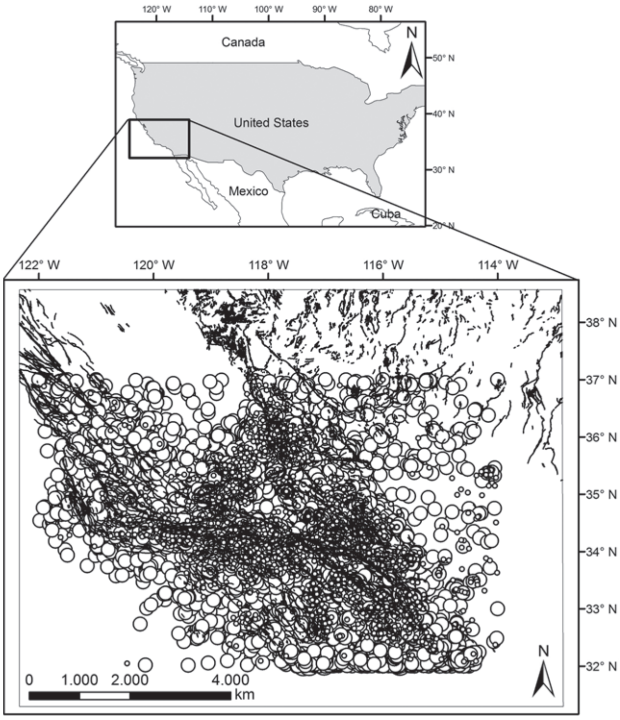

Fig. 2: Map of the seismicity in the SAF system, showing epicenters of earthquakes considered in this study (source SCEDC).

where the Boltzmann-Gibbs entropy, the usual exponential and logarithm are recovered if $q = 1$.

This theory has been successfully applied to many complex physical systems such as geological faults [10] and astrophysical systems [14–16]. In 2004, Tsallis [17] proposed the existence of a three-parameter set $(q_{stat}, q_{sen}, q_{rel})$, also known as q-Triplet, characterized by metastable states in nonequilibrium, where $q_{stat} > 1$, $q_{sen} < 1$ and $q_{rel} > 1$. When $(q_{stat}, q_{sen}, q_{rel}) = (1, 1, 1)$, the set denotes the B-G thermal equilibrium state. Burlaga and Viñas [18] used this triplet to describe the behavior of two sets of daily magnetic-field strength performed by Voyager 1 in the solar wind in 1989 and 2002. In 2009, de Freitas and De Medeiros [16] presented a physical corroboration of the q-Triplet, based on analyses of the behavior of three sets of daily magnetic-field strength observed by different solar indices. More recently, Ferri, Savio and Plastino [19] showed a physical implication of this triplet for the ozone layer in Buenos Aires, Argentina.

The main aim of this study is to analyze the behavior of physical parameters directly reflecting seismic activity in the context of Tsallis q-Triplet's formalism, and to compare the properties of this q-Triplet with those expected for a metastable or quasi-stationary dynamical system described by nonextensive statistics. In this context, we focus our attention on the magnitude values for SAF data $M(t)$ and their hourly variability $\mathrm{d}M_\tau(t)$. Following the ideas presented in ref. [20], we focus our investigation on the "return" or fluctuation $\mathrm{d}M_\tau(t) = M(t+\tau) - M(t)$, which denotes the differences between "avalanche" sizes obtained at time $t+\tau$ and at time t. With respect to seismic activity, this analysis also checks the validity of the q-Central Limit Theorem, the so-called q-CLT, recently conjectured by Umarov, Tsallis and Gell-Mann [21].

The remainder of this paper is organized as follows: in the second section, we present our seismic sample; the main results and discussions are presented in the third section; and, finally, conclusions are put forth in the last section.

The seismic data. – Figure 1 shows the time series for magnitude M of earthquakes along the SAF, in the interval $2 \leq M < 8$, with 134573 events. These were taken from the Southern California Earthquake Data Center (SCEDC) from 1932 to 2012. This range was chosen because for small magnitudes it has the limitation of seismic monitoring in the area, since many such events are unregistered. Figure 2 illustrates the distribution of events considering the SAF map.

Figure 2 shows the data and the San Andreas fault system. This system is more than 800 miles long and extends to depths of at least 10 miles. The fault is a complex zone of crushed and broken rock ranging from a few hundred feet to a mile wide. Many smaller faults branch from and join the San Andreas fault zone. Almost any road cut in the zone shows a myriad of small fractures, fault gouge (pulverized rock), and a few solid pieces of rock [4]. The movement that occurs along the fault is a right-lateral strike-slip forming the tectonic boundary between the Pacific Plate and the North American Plate.

Results and discussions. – In this section, we show the results after the estimation of the "q-Triplet" $\equiv \{q_{stat}, q_{sen}, q_{rel}\}$ based on SAF data from 1932 to 2012 (see fig. 1). These results are presented in three subsections, each associated to the properties of one of the q's.

On the behavior of the q-stationary parameter. For the time series $M(t)$, increment fluctuations due to its

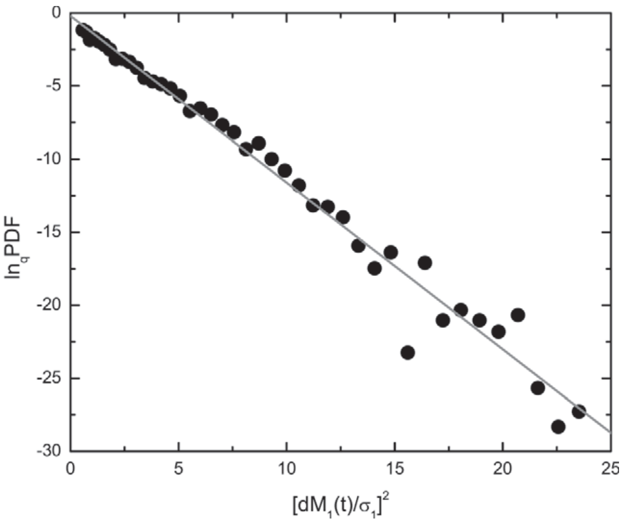

Fig. 3: Linear correlation between $\ln_q[PDF]$ and $[\mathrm{d}M_1(t)/\sigma_1]^2$, where $q_{stat} = 1.364 \pm 0.04$, with $R^2 = 0.992$ and $\chi^2/\mathrm{dof} = 7.0236 \times 10^{-6}$.

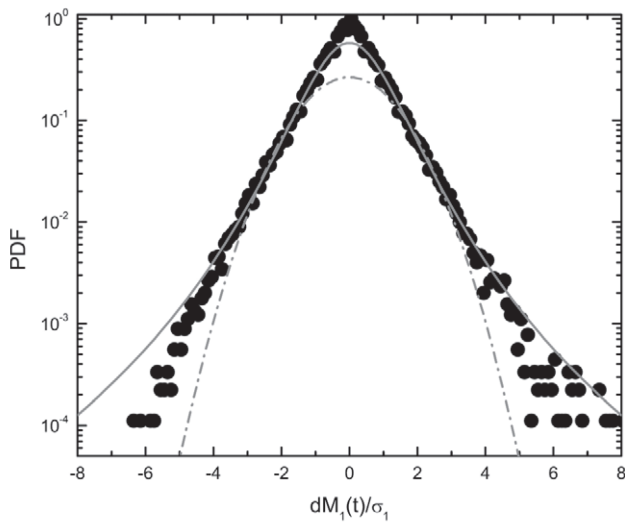

Fig. 4: Black circles: distribution of the increment for SAF data. Solid black line: the q_{stat}-Gaussian distribution based on eq. (4) with $B_q = 0.858 \pm 0.16$. Dashed line: the best fit with a standard Gaussian.

variability over the time scale τ are given as $\mathrm{d}M_\tau(t) = M(t+\tau) - M(t)$. The values of q_{stat} are derived from probability distribution functions (PDFs). These PDFs are obtained from the variational problem using the continuous version for the nonextensive entropy given by eq. (3),

$$PDF = A_q \left[1 + (q-1)B_q \mathrm{d}M_\tau(t)^2\right]^{\frac{1}{1-q}}, \qquad (4)$$

the *entropic parameter* q is related to the size of the tail in the distributions [15] and coefficients A_q and B_q for $q > 1$ are given by

$$A_q = \frac{\Gamma\left[\frac{1}{q-1}\right]}{\Gamma\left[\frac{3-q}{2q-2}\right]} \sqrt{\frac{q-1}{\pi} B_q} \qquad (5)$$

and

$$B_q = \frac{1}{\left[(3-q)\sigma_q^2\right]}, \qquad (6)$$

for further details see ref. [22].

Following the same procedure described by [19], we varied the index q between 1.0 and 2.0, making a linear adjustment in each computational iteration and evaluating the specific correlation coefficient R^2. The best linear fit is obtained for $q_{stat} = 1.364 \pm 0.04$ with $R^2 = 0.992$ as shown in fig. 3. It should be emphasized that this q_{stat} value is fully consistent with the bounds obtained from several independent studies involving the nonextensive Tsallis framework (see, *e.g.*, [23]). The PDF for the return $\mathrm{d}_\tau M(t)$ on scale $\tau = 1$ is shown in fig. 4. On this scale we can conduct a closer investigation of a possible correlation between events . Our study used the Levenberg-Marquardt method [24,25] to compute PDFs with symmetric Tsallis distribution from eq. (4). In this adjustment, we found $B_q = 0.858 \pm 0.16$. These results are consistent with the value expected for nonlinear systems, where the random variable is the sum of strongly correlated contributions [15,18,26]. In this respect, we showed that PDFs for the return $\mathrm{d}M_1(t)$ have fat tails with a q-Gaussian shape.

On the behavior of the q-sensibility parameter. Values of the q_{sen}-index are directly related to the system instability and the entropy growth. These values can be obtained from multifractal (or singularity) spectrum $f(\alpha)$, where α is the singularity strength or Hölder exponent. Spectrum $f(\alpha)$ is derived via a modified Legendre transform, through the application of the MFDFA5 method [27]. This method consists of a multifractal characterization of a nonstationary time series, based on a generalization of the detrended fluctuation analysis (DFA). MFDFA performs best when the signal is a noise-like time series. However, there is also difficulty in visualizing the difference between walk- and noise-like time series. As suggested by [28], before application, it is necessary to run a DFA and verify if the value of the Hurst exponent is less than 1.2. For SAF data we obtain a Hurst exponent of 0.87, indicating that the MFDFA method can be employed directly without transformation of the time series.

The q_{sen}-index denotes sensitivity at initial conditions. For the present purposes, we used the expression defined by Lyra and Tsallis [29] for the relation between q_{sen} and multifractality in dissipative systems, as follows:

$$\frac{1}{1-q_{sen}} = \frac{1}{\alpha_{min}} - \frac{1}{\alpha_{max}}, \qquad (7)$$

where α_{min} and α_{max} denote the roots of the best-fit.

The multifractal characterization of these data is shown in fig. 5. These spectra $f(\alpha)$, calculated for SFA data, show a narrow Hölder exponent interval with $\alpha_{min} = 0.924 \pm 0.04$ and $\alpha_{max} = 1.051 \pm 0.11$. For multifractal

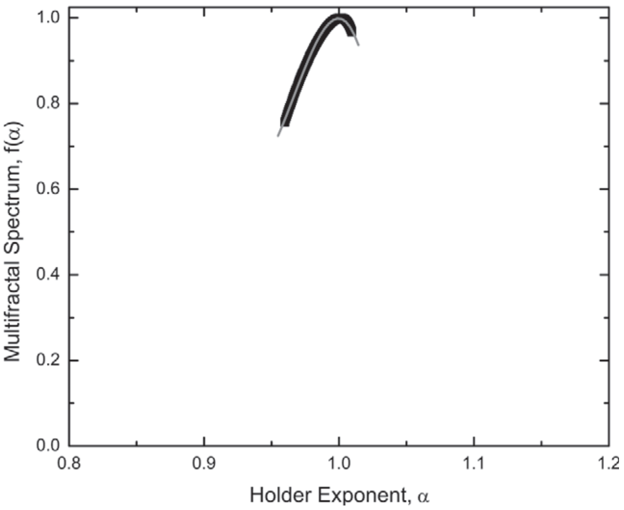

Fig. 5: The symbols are based on measurements of multifractal spectrum $f(\alpha)$ *vs.* α obtained from $M(t)$. We obtain for SAF $\alpha_{min} = 0.924 \pm 0.04$ and $\alpha_{max} = 1.051 \pm 0.11$, $q_{sen} = -6.747 \pm 0.35$. The curve represents the best ajustment using a cubic fit to the data.

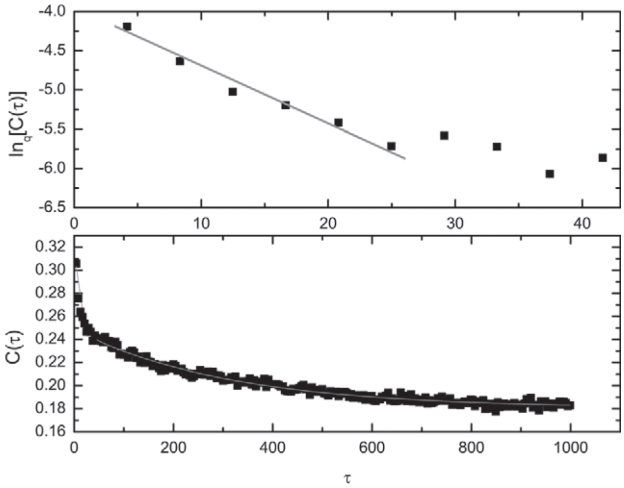

Fig. 6: Upper panel: $\ln_q$ of the autocorrelation coefficient $C(\tau)$ *vs.* time delay τ for SAF data. Lower panel: the symbols represent the autocorrelation function for our sample and the gray line represents a double exponential fit with characteristic times $t_1 = 8.42$ and $t_2 = 313.74$ yielding a ratio equal to about 37 between these two time scales ($R^2 = 0.964$, $\chi^2/\text{dof} = 1.4 \times 10^{-3}$ and time is expressed in order of hours).

spectrum width, we obtained $\Delta\alpha = \alpha_{max} - \alpha_{min}$, resulting in a value of 0.127. Using eq. (7), we found that $q_{sen} = -6.647 \pm 0.35$. This negative value indicates that its distribution exhibits weak chaos [17] in the full dynamical space of the system [17,18]. Furthermore, this figure reveals that the behavior of our sample is similar to that of a monofractal-like time series.

On the behavior of the q-relaxation parameter. The value of q_{rel}, which describes a relaxation process, can be computed from an autocorrelation coefficient as a function of scale τ defined by

$$C(\tau) = \frac{\langle [S(t_i + \tau) - \langle S(t_i)\rangle][S(t_i) - \langle S(t_i)\rangle]\rangle}{\langle [S(t_i) - \langle S(t_i)\rangle]^2\rangle}. \qquad (8)$$

In agreement with Tsallis statistics, we can estimate the value of q_{rel} by best fit on $\ln_q C(\tau)$ *vs.* scale τ, as shown in fig. 6 (upper panel), where $C(\tau)$ is given by eq. (8). In the nonextensive theory, this coefficient should decay following a power law, with increasing τ, where slope s is given by $s = 1/(1 - q_{rel})$. From this adjustment, we obtain $q_{rel} = 2.69 \pm 0.13$ for SAF data. Moyano [30] suggests that the above procedure for calculating q_{rel} only be used to describe stochastic processes with linear correlations. In other words, the autocorrelation coefficient $C(\tau)$ is not a good alternative to conveniently describe the nonlinearity of a sample [16].

On the other hand, in B-G statistics, in contrast to the nonextensive theory, the coefficient $C(\tau)$ should decrease exponentially with increasing τ, following a $C(\tau) = A_1 \exp(-\tau/t_1) + A_2 \exp(-\tau/t_2)$ relation, with t_1 and t_2 corresponding to the correlation or relaxation times. The fit shown in fig. 6 (lower panel) reveals that $t_2 \gg t_1$. As mentioned by [22], this behavior is related to local equilibrium, and then a much slower decay for larger τ. In agreement with these authors, this constitutes a necessary condition for the application of the superstatistical model, as described in ref. [31].

See [32] for further details and an extensive discussion about the estimation of the Tsallis q-Triplet.

Conclusions. – We used a new approach to nonextensive formalism for hourly measurements of earthquakes along the SAF from 1932 to 2012. From these data we were able to estimate the values of the nonextensive three-index. We found that $q_{stat} = 1.364 \pm 0.04$, $q_{sen} = -6.647 \pm 0.35$ and $q_{rel} = 2.69 \pm 0.13$. It is important to underscore that the result of the q_{stat} is consistent with the upper limit $q < 2$ obtained from several independent investigations [23]. In addition, the values of this triplet confirm the general scheme $q_{sen} \leq 1 \leq q_{stat} \leq q_{rel}$, according to the nonextensive scenario proposed by Tsallis [17]. These results reveal that this system is consistent with a nonequilibrium state, strongly suggesting that long-range correlations exist among the random variables involved in the physical process that controls seismic activity.

Finally, it is worth mentioning that the nonextensive three-index can be recalculated by considering a spatiotemporal analysis for earthquakes along the SAF. This issue will be addressed in a forthcoming communication.

* * *

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Capítulo 4

Nonextensivity at the Circum-Pacific Subduction Zones – Preliminary Studies

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Abstract

Following the fragment-asperity interaction model introduced by Sotolongo-Costa and Posadas (2004) and revised by Silva et al. (2006), we try to explain the nonextensive effect in the context of the asperity model designed by Lay and Kanamori (1981). To address this issue, we used data from the NEIC catalog in the decade between 2001 and 2010, in order to investigate the so-called Circum-Pacific subduction zones. We propose a geophysical explanation to nonextensive parameter q. The results need further investigation however evidence of correlation between the nonextensive parameter and the asperity model is shown, i.e., we show that q-value is higher for areas with larger asperities and stronger coupling.

I. Introduction

The study of earthquakes and geological faults is very complex since it involves many variables such as deformation, rupture, released energy, land features, heterogeneity in seismogenic plate interface, among others (Kawamura et al., 2012), even in planetary (Sarlis, 2011) and regional scale, considering not only the Earth (Vallianatos et al., 2011). In this concern, many different tools have been used for a better describing and understanding of earthquakes, many of them empirically developed, as the Omori law (Omori, 1894) for temporal distribution of aftershocks, the Gutenberg–Richter law (Gutenberg e Richter, 1944) for relationship between frequency and magnitude and the Bath law (Bath, 1965) for the constant difference in magnitude between a main shock and its largest aftershock.

Recently, the application of power law statistics was also used on earthquakes and faults studies. In 2012, an entire volume of Acta Geophysica (v. 60) was dedicated to

"Statistical Mechanics in Earth Physics and Natural Hazards", that testifies the relevance of the model developed by Tsallis (1988) as a "methodological tool to describe entities with fractal or multi-fractal distribution of their elements and where long-range interactions or intermittency are important, as in the Earth's systems are" (preface by Telesca and Vallianatos, 2012).

By starting from Boltzmann-Gibbs classical model, Tsallis (1988, 1995a, 1995b, 2009) developed a different model that can be applied to systems in non-equilibrium state, complex behavior and fractal pattern – characteristics present in earthquakes and geological faults.

The first connection between the nonextensive formalism in Seismology was done by Ian Main (1995). In this pioneering paper, the author used cumulative frequency to statistic evaluation of earthquakes, by classifying them in subcritical, critical, and supercritical behavior accordingly the heterogeneity and driving velocity. In studying aftershocks distributions, Abe and contributors made a review in the use of nonextensive approach (Abe and Okamoto, 2001), made the analysis of data from the full catalogue of California and Japan (Abe and Suzuki, 2003, 2005) and, more recently, introduced to that the concept of complex earthquakes network and non-Markovian nature (Abe and Suzuki, 2009, 2012). Since those works, many used Tsallis statistics to develop models in Seismology, e.g.: Sotolongo-Costa and Posadas (2004), Silva et al. (2006) and Darooneh and Mehri (2010) proposed earthquake energy distributions using Tsallis nonextensive approach, considering the energy released by each earthquake is proportional to the distribution of the size of fragments (assumed differently in each model) between tectonic plates; Kalimeri et al. (2008) evaluate pre-seismic emissions; Darooneh and Dadashinia (2008) applied it in spatial-temporal distribution between successive earthquakes; Vallianatos (2009) use it to estimate a risk function of natural hazards; Vallianatos and Sammonds (2011) developed and tested a model for the fault length distribution in the Valles Marineris extensional province, Mars; in 2013 they also suggest the existence of a coherent global scale intermediate-term nonextensive tectonic premonitory of impending mega-earthquake processes in the lithosphere; de Freitas et al. (2013) identified the Tsallis' q-Triplet (q_{stat}=1.36±0.04, q_{sen}=-6.65±0.35, q_{rel}=2.69±0.13) revealing a strong evidence that the seismic activity has a hierarchical structure on small scales.

Nevertheless, by accepting the nonextensive parameter q as a good adjustment parameter, in different approaches, any geophysical explanation has not been presented. A few considerations about the *q*-value indicate that it is a quantitative measure of the length scale of interactions: *q*~1 indicates short-range spatial correlations; as *q* increases, the physical state becomes more unstable, the internal energy which grows faster than the number of elements (Tsallis, 2012); high values of *q* mean the fault planes are not in equilibrium and more earthquakes can be expected (Sotolongo-Costa and Posadas, 2002). Villar et al. (2007) concluded that *q*-values for earthquakes data sets seemed to be always between 1.6 and 1.7. As the models found in both works can be considered a modification of the Gutenberg-Richter law, Vallianatos (2009) and Sarlis et al. (2010) reduced the model to find a relation between q and the parameter *b*. The b-value can be related to some important aspects as stress, material heterogeneity, focal mechanism and thermal gradients in fault region (Kulhanek, 2005). This is also suggested by laboratory experiments as seen in Vallianatos et al. (2012) as well as by the results of natural time analysis of seismicity (Varotsos et al., 2012).

At an attempt to find a relation between nonextensive effect and geophysical characteristics, in this paper we consider Lay and Kanamori (1981) empirical approach (see also the earlier work by Uyeda and Kanamori (1979) as well as the recent review by Uyeda, 2013) that define and describe some subduction zones along the Pacific Ring of Fire, the main seismic region on Earth. Each zone was described accordingly several characteristics and clustered in a general classification to define an asperity model. Therefore, our aim is to answer the following questions: is it the case that *q*-value has correlation with the circum-Pacific subduction zones? From the geophysical point of view, how to explain the connection between the nonextensive parameter and this asperity model? For this analysis, we considered 142,280 events in magnitude interval $1 \leq m \leq 9$, taken from the National Earthquakes Information Center Catalog (NEIC-USGS) during the decade from 2001 to 2010. The catalog offers data in different magnitudes types (M_W, M_B, M_S, M_L, M_D) for the same event and we choose to follow NEIC automatic ranking. We consider that use this sequence makes no significant impact on the final result of this paper because, in general, the differences between different magnitudes types are small. Any kind of impacts can be object of further investigation.

II. Nonextensive Formalism

Recollecting the theoretical background in statistical mechanics, it's know that in 1988, Tsallis proposed a generalized form of the Boltzmann-Gibbs (BG) entropy, given by

$$S_q = -k_B \sum_{i=1}^{W} p_i^q \ln_q p_i \qquad (4.1)$$

where k_B is Boltzman's constant, p_i is a set of probabilities and W is the total number of microscopic configurations. Indeed, in the limit q=1, we recover the celebrated BG entropy,

$$S_{BG} = -k_B \sum_{i=1}^{W} p_i \ln p_i \qquad (4.2)$$

But, differently of the BG entropy, S_q is said to be *nonadditive*. That means for two independent subsystems A and B:

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B) \qquad (4.3)$$

Given S_q is nonnegative, it follows that:

$S_q(A+B) \geq S_q(A) + S_q(B)$, if q<1 (case called superadditive); (4.3a)

$S_q(A+B) \leq S_q(A) + S_q(B)$, if q>1 (subadditive). (4.3b)

In order to investigate the connection between the nonextensive effects and the asperity model, let's now consider the main aspects of the nonextensive model for earthquakes. In this regards, Sotolongo-Costa and Posadas (2004) and Silva et al (2006) have proposed the q-entropy denoted by

$$S_q = -k_B \int p^q(\sigma) \ln_q p(\sigma) d\sigma \qquad (4.4)$$

where k_B is the Boltzmann constant, $p(\sigma)$ is the probability of find a fragment of surface σ. In the same way, when q=1, the equation becomes the entropy definition by Boltzmann-Gibbs. In particular, the entropic index q denotes a measure of the degree of nonextensivity in the system, caused by different processes, such as multifractality, long-range memory and interactions.

A nonextensive formalism is considered adequate for earthquake models since the process of violent fractioning is very probably a nonextensive phenomenon, leading to an accelerated grown of internal energy and long-range interactions among the parts of the object being fragmented (Tsallis, 2012). Sotolongo-Costa (2012) explains also that the nonextensivity becomes linked to stick-slip processes between tectonic plates.

As explained by Sotolongo-Costa and Posadas (2004) and Silva et al (2006), in the contact of the plates, surfaces are irregulars and there is constant formation and consumption of fragments in diverse shapes, what requires a special formalism that considers the size distribution of fragments. The distribution of energy by Silva et al. (2006) uses an energy scale of $\varepsilon \sim r^3$, i.e. the energy distribution of earthquakes generated by this mechanism can reflect the volumetric distribution of the fragments between plates. The model developed is similar to the modified Gutenberg-Richter law and given by:

$$\log(N_{>m}) = \log N + \left(\frac{2-q}{1-q}\right)\log\left[1-\left(\frac{1-q}{2-q}\right)\left(\frac{10^{2m}}{a^{2/3}}\right)\right] \qquad (4.5)$$

where $N_{>m}$ is the number of earthquakes with magnitude larger than m, N is the total number of earthquakes and a is the proportionality constant between the fragments volume and released energy.

Telesca (2010b, 2012), Telesca and Chen (2010), Telesca (2011), Telesca et al. (2012) and Valverde-Esparza et al (2012) used the same formulation to analyze the seismicity in Italy, Taiwan, Southern California, Morroco and Mexico, respectively. The same model was also used by Papadimitriou et al (2008) for preseismic electromagnetic emissions, by Telesca (2010a) for analyze seismic sequences and by Vallianatos et al. (2011 and 2013) in laboratory scale and analyzing volcanic seismicity respectively. In particular, Vallianatos et al (2014) found that the q value associated with spatial correlations exhibits a considerable increase when the order parameter of seismicity introduced in the frame of the new time domain, termed natural time (Varotsos et al. 2011), attains a critical value (Varotsos et al. 2008, Sarlis et al. 2008, Varotsos et al. 2011) showing the entrance of the system in the final pre-earthquake stage.

III. **The Asperity Model and the Circum-Pacific Subduction Zones**

After some laboratory experiments on frictional sliding, Byerlee (1970) proposed a first model based on the concept of asperity. Further, Sholz and Engelder (1976) deepened the idea showing that this mechanism is responsible for the various time and velocity dependent properties of rock friction, being an important mechanism for stick-slip sliding. The main suggestion in both works was the two sides of a fault are held together by asperities: areas with a higher stress than the surroundings on that fault plane. Lay and Kanamori (1981) appealed to this concept considering that on the basis of the rupture length of an earthquake it’s possible to categorize different subduction zones in major groups. Indeed, they considered earthquakes with rupture length over 200km that occurred in some specific regions on the circum-Pacific (figure 4.1) and concluded that the regional characteristics of each one can be modeled in terms of a stress distribution and the interaction of asperities. Asperity size and stress distribution govern the degree of loading of adjacent asperities when a large asperity does not stand, i.e. the failure of an asperity would cause an increase in stress on the adjacent asperities. They defined their model with four main categories (and a transitional one) as described in table 4.2 and figure 4.2. The general structure of categories in the extremes of this classification is illustrated by figure 4.3.

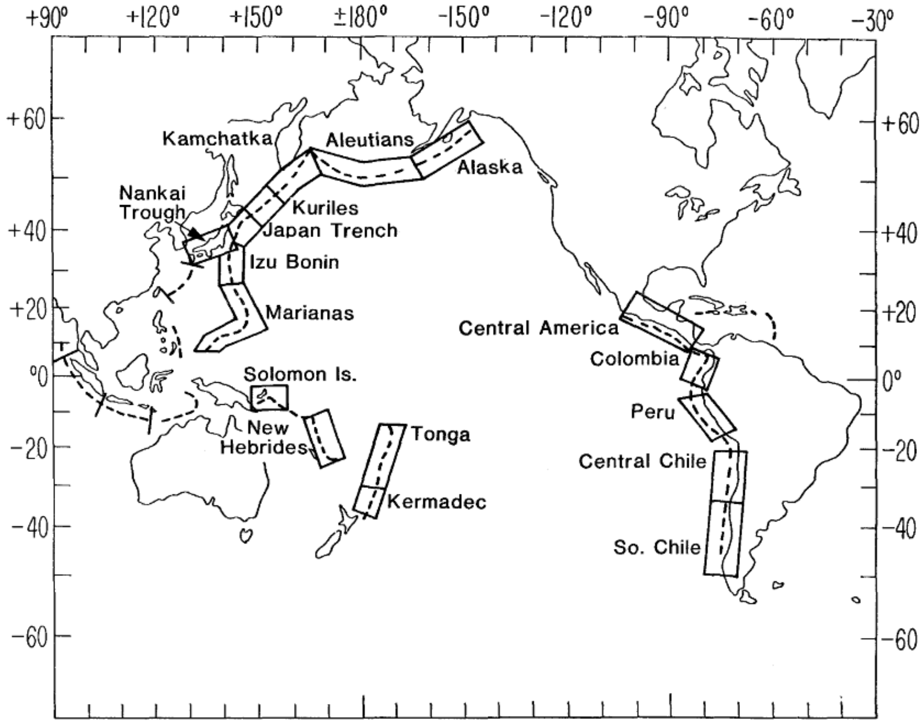


Figure 4.1 – Circum-Pacific Subduction Zones, indicating areas considered in Lay and Kanamori (1981, figure 3.1).

Areas	Latitude		Longitude	
	North	South	East	West
Tonga	-12	-30	-168	177
Kermadec	-30	-42	-174	172
New Hebrides	-9	-24	175	163
Solomon Islands	-2	-10	160	145
Marianas	28	8	150	135
Kuriles	48	44	158	145
Kamchatka	58	48	166	155
Aleutians	57	48	-165	166
Alaska	60	51	-150	-165
Central America	21	7	-74	-108
Colombia	7	-5	-74	-82
Peru	-5	-17	-68	-85
Central Chile	-17	-35	-65	-78
South Chile	-35	-49	-68	-78

Table 4.1 - Areas consonant the map at figure 4.1.

Categories	Areas	Characteristics
1	Southern Chile, Southern Kamchatka, Alaska, Central Aleutians	Regular occurrence of great ruptures (≥500km long). Large amount of seismic slip.
2	Western Aleutians (Rat Islands), Colombia, Nankai Trough, Solomon Islands	Variations in rupture extent, with occasional rupture 500km long. Close clustering of large events and doublets.
2-3	New Hebrides, Central America	Intermediate size and small events with no great earthquakes, but clustering of activity.
3	Kuriles Islands, Northeast Japan Trench, Peru, Central Chile	Repeated ruptures over limited zones. No great events. Large component of aseismic slip, or subducting ridges.
4	Marianas, Izu-Bonin, Southeast Japan Trench, Tonga, Kermadec	Large earthquakes are infrequent or absent. Back-ark spreading and large amounts of aseismic slip are inferred.

Table 4.2 – Subduction zones characteristics (from Lay and Kanamori 1981, table 4.2, with few alterations; used with second author's permission). In this study the Aleutians were considered as one area (zone 1), including western and central regions.

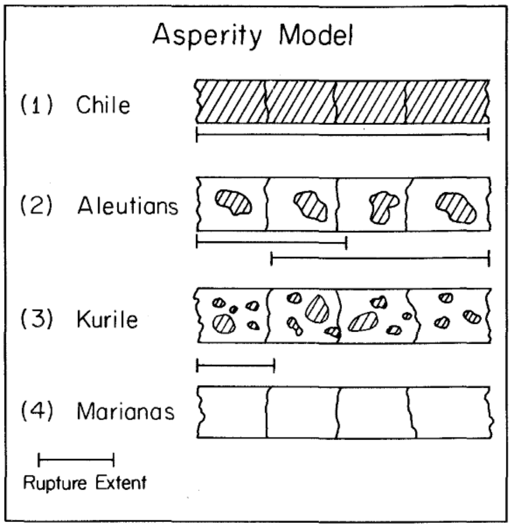
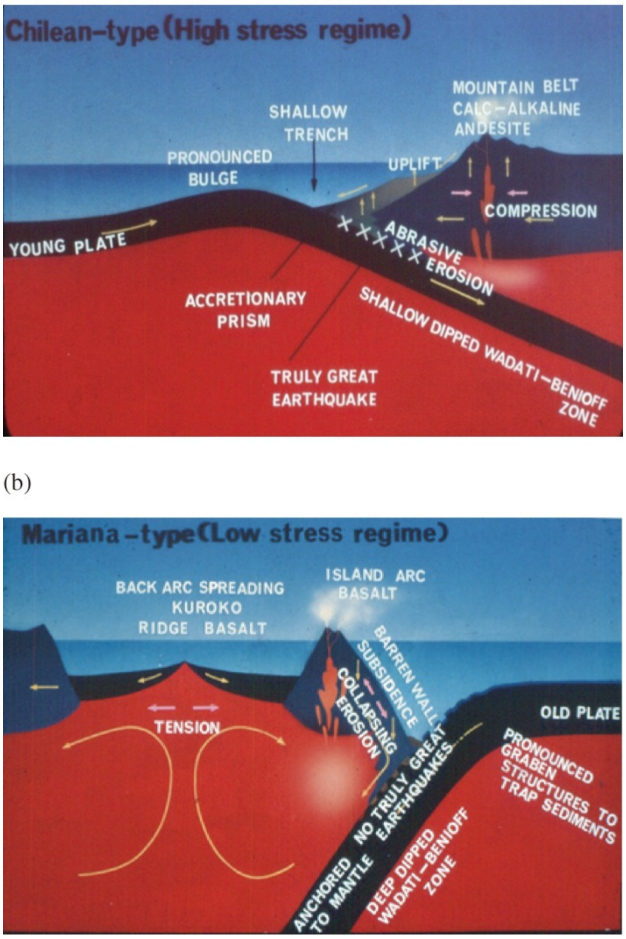


Figure 4.2 – An asperity model indicating the different nature of stress distribution in each subduction zone category. The hatched areas indicate the zones of strong coupling. (from Lay and Kanamori 1981, figure 4.4).

(a)



(b)

Figure 4.3 – Schematic comparison between the Chilean and the Mariana types subduction zones (from Uyeda, 2013).

A briefly description of each zone, as presented by Lay and Kanamori (1981) and Kanamori (1986) is shown:

- In the Chile-type behavior (zone 1), the lithospheric plates are strongly coupled, and the asperity distribution is basically uniform over the contact area, because of that, rupture occurs in great events. Sediments are scraped off on subduction and form an accretionary prism, what causes excess trend sediments. The trench and the dip angle of Wadati-Benioff are usually shallow.
- For Aleutians-type (zone 2 – considering the Western part), the asperities are comparatively large, but they are surrounded by weak zones. The relatively homogeneity causes some large ruptures but smaller ruptures also occur, possibly as doublets.

- Because of the relatively small size of asperities and heterogeneities in Kuriles-type zones (zone 3), there is an inhibition of large rupture development generating complicated ruptures and foreshock-aftershock activity.
- The last category (Marianas-type – zone 4) is characterized by no large asperities, so weak coupling and no large earthquakes. There is a heterogeneous contact plane that decreases the strength of mechanical coupling; it is called "host-and-graben structures". The trench and the dip angle of Wadati-Benioff are usually deeper. The back-arc basin is commonly found for this type of subduction zones.

We defined the borders of each area (table 4.1) considering the best rectangle accordingly figure 4.1 and using it as an approximation to download the data. This approach is general and work well for most of the areas. But for the areas over Japan and Kamchatka, the rectangle wasn't precise enough to delimit the area properly.

After more than 30 years, this model remains relevant and useful, as seen e.g. in Müller and Landgrebe (2012).

IV. Results and discussion

Now, let us discuss the connection between nonextensive models for earthquakes introduced by Sotolongo-Costa and Posadas (2004), Silva et al. (2006) and Vilar et al. (2007) and the asperity model designed by Lay and Kanamori (1981). For a better understanding of the q parameter, the data was separated accordingly the areas delimited in table 4.1 and adjusted. Considering our data set, we also found good adjustments and the results also sustain the limits for q between 1.6 and 1.7, suggested in previous results (see Vilar et al. 2007).

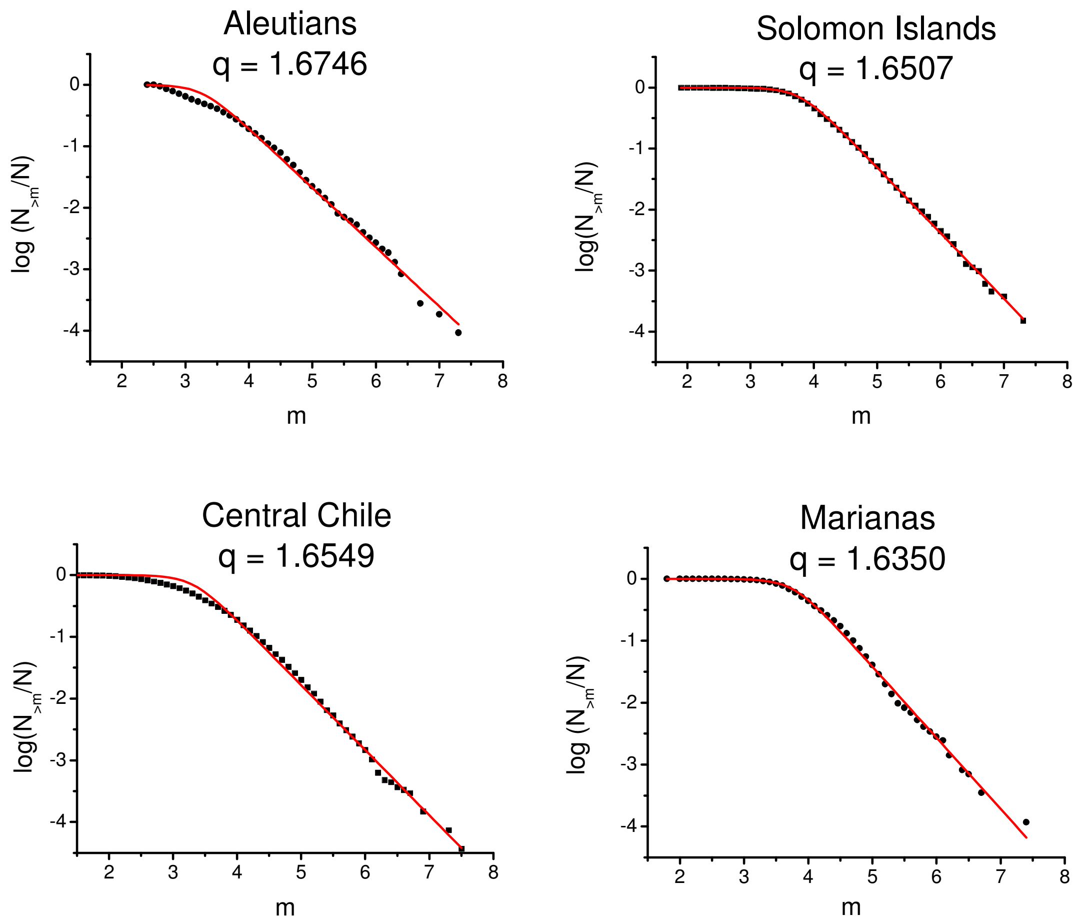


Figure 4.4 –The relative cumulative number of earthquakes as a function of the magnitude m. We show the graphics for Aleutians, Solomon Islands, Central Chile and Marianas, representing the zones 1, 2, 3 and 4, respectively.

The results are presented in table 4.3, considering the areas defined at table 4.1 and the classification presented at table 4.2. The table 4.3 contains the best fitting for q parameter and the fitting standard deviation (calculated using the Levenberg-Marquardt algoritm) for *q* which shows goodness of our fitting for the data sets.

Subduction Zone	Area	q	σ_q	a	σ_a
1	South Chile	1.6978	0.0091	6.28E+10	3.7E+10
1-2	Aleutians	1.6746	0.0035	1.80E+10	3.8E+09
1	Alaska	1.6656	0.0055	2.50E+09	7.5E+08
2-3	New Hebrides	1.6645	0.0023	1.06E+12	1.5E+11
3	Peru	1.6560	0.0048	1.45E+12	3.2E+11
3	Kuriles	1.6557	0.0025	1.48E+11	2.1E+10
3	Central Chile	1.6549	0.0026	2.19E+10	3.6E+09
2	Colombia	1.6548	0.0088	2.17E+11	8.5E+10
2	Solomon Islands	1.6507	0.0013	5.42E+11	3.8E+10
1	Kamchatka	1.6448	0.0041	1.08E+11	2.1E+10
4	Marianas	1.6350	0.0028	4.57E+11	6.1E+10
2-3	Central America	1.6341	0.0038	5.8E+10	1.2E+10
4	Tonga	1.6340	0.0037	5.5E+11	1.1E+11
4	Kermadec	1.6128	0.0040	5.7E+10	1.1E+10

Table 4.3. Values to q and a, ranked in decreasing order of q (σ_q and σ_a are the errors for the adjustments). The subduction zones for each area are indicated.

Indeed, a correlation between the subduction zones and the q-values to each area it seems to exist. The two exceptions are Kamchatka (whose chosen area wasn’t adequated, as included many data from Kuriles Islands) and Central America (transition zone with behavior completely different from the zone classified the same way – New Hebrides). Sorting the zones by q-values, zone 1 noticeably presents higher values of q, followed by zones 3, 2 and 4 respectively. As a matter of fact, the mechanical coupling between the plates does the role of the nonextensive effect, i.e. this coupling produces a mechanical interaction via the stress distribution. This statement is corroborated by Vallianatos and Sammonds (2010) that using nonextensive approach presents evidence that the self-organizing process is the prevailing aspect on evaluating the plates structure.

From the nonextensive framework point of view, that means the zones 1, 3, 2 and 4 present a fragment distribution with q-values decreasing with stress distribution between the plates. So, even if a heterogeneous stress distribution (zone 3) is not likely to generate very large ruptures or multiple events, as commented by Lay and Kanamori (1981), the strength of coupling in the area is remarkable in terms of a nonextensive behavior, which was higher than those calculated for zone 2 areas. Therefore, the q-values reinforce the sub-extensive nature of the phenomenon and present indications of a connection with Lay and Kanamori asperity model.

Considering the similarity between this model and the modified Gutenberg-Richter law, we can also evaluate the parameter b, that can be related to many relevant aspects. Carter and Berg (1981) defended a qualitative relation between b and stress considered the value presented a periodicity of 6-8 years. A mathematical relation between the parameters q and the b, for this model, was described for Sarlis et al. (2010) and is expressed in equation 4.6.

$$b_S = 2\,(2 - q)\,/\,(q - 1) \qquad (4.6)$$

Results for b-value are presented in the next table. Also it's presented the value calculated by Gutenberg-Richter law (b_{GR}), manually using Zmap (Wiemer, 2001) and considering only the range of data that presents an approximately linear behavior. They are typically higher than the values found by Carter and Berg (1981), using the maximum likelihood method with data from 1963 and 1975. But both b_S and b_{GR} approximately keeps showing higher values of *b* for zone 4 and lower values for zone 1.

Subduction Zones	Areas	q	b_S	b_{GR}
1	South Chile	1.698	0.866	0.93
1-2	Aleutians	1.675	0.965	0.92
1	Alaska	1.666	1.005	0.89
2-3	New Hebrides	1.665	1.010	1.01
3	Peru	1.656	1.049	0.98
3	Kuriles	1.656	1.050	1.70
3	Central Chile	1.655	1.054	1.07
2	Colombia	1.655	1.054	1.11
2	Solomon Islands	1.651	1.074	1.07
1	Kamchatka	1.645	1.102	1.09
4	Marianas	1.635	1.150	1.20
2-3	Central America	1.634	1.154	1.20
4	Tonga	1.634	1.155	1.21
4	Kermadec	1.613	1.264	1.13

Table 4.4 – *b*-values for each area defined in table 3.1.

That results evidence that the instability in the system can be described by nonextensive models, what means, as proposed by Tsallis (2012) that fast increase of the internal energy leads to a behavior that differs itself from the extensivity, as seen in expression (3b). And this deviation is greater the higher the asperitiy in the region, i.e. higher the asperity, more nonextensive is the system.

V. **Conclusions**

In this paper we aimed answer two questions using the nonextensive model developed by Sotolongo-Costa and Posadas (2004) and revised by Silva et al. (2006): is it the case that *q*-value has correlation with characteristics of subduction zones? From the geophysical point of view, how to explain the connection between the nonextensive parameter and the asperity model from Lay and Kanamori (1981)?

This preliminary study indicates the possibility of a geophysical interpretation of *q*-value, relating it with the subduction zones categorized by Lay e Kanamori (1981). These zones were identified empirically, considering the occurrence of ruptures, seismic / aseismic slips, coupling, among others regional characteristics, leading to a model based in terms of a stress distribution and the interaction of asperities. As seen in section IV, the q-value is higher for zone 1 and decreases in the following order: 3, 2 and 4, with few exceptions. It was shown, in agreement to previous studies (Sotolongo-Costa and Posadas 2004, Silva et al. 2006, Telesca 2010 and others) that earthquakes have a nonextensive behavior presenting $q>1$ (between 1.6 and 1.7, as indicated by Villar et al. 2007), what indicates nonextensive behaviour.

The explanation for the presentation of a higher value of q in zone 3 than zone 2 may be in the heterogeneity of the stress distribution for Kuriles-type zones that represents smaller ruptures but appears to present a significant mechanical coupling resulting in a more expressive formation and consumption of fragments when the whole contact area is considered.

Considering the characteristics of the asperity model by Lay and Kanamori and the results presented by this work, it seems the mechanical coupling between the plates plays a fundamental role to lead the system to a nonextensive behavior. The *q*-values for zones 1 and 4 are very distinctive and show clearly that zones with strong coupling presents higher values of q while zones with weak coupling have lower values of q. But the intermediary zones aren't so easily distinguished.

Further investigation is needed, especially in the study of intermediate zones (2 and 3) and a more accurate definition of the areas, but it is already clear that the *q* value isn't just a mathematical parameter but it can also give geophysical indication for understanding the behavior of a seismically active zone, especially in terms of coupling and stress distribution.

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Capítulo 5

Analysis of Four Brazilian Seismic Areas Using a Nonextensive Approach

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Analysis of Four Brazilian Seismic Areas Using a Nonextensive Approach

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Abstract

We analyze four seismic areas in Brazil using the nonextensive model revised by Silva et al (2006) and the data from the Brazilian Seismic Bulletin between 1720 and 2013. Two of those regions are contrasting zones, while the other two are dominated by seismic active faults. We notice that intraplate seismic zones present q-values similar to others fault zones, but the adjustment in contrast areas results in higher values for this parameter.

I. Introduction

Since Main (1995) suggested used cumulative frequency to statistic evaluation of earthquakes, the nonextensive formalism developed by Tsallis (1988) has been widely used in Seismology, as reviewed by Abe and Okamoto (2001). An interesting seismic model using Tsallis statistics was developed by Sotolongo-Costa and Posadas (2004), and then adapted in different ways, for example, by Silva et al. (2006), Telesca (2012), and Darooneh and Mehri (2010). As a matter of fact, all above models considered that there is a proportion between the release of energy in an earthquake and the size of the fragments.

Silva et al. (2006) applied its model in two intraplate areas: Samambaia fault (q=1.60) and New Madrid fault (q=1.63). Vilar et al. (2007) directed their work to fault systems. Vallianatos and Sammonds (2011) developed and tested a model for the fault population in Mars. More recently, Scherrer et al. (2014), have shown an relation between the non-extensive parameter q and the nature of stress distribution in different subduction zones, especially considering areas with very strong and very weak coupling, it has been investigated the role of statistical correlations applying Silva et al. (2006) model in dominant fault systems on stable regions. This paper aims to use non-extensive formalism to describe some of those areas. Two of them are contrasting zones: Brazilian Fold Belts versus Craton (BFBvC) and Margin Passive in oceanic and continental crust (MP). The other two are dominated by seismic active faults: Transbrazilian Lineament (LT) and the Borborema Province (PB).

We used data between 1720 and 2013, from the Brazilian Seismic Bulletin. This catalog is built with data from the universities of São Paulo (USP), Brasília (UnB), Rio Grande do Norte (UFRN), and the Technological Research Institute (IPT) of the state of São Paulo and contribution also from State University of São Paulo (UNESP) and National Observatory (ON). The range of magnitudes is from 2.0 to 6.2.

The remainder of this work is summarized as follows: in the next section, we present the nonextensive earthquake model; in the third section, we show our working seismic sample; the main results and discussions are exhibited in the following section; and, finally, conclusions are put forth in the last section.

II. Nonextensive Formalism

The nonextensive approach developed by Tsallis (1988) is based on the mathematical generalization of entropy,

$$S_q = -k_B \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1} \qquad (5.1)$$

where k_B is Boltzman's constant, p_i is a set of probabilities and W is the total number of microscopic configurations. Indeed, in the limit q=1, we recover the celebrated BG entropy,

$$S_{BG} = -k_B \sum_{i=1}^{W} p_i \ln p_i \qquad (5.2)$$

For earthquakes, Sotolongo-Costa and Posadas (2004) and Silva et al. (2006) have proposed to describe q-entropy as

$$S_q = -k_B \int p^q(\sigma) \ln_q p(\sigma) d\sigma \qquad (5.3)$$

where k_B is the Boltzmann constant, $p(\sigma)$ is the probability of find a fragment of surface σ. Again, when q=1, the equation becomes the entropy definition by Boltzmann-Gibbs.

The *q-value* outlines the degree of nonextensivity in the system which can be generated by different processes, such as multifractality, long-range memory and interactions.

We consider here the model by Silva et al. (2006), considering that the energy distribution of earthquakes is proportional to the volumetric distribution of the fragments in the fault. The key expression of the model is the so-called relative cumulative number of earthquakes as a function of the magnitude m, i.e.

$$\log(N_{>m}) = \log N + \left(\frac{2-q}{1-q}\right)\log\left[1-\left(\frac{1-q}{2-q}\right)\left(\frac{10^{2m}}{a^{2/3}}\right)\right] \qquad (5.4)$$

where $N_{>m}$ is the number of earthquakes with magnitude larger than m, N is the total number of earthquakes and the parameter a is the proportionality constant between the fragments volume and released energy. Considering magnitudes above the magnitude of completeness, the behavior here described is similar to the Gutenberg-Richter law.

III. The Studied Areas

In intraplate seismicity there is influence from the basement structure, the formation process, deposits, among others external factors that can alter the stress distribution. Ferreira et al. 2008 lists the major difficulties in the study of those areas related with few information about many aspects: (1) the type of weakness zone being reactivated, (2) the attitude, geometry, and location of preexisting weakness zones, (3) the reactivation history of faults, and (4) the variation in faulting regime expressed by a diversity of focal mechanisms. In particular, in Brazil the stress field is still poorly known due to the small number of well-determined focal mechanisms and few in-situ stress measurements (Assumpção et al. 2014).

Almeida et al. (1981) divided Brazilian territory in 10 structural provinces based on the basement rocks and the sedimentary cover. This division was revised in Almeida et al. (2000), that presented the state-of-the-art of the geological knowledge on the origin and evolution of the South American Platform, but kept the same provinces. For this paper, considering that the Brazilian Seismic Bulletin still has a low spatial data density, just the four mentioned in the introduction were selected considering their seismicity: Brazilian Fold Belts versus Craton (BFBvC), Margin Passive in oceanic and continental crust (MP), Transbrazilian Lineament (LT) and the Borborema Province (PB). They can be easily identified by figure 4.1 and their characteristics are briefly described in table 4.1 (more details can be found in Almeida et al. 1981, Berrocal et al. 1984, Almeida et al. 2000, Byzzi et al. 2003, Assumpção et al. 2014).

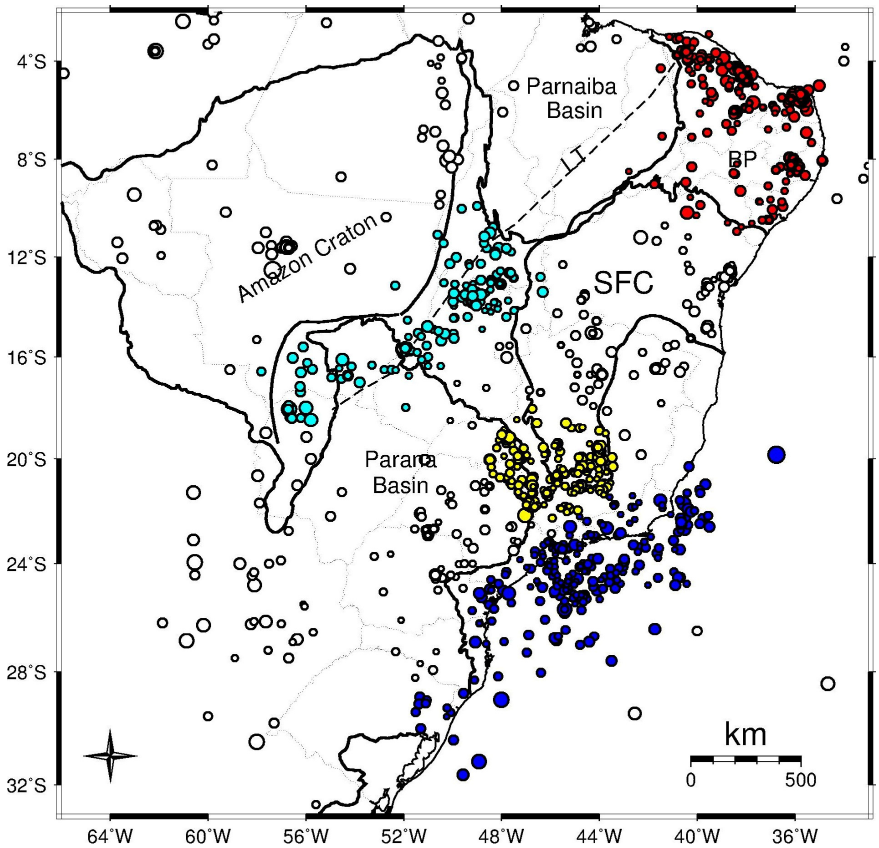

Figure 5.1 – Seismological Map of Brazil. Brazilian Fold Belts versus Craton (BFBvC – in yellow), Passive Margin in oceanic and continental crust (MP – in dark blue), Transbrazilian Lineament (LT – events in light blue and dashed line marks the dominant feature) and Borborema Province (PB – in red). SFC is the craton San Francisco.

Studied Zones	Geology	Tectonic		Type
Borborema Province (PB)	Constituted by branching systems of orogens developed in Neoproterozoic (Brazilian domain). Position intercratons and complex embasement is caracteristic. Presents rich domains in supracrustals. It's tectonics is ductil and ruptil, regeneering the cratons borders.	Important faulted and shear zones. Recurrent swarms and aftershock sequences, some lasting for several years, are very common.	Active fault system	Atlantic Shield
Brasília fold belt (BFBvC)		Mainly affected by Braziliano folding cycle.The foreland domain of the Brasília foldbelt has thicker crust.	Contrast craton and fold belt	Atlantic Shield
Transbrasilian lineament (LT)		Oldest rocks in the center and metamorphic sequences at eatern and western borders. Almost no Phanerozoic deposits.	Active fault system	Central Brazilian Shield
Passive Margin (MP)	Formed in Meso-Cenozoic (the youngest of Brazilian Provinces). It's tectonic formation is the break up of Pangea.	It has 70% more earthquakes (magnitudes above 3.5) than the average stable continental region.	Contrast oceanic and continental crust	Sedimentary cover

Table 5.1 – General characteristics of each zone considered in this study.

IV. Data analysis

Four data sets were adjusted: one for each of areas described in the previous sections, between 1720 and 2013. We found good adjustments with q-values ranged from 1.65 to 1.75. The fault regions (LT and PB) present values that are close to those found in Vilar et al. (2007). But for the contrasting zones, the nonextensive parameter is quite higher. The results are shown in table 5.2, figures 5.2 and 5.3.

	q	σ_q	a	σ_a
Brazilian Fold Belt	1.7080	0.0083	2.29E+08	8.2E+07
Passive Margin	1.7469	0.0049	2.50E+07	7.4E+06
Transbrazilian Lineament	1.6634	0.0070	6.6E+07	1.6E+07
Borborema Province	1.6574	0.0070	2.59E+08	7.5E+07

Table 5.2 – Values of *q* and *a* for the adjustment in each area and their respective errors.

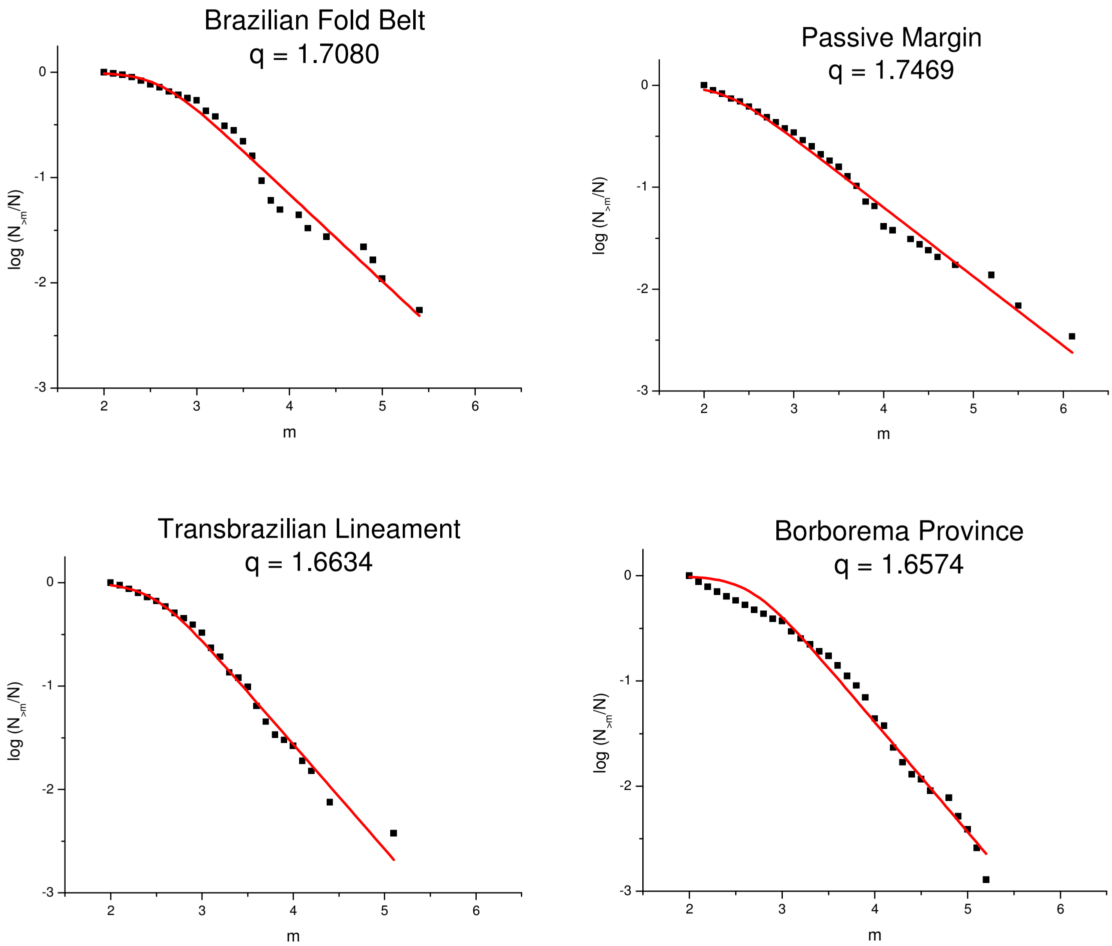

Figure 5.2 - The relative cumulative number of earthquakes as a function of the magnitude *m*, for each region: Brazilian Fold Belts versus Craton (BFBvC), Margin Passive in oceanic and continental crust (MP), Transbrazilian Lineament (LT) and Borborema Province (PB).

It's noticeable that the contrast areas present superior values for the nonextensive parameter than the fault areas. Data from those areas also oscillated more around the adjusted curve.

As this model is similar the modified Gutenberg-Richter law, a mathematical relation between the parameters *q* and the *b*, was described for Sarlis et al. (2010) and is expressed by:

$$b_S = 2\,(2 - q)\,/\,(q - 1) \qquad (5.5)$$

Results for b-value were calculated using equation 5 and also Gutenberg-Richter law (b_{GR}), manually using Zmap (Wiemer, 2001) and considering only the data from de magnitude of completeness (M_c) on. The results are presented in table 5.3.

	q	b_S	b_{GR}	M_c
Brazilian Fold Belt	1.7080	0.8249	0.83	3
Passive Margin	1.7469	0.6777	0.67	2
Transbrazilian Lineament	1.6634	1.0148	1.02	3
Borborema Province	1.6574	1.0423	0.88	2

Table 5.3 – *q* and *b*-values (calculated by equation 5 and for the Gutenberg-Richter law) for each area defined in table 4.1, and the magnitude of completeness considered to calculate b_{GR}.

V. Conclusions

We have addressed the earthquake model developed in Sotolongo-Costa and Posadas (2004) and Silva et al. (2006) in the context of four seismic areas in Brazil. By using the data from the Brazilian Seismic Bulletin between 1720 and 2013, we have shown that for values of the nonextensive parameter of the order of 1.65 to 1.75, the model provides an excellent fit to the fault regions (LT and PB) and to the contrasting zones (BFBvC and MP). We have also noted that the predicted values for q are very similar to ones obtained in Vilar et al. (2007), however, when considered the contrasting zones, the nonextensive parameter is quite higher. Indeed, the higher q-value for the regions with great contrast indicates that the density difference may produce more fragmentation and instability, with a major influence in nonextensive behavior. As q differs from unity, the physical state goes away from equilibrium states, indicating that the analyzed cases are out of equilibrium and more earthquakes can occur.

Finally, it is worth mentioning that the nonextensive parameter calculated for the four seismic areas in Brazil is in full agreement with the upper limit $q < 2$ obtained from several independent studies involving the Tsallis nonextensive framework (see, e.g. Carvalho et al. (2009), Liu and Goree (2008), Khachatryan et al. (CMS Collaboration, 2010), etc). In addition, these results reveal the nonextensive approach adjusts very satisfactorily and robustly the real case also for earthquake intraplate, showing that the Tsallis formalism is unquestionably a powerful tool to the analysis of the equilibrium phenomena and complex systems.

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Capítulo 6

Nonextensivity at the Circum-Pacific Subduction Zones
The Influence of Magnitudes Types

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Nonextensivity at the Circum-Pacific Subduction Zones – The Influence of Magnitudes Types

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Abstract

Following Scherrer et al. (2014), we reanalyze the nonextensive behavior over the circum-Pacific subduction zones evaluating the impact of using different types of magnitudes in the results. We used the same data range of our previous work, from the NEIC catalog in the decade between 2001 and 2010. Even considering different data sets, the correlation between q and the subduction zones is perceptible, but the values found for the nonextensive parameter in the data sets considered present an expressive variation. The data set with surface magnitude exhibits the best adjustments.

I. Introduction

In our previous work (Scherrer et al., 2014) we presented a brief review about the use of Tsallis Statistics in Seismology and used a model based on this approach (developed by Sotolongo-Costa and Posadas, 2004 and revised by Silva et al., 2006) to relate the nonextensive paramenter with the subduction zones along the Pacific Ring of Fire as described by Lay and Kanamori (1981). For that study, we used data of 142,280 events in magnitude interval $1 \leq m \leq 9$, taken from the National Earthquakes Information Center Catalog (NEIC-USGS) during the decade from 2001 to 2010 and we followed the NEIC automatic ranking, independent of the magnitude type (M_W, M_B, M_S, M_L, among others) used to measure each event. We consider at time that this would makes no significant impact on the final result because, in general, the differences between different magnitudes types are small. But, what else there is a difference? What if a specific magnitude type provides a better representation of the non extensive behavior in earthquakes? In this paper we considered four different types of magnitude (M_W, M_B, M_S, M_L) and made the nonextensive model fitting to see if the results present significate variation from what was found in Scherrer et al. (2014).

II. Nonextensive Formalism

Starting from Boltzmann-Gibbs entropy, Tsallis (1988, 1995a, 1995b, 2009) developed a different model that can be applied to systems in non-equilibrium state, complex behavior and fractal pattern – characteristics present in earthquakes and geological faults. The entropy is calculated in this model as

$$S_q = -k_B \sum_{i=1}^{W} p_i^q \ln_q p_i \qquad (6.1)$$

where k_B is Boltzman's constant, p_i is a set of probabilities and W is the total number of microscopic configurations. It's easily verified that is a generalization of Boltzmann-Gibbs entropy, as in the limit q=1, we recover the classical model,

$$S_q = -k_B \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1} \qquad (6.2)$$

In order to investigate the impact of using different types of magnitudes, we used the same model revised by Silva et al. (2006) for earthquakes, in which the q-entropy is denoted by

$$S_q = -k_B \int p^q(\sigma) \ln_q p(\sigma) d\sigma \qquad (6.3)$$

where k_B is the Boltzmann constant, $p(\sigma)$ is the probability of find a fragment of surface σ. In the same way, when q=1, the equation becomes the entropy definition by Boltzmann-Gibbs. It's considered the energy scale of $\varepsilon \sim r^3$, i.e. the energy distribution from earthquakes reflects the volumetric distribution of the fragments between plates. The model is given by:

$$\log(N_{>m}) = \log N + \left(\frac{2-q}{1-q}\right) \log\left[1 - \left(\frac{1-q}{2-q}\right)\left(\frac{10^{2m}}{a^{2/3}}\right)\right] \qquad (6.4)$$

where $N_{>m}$ is the number of earthquakes with magnitude larger than m, N is the total number of earthquakes and a is the proportionality constant between the fragments volume and released energy.

III. The Asperity Model and the Circum-Pacific Subduction Zones

A more complete description of the zones can be found in Lay and Kanamori (1981), Kanamori (1986), Müller and Landgrebe (2012), Uyeda (2013) and Scherrer et al. (2014). Here we will just present the basic information to identify the areas analyzed and allow a clearly understanding of the section V. At the table 6.1, we present the limits considered for each area shown in figure 6.1.

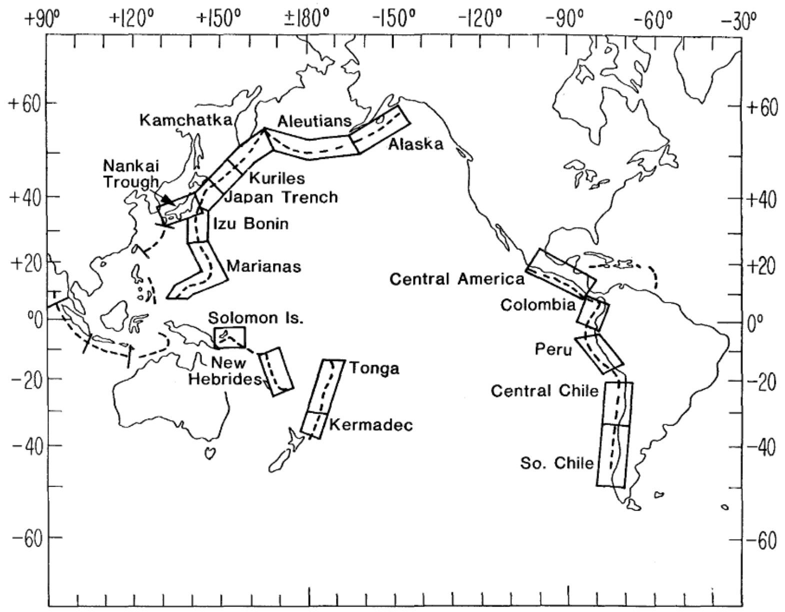

Figure 6.1 – Circum-Pacific Subduction Zones, indicating areas considered in Lay and Kanamori (1981, figure 5.1).

Areas	Latitude		Longitude	
	North	South	East	West
Tonga	-12	-30	-168	177
Kermadec	-30	-42	-174	172
New Hebrides	-9	-24	175	163
Solomon Islands	-2	-10	160	145
Marianas	28	8	150	135
Kuriles	48	44	158	145
Kamchatka	58	48	166	155
Aleutians	57	48	-165	166
Central America	21	7	-74	-108
Colombia	7	-5	-74	-82
Peru	-5	-17	-68	-85
Central Chile	-17	-35	-65	-78
South Chile	-35	-49	-68	-78

Table 6.1. Areas considered in this study (from Scherrer et al., 2014).

Categories	Areas	Characteristics
1	Southern Chile, Southern Kamchatka, Alaska, Central Aleutians	Regular occurrence of great ruptures (≥500km long). Large amount of seismic slip.
2	Western Aleutians (Rat Islands), Colombia, Nankai Trough, Solomon Islands	Variations in rupture extent, with occasional rupture 500km long. Close clustering of large events and doublets.
2-3	New Hebrides, Central America	Intermediate size and small events with no great earthquakes, but clustering of activity.
3	Kuriles Islands, Northeast Japan Trench, Peru, Central Chile	Repeated ruptures over limited zones. No great events. Large component of aseismic slip, or subducting ridges.
4	Marianas, Izu-Bonin, Southeast Japan Trench, Tonga, Kermadec	Large earthquakes are infrequent or absent. Back-ark spreading and large amounts of aseismic slip are inferred.

Table 6.2. Subduction zones characteristics (from Lay and Kanamori 1981, table 5.2, with few alterations). In this study the Aleutians were considered as one area, including western and central regions.

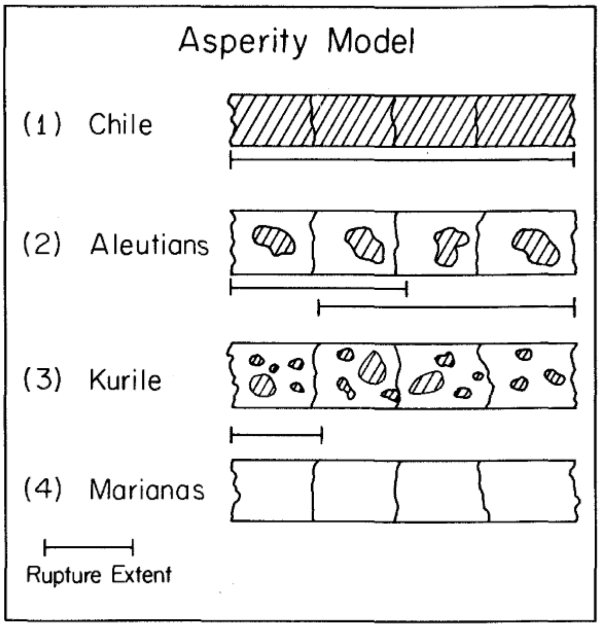

Figure 6.2 – An asperity model indicating the different nature of stress distribution in each subduction zone category. The hatched areas indicate the zones of strong coupling. (from Lay and Kanamori 1981, figure 5.4).

A briefly description of each zone is shown in table 6.2 and featured in figure 6.2. As described in Lay and Kanamori (1981) and Kanamori (1986):

- In the Chile-type behavior (zone 1), the lithospheric plates are strongly coupled, and the asperity distribution is basically uniform over the contact area, because of that, rupture occurs in great events. Sediments are scraped off on subduction and form an accretionary prism, what causes excess trend sediments. The trench and the dip angle of Wadati-Benioff are usually shallow.
- For Aleutians-type (zone 2 – considering the Western part), the asperities are comparatively large, but they are surrounded by weak zones. The relatively homogeneity causes some large ruptures but smaller ruptures also occur, possibly as doublets.
- Because of the relatively small size of asperities and heterogeneities in Kuriles-type zones (zone 3), there is an inhibition of large rupture development generating complicated ruptures and foreshock-aftershock activity.
- The last category (Marianas-type – zone 4) is characterized by no large asperities, so weak coupling and no large earthquakes. There is a heterogeneous contact plane that

decreases the strength of mechanical coupling; it is called “host-and-graben structures”. The trench and the dip angle of Wadati-Benioff are usually deeper. The back-arc basin is commonly found for this type of subduction zones.

IV. Magnitude Types

A wider description of the types of magnitude can be found in Kanamori (1983) and Båth (1981), including the mathematical relations between them. We present just the basic characteristics and limitations of the most commonly used and also considered in this work.

The first magnitude scale in Seismology was developed by Richter (1935) and called the local magnitude (M_L) or Richter magnitude. It’s measure by records of the standard Wood-Anderson torsion seismograph and it’s influenced by each region attenuation characteristics. It’s applicable just until 600km of distance.

In 1945, Gutenberg (1945 a and b) introduced more two scales:

- Surface-wave magnitude (M_S), considering shallow earthquakes, defined by:

$$M_S = \log A_S + 1.656 \cdot \log \Delta^o + 1.818 \qquad (6.5)$$

where A_S is the surface wave amplitude and Δ is the distance from the shallow epicenter in degrees.

- And the body-wave magnitude, M_B, that considered a wave group with different seismic phases and could be used to measure shallow and deep events, calculated by:

$$M_B = \log \frac{A_B}{T} + Q + C \qquad (6.6)$$

where A_B is the wave maximum amplitude, T is the wave period, Q is an attenuation factor and C is the station correction. But, this estimation can produce anomaly high values for distances under 2000 km in continental regions of lower seismicity (Berrocal et al. 1984).

Both scales have limited range and applicability and are saturated when the magnitude is higher than 8.

The moment magnitude (M_W) scale, based on the concept of seismic moment of the earthquake, which is equal to the rigidity of the Earth multiplied by the average amount of slip on the fault and the size of the area that slipped, can be useful to measure all sizes of earthquakes but is more difficult to compute than the other types.

$$M_w = \frac{2}{3}\log_{10}(M_0) - 6.0 \qquad (6.7)$$

where M_0 is the seismic moment in $N{\cdot}m$. The constant values in the equation are chosen to achieve consistency with the magnitude values produced by M_L and M_S.

These scales were conceived as intercalibrated and should yield approximately the same value for any given earthquake, however, Kanamori (1983) points out that, "because of the difference in the type of seismic waves and wave period, complete calibration cannot be made. That's not necessarily a problem, since different scales may represent fundamentally different properties of the source".

V. Results and discussion

As shown in Scherrer et al. (2014), we found good adjustments for the catalog data set (indicated here by the subscription *cat*), but now the original data for each area was adjusted, considering also new sets as described: all events considering the following priority of magnitude types – M_W, M_B, M_S, M_L, M_D, M_G (subscripted as *pr*), and events measured by magnitudes M_S, M_W, M_B and M_L separately.

Considering the similarity between this model and the modified Gutenberg-Richter law, we also calculated the parameter *b*, as was described for Sarlis et al. (2010):

$$b = 2\,(2 - q)\,/\,(q - 1) \qquad (6.8)$$

Results are shown in table 6.3 and figures 6.3 to 6.6. The q-values found presented a wider range (specially for the fitting with M_L), from 1.18 to 1.69. In all the adjustments, zone 1 has q-values typically higher. For the data sets catalog, priority, Ms and M_B, the areas from zone 4 has typically lower values. However, it's necessary to stand out that the range between the higher and lower q-value is smaller in catalog and M_B data sets. But again, it was hard to categorize the intermediate zones. Also, the q-value for Kamchatka region for all data sets doesn't follow the values for the other areas in subduction zone 1.

As can be seen in table 6.3, in each set and for each area even in just a 10 years period a significant number of events was considered in the analysis. The only exception was M_W and we considered it's not possible to reach a plausible conclusion with this data set.

Each data set presents very different inclination of the curves, but inside the same data set these inclinations are similar. In general, the M_S data set presented the best adjustments.

Subduction Zone	Area	q_{cat}	b_{cat}	number of events	q_{pr}	b_{pr}	number of events
1	South Chile	1.6978	0.8661	4038	1.509	1.9331	3832
1	Alaska	1.6656	1.0050	5671	1.448	2.4679	4763
1	Kamchatka	1.6448	1.1019	5648	1.419	2.7730	5648
1-2	Aleutians	1.6746	0.9646	10810	1.444	2.5015	10288
2	Colombia	1.6548	1.0543	2309	1.435	2.5968	2304
2	Solomon Islands	1.6507	1.0737	13125	1.426	2.6913	13125
2-3	New Hebrides	1.6645	1.0097	9818	1.455	2.3952	9768
2-3	Central America	1.6341	1.1539	19488	1.441	2.5301	9043
3	Kuriles	1.6557	1.0503	7428	1.434	2.6088	5306
3	Central Chile	1.6549	1.0540	27106	1.428	2.6743	24642
3	Peru	1.6560	1.0489	2331	1.435	2.5932	2311
4	Kermadec	1.6128	1.2638	21655	1.388	3.1612	21302
4	Marianas	1.6350	1.1499	8560	1.427	2.6883	8486
4	Tonga	1.6340	1.1546	23462	1.433	2.6154	23462

Table 6.3a. Values to *q* and *b*, calculated with magnitude as the catalog presents, considering magnitudes in priority: M_W, M_B, M_S, M_L, M_D, M_G.

Subduction Zone	Area	q_S	b_S	number of events	q_W	b_W	number of events
1	South Chile	1.61	1.2804	209	1.593	1.3706	89
1	Alaska	1.547	1.6577	335	1.329	4.0755	28
1	Kamchatka	1.521	1.8394	599	1.481	2.1557	21
1-2	Aleutians	1.537	1.7265	1200	1.437	2.5788	67
2	Colombia	1.502	1.9827	753	1.657	1.0437	12
2	Solomon Islands	1.54	1.7059	2809	1.487	2.1107	159
2-3	New Hebrides	1.541	1.6945	2601	1.536	1.7287	203
2-3	Central America	1.521	1.8379	1765	1.483	2.1410	444
3	Kuriles	1.554	1.6071	1309	1.524	1.8195	48
3	Central Chile	1.537	1.7227	944	1.483	2.1410	444
3	Peru	1.578	1.4591	652	1.365	3.4747	36
4	Kermadec	1.506	1.9505	1162	1.4	2.9946	52
4	Marianas	1.526	1.8014	1558	1.534	1.7476	99
4	Tonga	1.508	1.9374	3091	1.5	1.9990	297

Table 6.3b. Values to *q* and *b*, calculated only with M_S and only with M_W. The subduction zones for each area are indicated.

Subduction Zone	Area	q_B	b_B	number of events	q_L	b_L	number of events
1	South Chile	1.421	2.7509	1517	1.502	1.9878	2482
1	Alaska	1.399	3.0152	1427	1.615	1.2515	3476
1	Kamchatka	1.434	2.6069	2351	1.183	8.9487	3311
1-2	Aleutians	1.404	2.9559	4233	1.427	2.6802	6673
2	Colombia	1.415	2.8216	2298	1.516	1.8782	64
2	Solomon Islands	1.384	3.2119	13119	1.348	3.7409	1255
2-3	New Hebrides	1.386	3.1865	9102	1.42	2.7658	948
2-3	Central America	1.447	2.4696	6557	1.312	4.4133	2430
3	Kuriles	1.419	2.7759	5291	1.209	7.5662	259
3	Central Chile	1.407	2.9128	7579	1.406	2.9307	17548
3	Peru	1.426	2.6968	2301	1.41	2.8746	195
4	Kermadec	1.372	3.3785	6645	1.32	4.2584	15257
4	Marianas	1.411	2.8671	8479	1.357	3.5968	225
4	Tonga	1.401	2.9814	23448	1.435	2.5996	504

Table 6.3c. Values to *q* and *b*, calculated only with M_B and only with M_L. The subduction zones for each area are indicated.

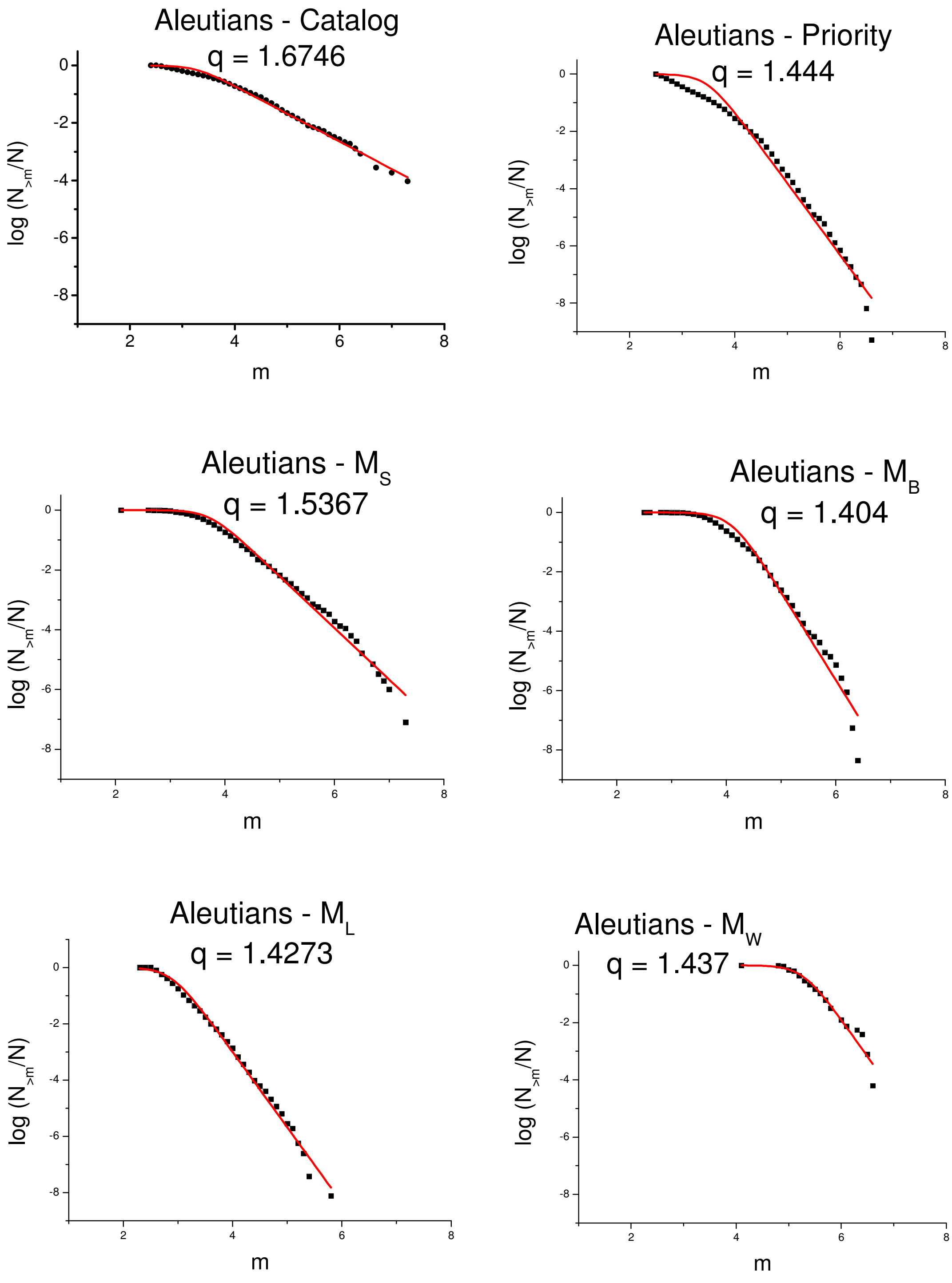

Figure 6.3 - The relative cumulative number of earthquakes as a function of the magnitude m for Aleutians, representing zone 1. We show the graphics for using catalog magnitudes, priority magnitudes, M_S, M_B, M_L, M_W respectively.

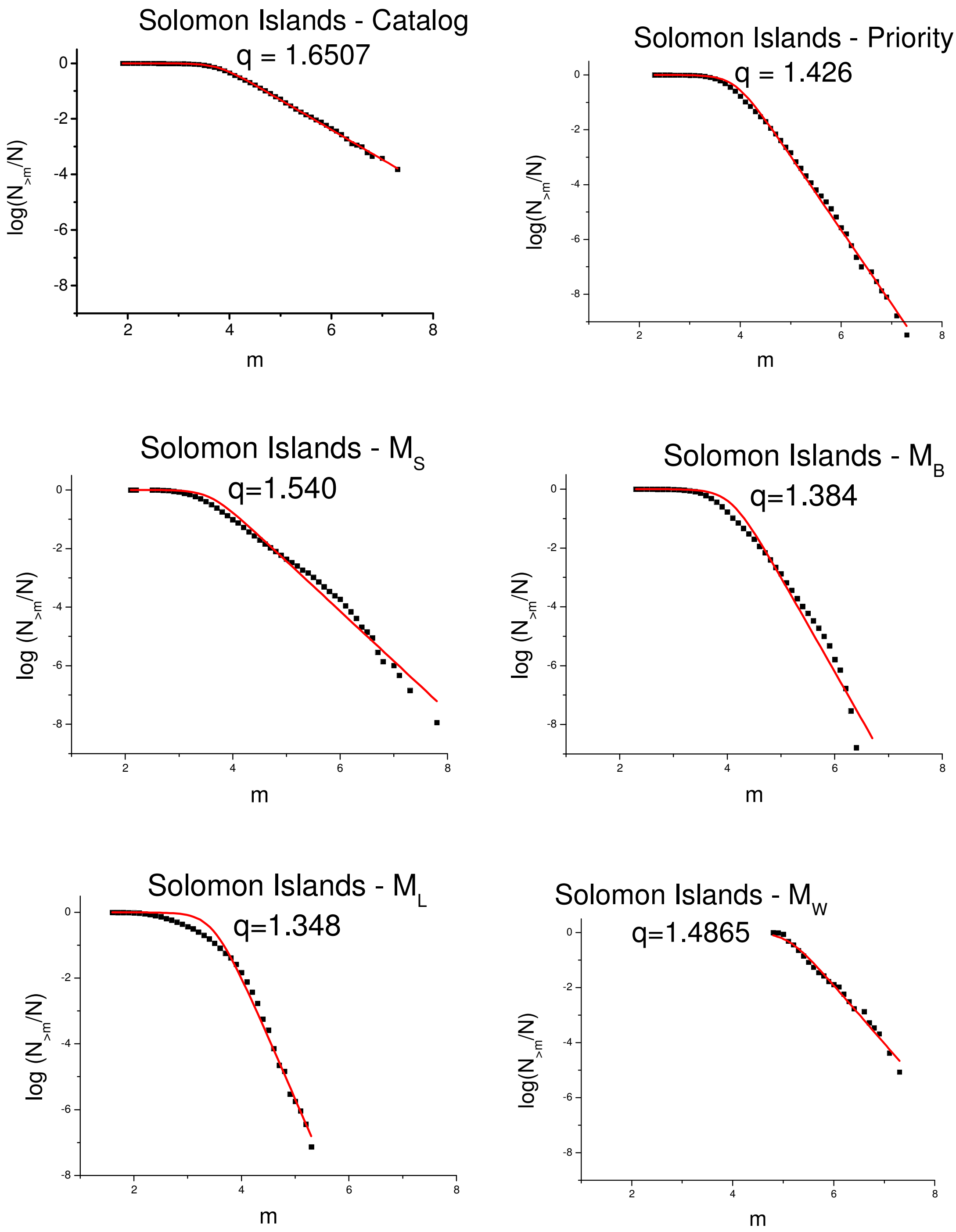

Figure 6.4 - The relative cumulative number of earthquakes as a function of the magnitude m for Solomon Islands, representing zone 2. We show the graphics for using catalog magnitudes, priority magnitudes, M_S, M_B, M_L, M_W respectively.

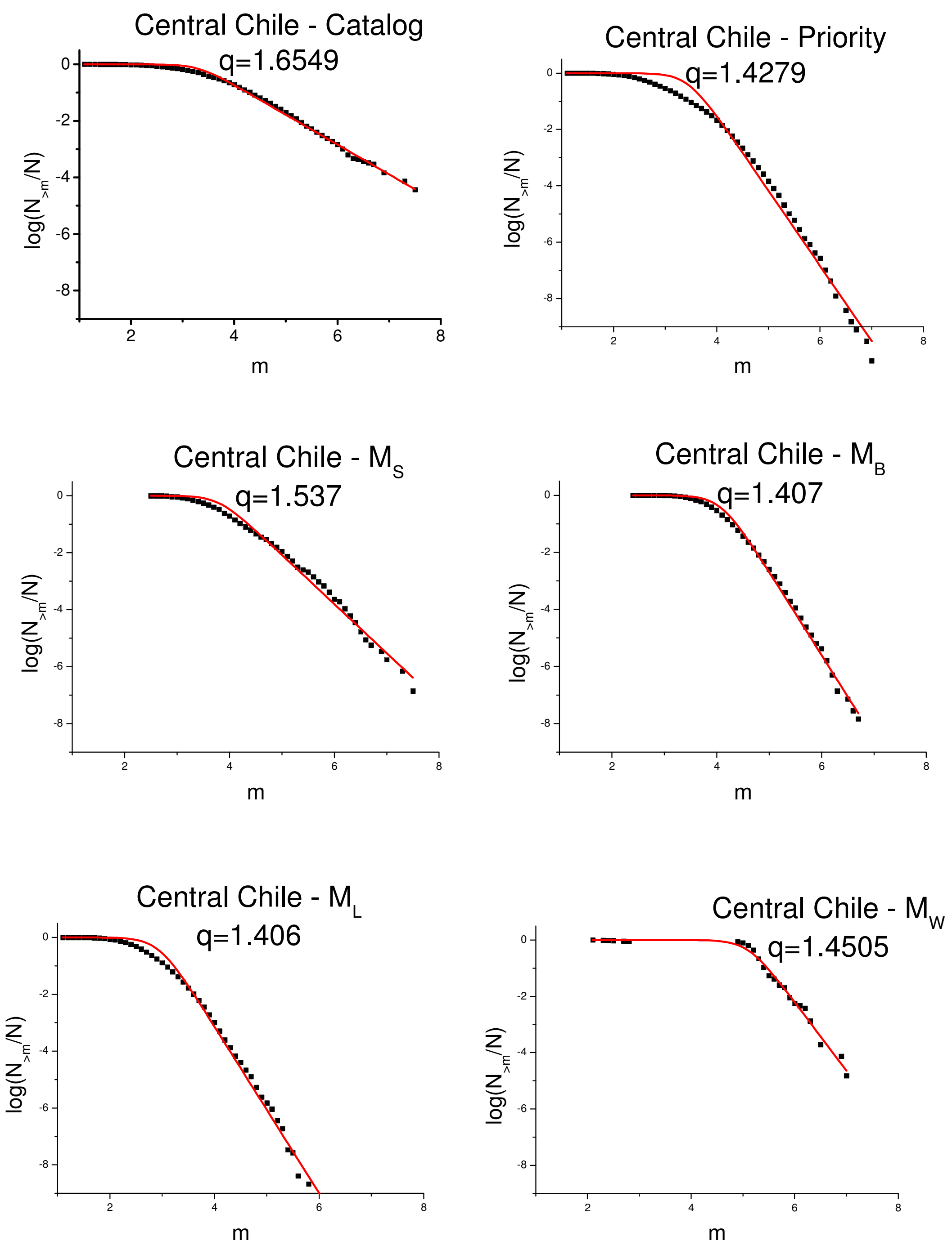

Figure 6.5 - The relative cumulative number of earthquakes as a function of the magnitude m for Central Chile, representing zone 3. We show the graphics for using catalog magnitudes, priority magnitudes, M_S, M_B, M_L, M_W respectively.

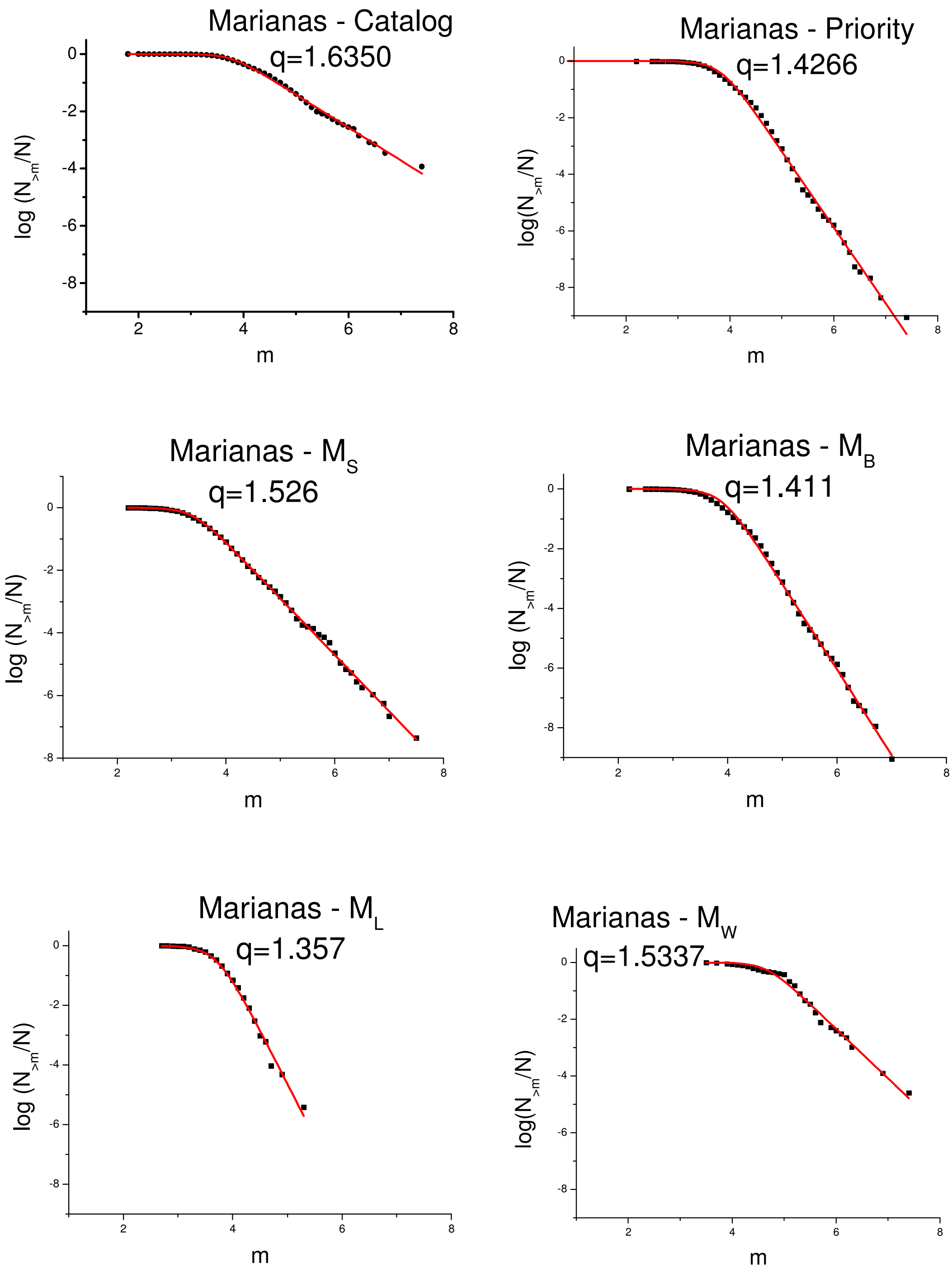

Figure 6.6 - The relative cumulative number of earthquakes as a function of the magnitude m for Marianas, representing zone 4. We show the graphics for using catalog magnitudes, priority magnitudes, M_S, M_B, M_L, M_W respectively.

VI. Conclusions

Other than expected, the variation of the q-value calculated considering different magnitudes types was elevated and the cumulative distribution of the data present very distinct inclination on each case. In general, q_{cat} and q_S are those that better correlate with the subduction zones, zone 1 presents higher values and zone 4 lower values. But for the intermediate areas it's still not possible to separate the categories considering these parameters. So, the influence of coupling for the nonextensive parameter is reaffirmed. The good adjustment with q_S may be due to the relevance of the fragmentation process in nonextensive behaviour. The calculation with M_B also presents a good correlation with the subduction zones.

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Capítulo 7 - Conclusões

Esse trabalho buscava responder algumas questões usando o modelo não-extensivo desenvolvido por Sotolongo-Costa and Posadas (2004) e revisado por Silva et al. (2006): há significado físico no parâmetro não extensivo? Há correlação entre q e grandezas geofísicas? O valor de q varia com diferentes características tectônicas ou é apenas um ajuste matemático? Aplica-se em situações de contato de placas tectônicas e também intraplacas? O uso de diferentes escalas de magnitude, que consideram diferentes aspectos das ondas geradas e do ambiente de propagação delas, tem influência no valor desse parâmetro?

Destaca-se que em todos os casos apresentados a aplicação da estatística não extensiva permitiu um ajuste adequado e coerente com os dados analisados e com os resultados esperados para a metodologia.

O estudo do q-tripleto em San Andreas confirmou o comportamento do sistema como sendo consistente com um estado de não equilíbrio e sugerindo correlações de longo prazo. Ainda, revelou forte evidência de que a atividade sísmica nessa região tem uma estrutura hierárquica em pequenas escalas, ou seja, que o tamanho, distribuição e processo de fragmentação, são dominantes para o estado de não equilíbrio.

O estudo indica que há a possibilidade de uma interpretação geofísica para o valor de q ao relacioná-lo com as zonas de subducção definidas empiricamente pelo modelo de aspereza de Lay e Kanamori (1981). Percebe-se que o valor do parâmetro não extensivo é maior para a zona 1 e decresce na seguinte ordem: 3, 2 e 4, com exceção de algumas áreas (Kamchatka e América Central). Isso indica que o acoplamento mecânico entre as placas tem papel fundamental no comportamento não extensivo do sistema. A diferença entre os valores encontrados para a zona 1 e 4 mostram claramente que zonas com alto grau de acoplamento tem comportamento não extensivo mais intenso que as de baixo acoplamento. Ressalta-se, entretanto, que as zonas intermediárias não foram facilmente distinguidas.

Apesar da clara indicação da influência de fatores geofísicos no valor de q, uma forma de verificar essa informação ou de melhor especificar quais fatores têm maior influência no cálculo desse parâmetro seria aplicar essa abordagem em outras áreas, com características diferenciadas, ou ainda sobre as mesmas áreas, considerando separadamente as diferentes escalas de magnitudes.

Avaliando-se os valores calculados em regiões intraplaca no Brasil, mesmo em mecanismos sísmicos diferenciados, percebe-se um valor de q semelhante entre as regiões de falha, e maior nas regiões de contraste de estruturas. Destaca-se de uma forma especial que, a margem passiva, que apresenta 70% mais sismos acima de 3,5 que a média das regiões continentais estáveis (Assumpção et al., 2014), apresentou um valor de q significativamente mais elevado.

Já ao considerarmos os diferentes tipos de magnitude, a variação dos valores calculados para o parâmetro q é grande e a distribuição cumulativa dos dados apresenta inclinações muito diferenciadas. De forma geral, os valores de *q* calculados a partir do catálogo e da magnitude de superfície são os que mais mantêm a correlação com as zonas de subducção. Não significa que as outras magnitudes não podem descrever características não extensivas, mas que as características relacionadas à aspereza das regiões são melhor descritas na análise por meio das ondas de superfície, possivelmente devido à processos de fragmentação. O cálculo de *q* a partir de M_B e usando a prioridade estabelecida (M_W, M_B, M_S, M_L, M_D, M_G) também apresentaram boa correlação. Mas, em todos os conjuntos de dados, as zonas intermediárias não obtiveram valores de *q* que permitissem uma separação clara entre as zonas de subducção.

Ainda, a partir do valor de q, foi calculado também o parâmetro b de Gutenberg-Richter, tanto por máxima verossimilhança (determinado em Sarlis et al. 2010) como calculado manualmente pelo Zmap (Wiemer, 2001). Observa-se que há coerência entre os valores.

Uma questão que pode ser levantada é se a distribuição dos dados do Círculo de Fogo pode ser considerada representativa do todo, considerando-se que foi avaliado um período aparentemente curto de 10 anos. Assim, se por exemplo, muitos eventos de intensidade acima do normal tivesse ocorrido exatamente dentro do período considerado, estes poderiam provocar um deslocamento do ajuste, resultando em valores tendenciosos para o parâmetro não extensivo. Destaca-se, entretanto, que a base de dados é vasta, devido à intensa atividade sísmica na região e assim considera-se que, caso tal efeito ocorra na base de dados usada neste trabalho, ele não será tão significativo.

Ressalta-se que todos os valores encontrados respeitam os limites estipulados pela estatística de Tsallis e estão de acordo com valores encontrados em outros ajustes por este método, como o limite superior de $q < 2$ obtido em diversos estudos independentes que usam a estatística de Tsallis (*e.g.* Carvalho et al. (2009), Liu and Goree (2008), Khachatryan et al. (CMS Collaboration, 2010), dentre outros).

Considerando todas as análises ora apresentadas, destaca-se que aparentemente o processo de fragmentação em pequenas escalas apresenta o impacto mais significativo no valor do parâmetro não extensivo. Assim, a análise de cada uma dessas áreas com o q-tripleto como também uma nova revisão do modelo, incluindo as características multifractais desse processo no ajuste podem prover informações mais detalhadas.

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