REFERÊNCIA

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Research of a Floquet's reference frame through a first order recurrence equation

ABSTRACT

The authors have previously developed a method to solve periodic coefficient ordinary differential equations through the use of second order recurrent relations which yield the values of the Floquet's s multipliers. In the present pacer, they examine the possibility of using a first order recurrent relation; they study one example where this is possible.

INTRODUCTION

In previous papers [3],[5], we have treated the case of the following ordinary differential equation with periodic coefficients :

$$(1) \qquad RI + d(LI)/dt = 0$$

where L is a periodic matrix.

By use of the Floquet's theorem, we have shown that

$$(2) \qquad I = \sum_h K_h \exp(-\alpha_h t).\sum_n I_{nh} \exp(jn\theta)$$

where the α_h's, the I_{nh}'s and K_h's are to be determined.

Substituting (2) into (1) yields the following second order recurrence relation :

$$(3) \qquad A.I_{n+1} + B_n.I_n + C.I_{n-1} = 0$$

where A and C are constant m×m matrices, B_n is a m×m matrix which is function of α and n.

The key of the method is to show that (3) yields a condition which ensures the convergence of expansion (2) ; from this condition, m values of α can be determined. Numerical applications have been given.

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Now, it is a common use to replace second order differential equations by first order ones, at the expense of a larger number of unknowns. Therefore, it is pertinent to raise the question : would it be easier to replace (3) by a first order recurrence relation such as :

$$(4a) \qquad X_{n+1} = M_n . X_n$$

where M_n would be a 2m×2m matrix. If this is possible, is it possible to make use of (4a) to determine the α_h's ?

OUTLINE OF THE METHOD

Indeed, if A is non singular, it is possible to write :

$$(5a) \qquad I_{n+1} = -(A^{-1}.B_n).I_n - (A^{-1}.C).I_{n-1}$$

Since it is always possible to write :

$$(5b) \qquad I_n = |1|.I_n$$

where $|1|$ is the m×m unit matrix, we obtain :

$$(4b) \qquad \begin{vmatrix} I_{n+1} \\ I_n \end{vmatrix} = \begin{vmatrix} -A^{-1}.B_n & -A^{-1}.C \\ 1 & 0 \end{vmatrix} \begin{vmatrix} I_n \\ I_{n-1} \end{vmatrix}$$

Therefore, it is possible to reduce the second order recurrence to a first order one, at least if A is non singular. It is to be noted that M_n is always a function of α for finite values of n, and may be independent of α when "n" goes to plus or minus infinity.

When "n" goes to $\pm\infty$, some of the eigenvalues of M_n are larger than unity, and others ones are smaller. For the expansion (4) to be convergent, it is necessary that the constituent vectors lay in the subspace associated with eigenvalues larger than one for $n = -\infty$, and in the subspace associated with eigenvalues smaller than one for $n = \infty$. The role of the intermediate (i.e. finite) values of "n" is to ensure a transition between those two extreme cases. And since M_n is a function of α when n is finite, the transition between the two extreme cases cannot be ensured unless α assumes some particular values; this remark thus provides the equation which determines the α_h's.

This very simple procedure will be now explained on a practical example.

PRESENTATION OF THE EXAMPLE

Let us consider (figure 1) two coils whose resistances and inductances are constant, but mutual inductance is a periodic function of time.

The equations are :

$$(6a)\quad \begin{aligned} &Ri_a + L\,di_a/dt + d(i_f M\cos\theta)/dt = 0 \\ &ri_f + \ell\,di_f/dt + d(i_a M\cos\theta)/dt = E \end{aligned}$$

(6b) with $\theta = \theta_0 + wt$

w and θ_0 being constants. At time "t = 0" when switch S is closed :

$$(6c)\quad \begin{aligned} &i_a(0) = 0 \\ &i_f(0) = E/r \end{aligned}$$

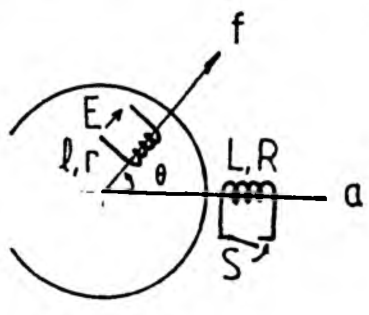

Fig. 1 : Undamped uniform air gap machine

Note that (6a) is inhomogeneous, while (1) is homogeneous. Relation (2) yields :

$$(7a)\quad \begin{vmatrix} i_a(t) \\ i_f(t) \end{vmatrix} = \sum_{h=0}^{2} k_h \exp(-\alpha_h t) \sum_{n=-\infty}^{\infty} \begin{vmatrix} i'_n \\ i''_n \end{vmatrix} \exp(jn\theta)$$

where h = 0 corresponds to the particular solution of the inhomogeneous equation ($\alpha_0 = 0$). From now on, we shall drop the index "h", in order to alleviate the expressions. In addition, we shall use the following variables :

$$(7b)\quad I_n = \begin{vmatrix} i'_n{}^*\sqrt{L} \\ i''_n{}^*\sqrt{\ell} \end{vmatrix}$$

which allow (6a) to be rewritten in terms of the time constants and minimum dispersion of the two windings :

$$(7c)\quad \begin{aligned} &\delta_a = R/L \qquad \delta_f = r/\ell \\ &\beta = M/\sqrt{L\ell} = (1-\sigma)^{1/2} \text{ and } \sigma = 1 - M^2/L\ell \end{aligned}$$

Indeed with those notations, we get (for non zero values of both "n" and "h") :

$$(8a)\quad \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} I_{n+1} + \begin{vmatrix} a_n & 0 \\ 0 & b_n \end{vmatrix} I_n + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} I_{n-1} = 0$$

where

$$(8b)\quad \begin{aligned} &a_n = 2/\beta^*[1 + \delta_a/(jnw - \alpha)] \\ &b_n = 2/\beta^*[1 + \delta_f/(jnw - \alpha)] \end{aligned}$$

so that (4a) and (4b) become :

$$(9)\quad \begin{vmatrix} I_{n+1} \\ I_n \end{vmatrix} = \begin{vmatrix} 0 & -b_n & -1 & 0 \\ -a_n & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix} \begin{vmatrix} I_n \\ I_{n-1} \end{vmatrix}$$

which is equivalent to

$$(10a)\quad X_{n+1} = M_n \cdot X_n$$

with

$$(10b)\quad X_n = \begin{vmatrix} I_n \\ I_{n-1} \end{vmatrix}$$

and

$$(11a)\quad M(\infty) = \begin{vmatrix} 0 & -2/\beta & -1 & 0 \\ -2/\beta & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}$$

If we let :

$$(11b)\quad \lambda = \sqrt{(1+\sqrt{\sigma})/(1-\sqrt{\sigma})}$$

the eigenvalues of $M(\infty)$ are λ, $-\lambda$, $1/\lambda$, $-1/\lambda$, which correspond to the following eigenvectors :

$$(11c)\quad u_1 = \begin{vmatrix} 1 \\ -1 \\ 1/\lambda \\ -1/\lambda \end{vmatrix},\ u_2 = \begin{vmatrix} 1 \\ 1 \\ -1/\lambda \\ -1/\lambda \end{vmatrix},\ u_3 = \begin{vmatrix} 1/\lambda \\ -1/\lambda \\ 1 \\ -1 \end{vmatrix},$$

$$u_4 = \begin{vmatrix} -1/\lambda \\ -1/\lambda \\ 1 \\ 1 \end{vmatrix}$$

respectively.

Since $\lambda > 1$, it immediately appears that convergence of (7a) cannot be ensured if X_n has a component along u_1 or u_2, for very large values of "n" ; conversely, convergence of (7a) cannot be ensured if X_n has a component along u_3 or u_4, for $n < 0$ (and $|n|$ very large). This means that X_n must lay in the plane (u_3, u_4) for $n > 0$, and X_n in the plane (u_1, u_2) for $n < 0$. in other words :

$$(12) \qquad X_\infty = \begin{vmatrix} C/\lambda \\ -D/\lambda \\ D \\ -C \end{vmatrix}, \quad X_{-\infty} = \begin{vmatrix} E \\ -F \\ F/\lambda \\ -E/\lambda \end{vmatrix}$$

Writing (4a), (9) and (10a) as :

$$(13) \qquad X_{N+1} = M(N). \ \dots \ .M(1).M(0).M(-1). \ \dots \ .M(-N).X_{-N}$$

where "N" is a very large value of "n" will therefore yield one four line equation to determine α, C, D, E, F, that is to say to determine four unknowns (since one among C, D, E, F can be chosen arbitrarily). This equation is non linear ; we know that there are two time constants, therefore we must find two convenient sets of (α, C, D, E, F).

We have completed the solution in the case of a very high $d\Theta/dt$, then for various values of $d\Theta/dt$.

CASE OF HIGH ROTATION SPEED

If "w" is large enough, α will be neglegible when compared to "jnw", and

$$(14) \qquad 1/(jnw - \alpha) = 0$$

even for n = 1.

Therefore, equation (13) can be used either as

$$(15a) \qquad X_2 = M(1).M(0).M(-1).X_{-1}$$

or as

$$(15b) \qquad X_\infty = M_\infty.M(0).M_{-\infty}.X_{-\infty}$$

Since the directions of X_∞ and $X_{-\infty}$ are known, and since M(0) is a function of α, the following equation defines C, D, E, F and α :

$$(16a) \quad M_\infty^{-1} \begin{vmatrix} 1/\lambda & 0 \\ 0 & -1/\lambda \\ 0 & 1 \\ -1 & 0 \end{vmatrix} \begin{vmatrix} C \\ D \end{vmatrix} - M(0).M_\infty \begin{vmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 1/\lambda \\ -1/\lambda & 0 \end{vmatrix} \begin{vmatrix} E \\ F \end{vmatrix} = 0$$

Recalling that a_0 and b_0 are functions of α and rearranging, we get

$$(16b) \quad \begin{vmatrix} -\lambda b_0+1 & 1 & 0 & 0 \\ \lambda & -\lambda & 0 & 0 \\ 0 & 0 & \lambda & -\lambda \\ 0 & 0 & -1 & \lambda a_0-1 \end{vmatrix} \begin{vmatrix} E \\ D \\ C \\ F \end{vmatrix} = 0$$

The determinant of (16b) obviously cancels when either one of the following conditions is fulfilled :

$$(17a) \qquad a_0 = 2/\lambda \qquad b_0 = 2/\lambda$$

which in turn yield either one of the following values of α :

$$(17b) \qquad \alpha_1 = \delta_a/\sqrt{\sigma} \qquad \alpha_2 = \delta_f/\sqrt{\sigma}$$

which are the well known values given by Boucherot [1].

CASE OF LOW ROTATION SPEEDS

For low values of w, it is permissible to use again (13) with a "large enough" value of N. Let us call :

$$(18) \qquad P^+ = M(N).M(N-1). \ \dots \ .M(1) \qquad P^- = M(-1).M(-2). \ \dots \ .M(-N)$$

P^+ and P^- are function of both α and N, and may be written $P^+(\alpha,N)$ and $P^-(\alpha,N)$. Thus

$$(19a) \qquad \begin{vmatrix} C/\lambda \\ -D/\lambda \\ D \\ -C \end{vmatrix} = P^+(\alpha,N).M_0(\alpha).P^-(\alpha,N) \begin{vmatrix} F \\ -F \\ F/\lambda \\ -E/\lambda \end{vmatrix}$$

is the equation which gives α and the values of C, D, E, F. This equation will be solved by successive iterations. To this end, let us call α'' an approximate value of α, and α' a value of α which is a better approximation than α''. In (19a) we may replace the square matrix by :

$$(19b) \qquad P^+(\alpha'',N).M_0(\alpha').P^-(\alpha'',N)$$

and then we rearrange (19a) as :

$$(19c) \qquad \mu(\alpha'',\alpha') \begin{vmatrix} E \\ D \\ C \\ F \end{vmatrix} = 0$$

which is more general than (16b). Thus, if we have an approximation of α, (19c) will yield another one. If we start with one of the values (17), convergence is obtained after 2 or 3 iterations. We shall say that the value of N which has been used was "large enough" if the process converges to the same value of α for N and for N+1.

It is to be noted that the eigendirection (E,D,C,F) provided by (19c) must obviously be the same as the eigendirection provided by (16b). Therefore, the solution of (19) for α' when α'' is known may be greatly simplified.

This process has been used for σ = 0.09, δ_a = 2.s, δ_f = 1.s ; for each value of w, two values α_1 and α_2 of α are found : they are plotted in figure 2.

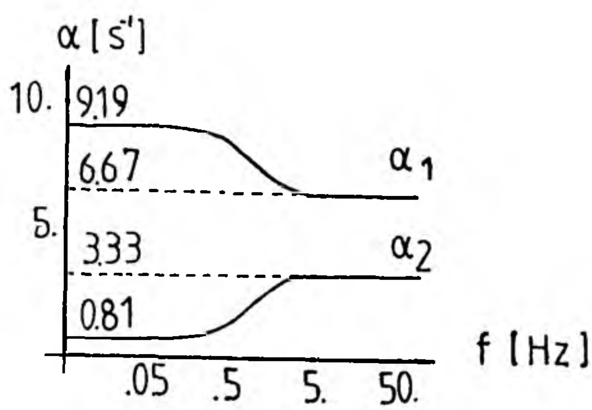

Fig. 2 : Variation of α as a function of frequency

CONCLUSION

Some linear first order periodic coefficient ordinary differential equations may be solved through a second order recurrent relation which we have studied in our previous work. In the present paper, we have shown that it is possible to replace this relation by a first order one, under the condition that a given matrix be non singular. We have shown how to solve such a first order recurrent relation. We hope that this will simplify the study of more complicated problems.

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