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The application of a mesoscopic scale approach in fretting fatigue

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ABSTRACT

This work concerns the application of a mesoscopic scale fatigue criterion to predict the initiation of cracks in components under fretting conditions. A verification of the analysis is carried out by considering available experimental data for fretting fatigue in a high strength Aluminium alloy commonly used in the aerospace industry. These experiments revealed there is a contact size effect in fretting fatigue life. The results show that the mesoscopic scale fatigue criterion can correctly predict the initiation of fretting cracks for larger contact configurations. It is concluded that the reason for the poor performance of the criterion in predicting failure at smaller contacts may well be related to the effect of the stress gradient, a variable not accounted for in the mesoscopic criterion here assessed.

Keywords: Fretting fatigue, multiaxial fatigue, contact size effect

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Introduction

Fretting is a process which invariably occurs on contacting surfaces of tightly fitting metal joints subjected to vibrational loading. The superficial damage caused by the rubbing of the contacting surfaces together with the severe stress gradient under the contact region accelerate the nucleation and early growth of cracks, which can lead to catastrophic failure if there is at least one of the components of the fitting subjected to a remote fatigue load. This is the so called fretting fatigue phenomenon. S-N curves of fretted specimens have shown reductions between 50 and 90% in the fatigue strength or endurance limit of the material (Shaffer and Glaeser, 1994). This reduction in component life has challenged engineers and researchers to enhance fatigue models and decrease the amount of conservatism currently employed in designing against fretting fatigue. The aircraft industry is perhaps one of the most affected by the fretting fatigue problem. Fretting failure of components such as the dovetail fixing between blade and disc in fans of aeroengines and riveted skins of the aircraft fuselage have become a major concern and further efforts are necessary to enhance fatigue life predictions. Although there have been attempts to predict fretting fatigue damage for such complex configurations (Ruiz and Chen, 1986, Harish and Farris, 1998), experiments are still the basis of fatigue life prediction. There is no doubt that experimental testing of representative geometries provides more realistic information, which is crucial for the design process, however, these tests are costly and lengthy and should be kept at a minimum. In the longer term there is a need to gain a more fundamental understanding of the fretting fatigue process, especially in the initiation and early propagation stages. In principle, this can be achieved by quantifying and understanding the effect of the contact stress field on the fretting fatigue life of simpler contact configurations. In this setting, a number of tests available for "dog bone" tensile specimens fretted by cylindrical pads will be considered in order to assess the performance a multiaxial fatigue criterion in predicting failure. Multiaxial fatigue criteria are usually divided into i) empirical methods (e.g. Gough et al., 1951), ii) equivalent stress/strain criteria, (e.g. Sines and Ohgi, 1981, Crossland, 1956), iii) critical plane approaches (e.g. Brown and Miller, 1973, Socie, 1987, Fatemi and Socie, 1988, Carpinteri and Spagnoli, 2001) iv) energy or plastic work methods, (e.g. Garud, 1979, Park and Nelson, 2000, Varvani-Farahani, 2000) and v) mesoscopic scale criteria (e.g. Dang Van et al., 1989, Papadopoulos et al., 1997). Mesoscopic models have gained increasing interest due to its strong physical basis associated with good predictions of fatigue crack initiation in laboratory as well as in practical applications (Dang Van et al., 1989). In this work we intend to evaluate the performance of the mesoscopic fatigue criterion proposed by Dang Van in predicting the failure of fretted specimens subjected to a rapidly varying contact stress field condition. A crucial part of the work is to evaluate the ability of such model to capture the effects that different stress gradients may have on the fretting life.

Dang Van's Mesoscopic Fatigue Criterion

Suppose that a volume of material containing a number of grains with arbitrary orientations is subjected to a cyclic elastic stress state, which is assumed to be known at any time t. Although the stress state is macroscopically elastic, localized plasticity may take place within some favourably oriented grain. Under high cycle fatigue conditions, this localized plastic deformation in the grain is contained by the bulk material, which remains elastic. According to Dang Van et al. (1989) crack initiation will take place if a state of plastic shakedown is achieved within this grain. Hence, the boundary between infinite life and failure corresponds to conditions of elastic shakedown at mesoscopic level. In this setting, Dang Van proposes that crack initiation will occur if

$$f(\sigma(t)) \geq 0, \qquad (1)$$

where $f(\sigma)=0$ is some appropriate function of the local stress state. As cracks are expected to form due to a permanent slip of crystallographic planes, an important variable controlling the fatigue process is the microscopic deviatoric stress tensor, **s**(t), which is given as:

$$\mathbf{s}(t) = \mathbf{S}(t) - \rho \quad , \qquad (2)$$

where **S**(t) is the deviatoric stress tensor at any time t, and ρ is the stabilized deviatoric residual stress tensor after the state of shakedown is achieved. An appropriate algorithm for the determination of ρ is presented in the appendix (interested readers should also consult Bernasconi (2001) for different algorithms). Another important variable governing the initiation of a crack is the pressure p_h, since a positive p_h will encourage crack opening. A simple fatigue criterion relating these variables was then proposed as:

$$f = \tau \pm m p_h \mp n, \qquad (3)$$

where τ is

$$\tau = \frac{1}{2} Tresca(\mathbf{s}(t)). \qquad (4)$$

The material constants *m* and *n* are evaluated by considering the fatigue limits in bending (σ_{fl}) and torsion (τ_{fl}), yielding:

$$n = \tau_{fl} \text{ and } m = \frac{6\sigma_{fl} - 3\tau_{fl}}{2\sigma_{fl}}, \qquad (5)$$

Notice that Eq. (3) leads to two intersecting lines in the τ versus p_h plane (Fig. 1). If the local stress cycle, plotted on this plane stays within these lines the component is safe, if it crosses one or both of the lines failure will result. The use of Eq. (4) assures that the loading path is always above the p_h axis.

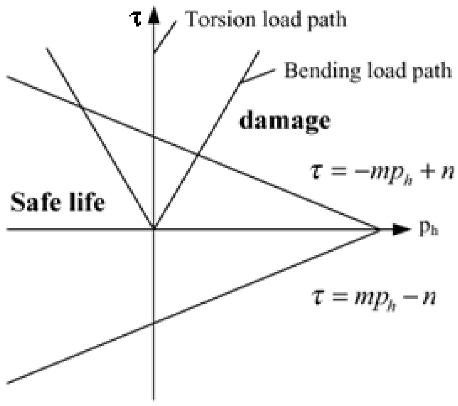

Figure 1. τ **x** p_h **plane showing the torsion and bending load paths.**

From the foregoing a crack nucleation risk factor can be defined for the Dang Van criterion.

$$DV = Max\left[\frac{\tau(t)}{\tau_{fl} - mp_h(t)}\right]. \qquad (6)$$

The maximization is to be carried out on time (t), and if DV is greater than 1 fatigue failure is expected to occur. An schematic view of the Dang Van criterion containing its different steps is shown in Fig. 2.

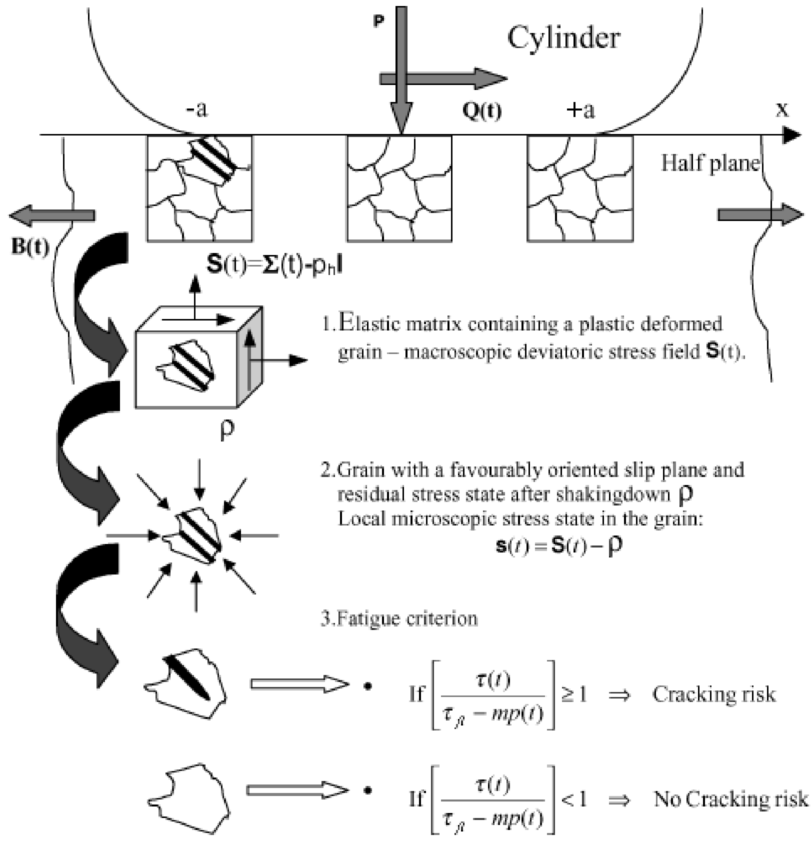

Figure 2. Different steps of the high cycle fatigue criterion proposed by Dang Van et al. (1989).

Experimental Data

To validate the analysis, experimental data published by Nowell (1988) will be considered. A schematic diagram of the experimental geometry is shown in Fig. 3. The main specimen is a "dog's bone" tensile test piece held between two movable jaws. This is clamped by two cylindrical pads subjected to a static normal load per unit specimen width, P. The specimen was then subjected to an oscillatory load $B\sin(wt)$. This causes extension and contraction of the specimen. As the pads are in contact with the specimen and are restrained by springs a cyclic tangential fretting force, $Q\sin(wt)$, is also developed. Five series of tests were carried out under this configuration. Within each data series the tests were conducted so that the contact size, a, was varied while the peak contact pressure, p_0, was held constant. This was possible because of the Hertzian nature of the experimental configuration, where the peak contact pressure, p_0, is proportional to $(P/R)^{1/2}$ and the contact semi -width, a, is proportional to $(PR)^{1/2}$, as shown in eqs (7) and (8)

$$a = \sqrt{\frac{4PR}{\pi E^*}}, \qquad (7)$$

$$p_0 = \sqrt{\frac{PE^*}{\pi R}}, \qquad (8)$$

where P is the applied normal force, R is the relative radius of curvature of the contacting surfaces and E* is the equivalent elastic limit (eqs. (9) and (10)).

$$R = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}, \qquad (9)$$

$$E^* = \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)^{-1}. \qquad (10)$$

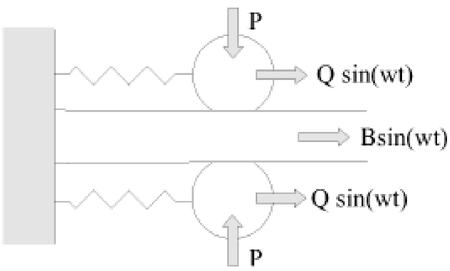

Figure 3. Schematic of experimental fretting fatigue configuration.

Hence, it is possible to vary P and R keeping their ratio constant. The advantage of conducting such tests is that it was possible to produce a data series where the magnitude of the stress field was held constant on the surface but it varied in extent from test to test. The fretting pads and specimens were made of A14%Cu (HE15-TF). Basic mechanical properties for this material are: Young's modulus, E=74 GPa, yield stress, σ_{ys}=465 MPa, ultimate tensile strength, σ_{us}=500 MPa and Poisson's ratio ν=0.33.

In order to provide a range of contact sizes, eight pairs of fretting pads were chosen with radii of curvature varying from 12.5 mm to 125 mm. An average of eight tests were run within each data series keeping the salient parameters p_0, Q/P and σ_0 constant, where σ_0 is the normal stress generated in the specimen due to the application of the bulk load, B. Table 1 summarises the characteristic contact parameters for the data series analysed here, while Table 2 records the average total life and corresponding contact size and pad radius for each test condition. From Table 2 it is clear that within each data series small contacts produced a life greater than 10^7 cycles, whereas larger contacts produced lives in the region of 10^5 to 10^6 cycles. A typical set of results is shown in Fig. 4, where the critical contact size range, a_0 is clearly observed. For Al series 2 data, the salient parameters gave rise to a condition of reverse slip. This requires a more complex approach to raise the subsurface stress field, hence this data series will not be considered in this work.

Table 1. Contact parameters for the experimental series analysed.

Series No.	p_0 (MPa)	σ_0 (MPa)	Q/P	Friction coefficient, f
1	157	93	0.45	0.75
3	143	93	0.45	0.75
4	143	77	0.45	0.75
5	120	62	0.45	0.75

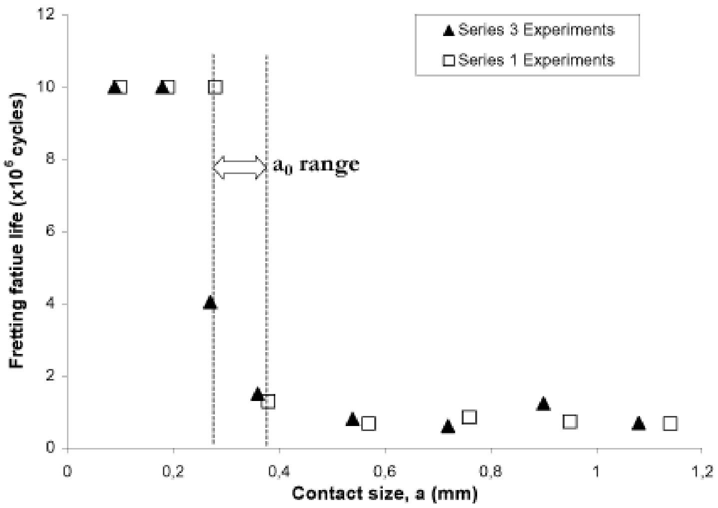

Figure 4. Results of Nowell's (1988) fretting fatigue experiments: variation of life with contact size.

In summary, these set of Hertzian tests show that is possible to combine contact loads and geometry of pads so that within a same series of tests the stress field remains identical on the surface but vary in extent from test to test as the contact size changes. Moreover, it was observed the number of cycles necessary to initiate and propagate a crack till complete failure of the specimen was somehow dependent on the contact size, even though the magnitude of stress state for corresponding surface points did not change from test to test. Such set of results constitute a good challenge for the validation of a multiaxial fatigue criterion. The next step is to assess if the mesoscopic scale criterion chosen for the analysis is capable of predicting such contact size effect in fatigue life.

Cyclic Stress Field

In order to apply the Dang Van criterion to the fretting problem it is first necessary to evaluate the cyclic stress field developed under the contact in the experimental configuration. These quantities may be evaluated using a well-established technique and only an overview will be presented here. The first step towards a solution for the subsurface stress field is to solve the contact problem itself, i.e., to find the magnitude and distribution of the surface tractions. The shear tractions will not cause any disturbance in the Hertzian pressure distribution if the contacting surfaces are elastically similar. Thus, since the normal load is constant, the normal traction will also be independent of time. The cyclic shear load on the other hand will give rise to history dependent tractions, as described by Cattaneo (1938) and Mindlin (1949). Once the normal and shear tractions have been found, the cyclic stress field can be obtained at each load step by using Muskhelishvili's potential theory (Muskhelishvili, 1953).

Results

The Dang Van criterion is now used to assess the crack nucleation risk for Nowell's experiments with A14%Cu. Application of the criterion requires the knowledge of the fatigue limit in bending, σ_{fl}, and torsion, τ_{fl} The endurance limit in bending for this alloy is reported to be σ_{fl}=124 MPa (Nowell, 1988). Specific torsional data has not been found for such material, however τ_{fl} can be predicted approximately from the ratio τ_{fl}/σ_{fl}, which is available in the literature for a wide range of Aluminium alloys (Forrest, 1962, Sauer and Lemmon, 1949, Nishihara and Kawamoto, 1941, Mathaes, 1932, Ludwik, 1931,). These data reveals that the average value of τ_{fl}/σ_{fl} for wrought aluminium alloys is 0.55. This yields τ_{fl}=68.2 MPa.

The cracking risk *DV* was evaluated on and under the surface at spatial intervals $\Delta x/a = \Delta y/a = 0.01$. At each point sixteen different load steps in the complete cycle were examined. The analysis revealed that the larger values of *DV* were found on the surface.

Fig. 5 depicts the variation of the parameter on the contact surface for Al series 1 data. This plot shows that the maximum global value of the parameter, DV_{max}, is reached on the surface at trailing edge of the contact zone, x/a=-1.00. As the parameter exceeds unity at this position high cycle fatigue failure is predicted.

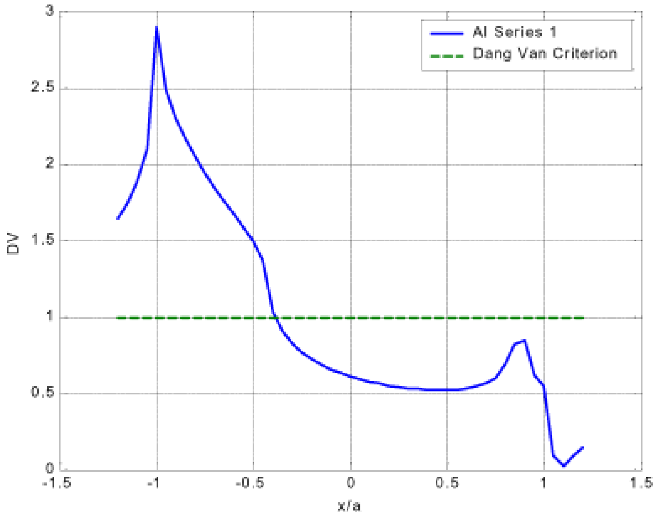

Figure 5. Surface variation of the Dang Van cracking risk parameter DV for Al series 1.

Perhaps a better way to visualize the application of the fatigue criterion is to plot on the same graph the history of the local stress state and the cracking risk line dividing the safe life and damage regions. This is depicted in Figs. 6 and 7. The curves shown are for Al series 1 and 5 data, respectively. A summary of the application of the Dang Van criterion to all tests with Al is presented in Table 3. This records the maximum global value of the cracking risk parameter DV_{max} and its corresponding location of occurrence on the surface, $(x/a)_{max}$, for each data series. Notice that application of the criterion for each different contact size within the same data series is not necessary once the magnitude of stress field on the surface does vary for tests under the same p_0. Values greater than one are predicted for DV_{max} in all the different series data assessed. However, as previously reported the experimental data show that for smaller contact sizes infinite lives were achieved while for larger contacts failure occurred. Here the reader should be reminded that smaller contacts present a more rapidly varying stress field than larger contacts if they are subjected to the same peak pressure. On the other hand, the Dang Van's cracking risk parameter assumes that fatigue damage is controlled by local microscopic stresses of superficial critically stressed points, hence, the Dang Van criterion can not account for the presence of sharp stress gradients.

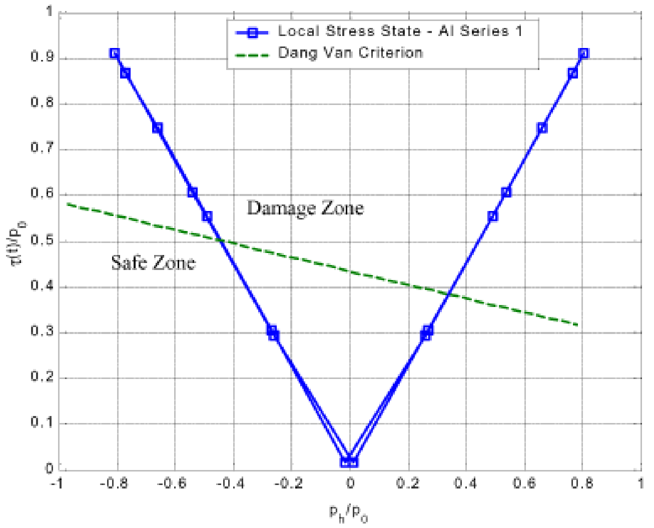

Figure 6. History of local stress state and Dang Van fatigue criterion for Al series 1 data at (x,y)=(-1, 0).

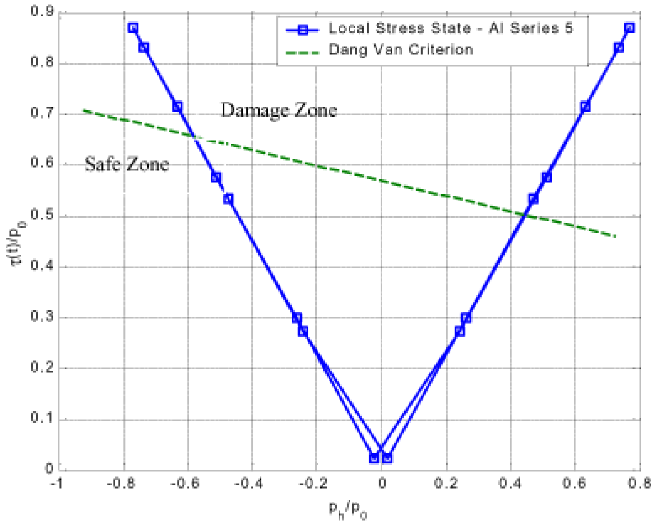

Figure7. History of local stress state and Dang Van fatigue criterion for Al series 5 data at (x,y)=(-1.0, 0).

Table 3. Summary of the Dang Van cracking risk predictions for tests with Al4% Cu.

Series	DV_{max}	x/a
1	2.90	-1.0
3	2.67	-1.0
4	2.44	-1.0
5	1.91	-1.0

Discussion and Conclusions

The analysis of the results previously presented reveals that the Dang Van criterion can not explain the pad size effect in Nowell's experiments with A14%Cu (Nowell, 1988) if the evaluation of the cracking risk parameter is based on the microscopic local stress state of single superficial severely stressed points. The reason for such phenomenon may be associated with the steeper stress gradient present at smaller contacts. As previously discussed, the tests carried out by Nowell (1988) were designed so that the contact size could be varied whilst the peak pressure was kept constant. This essentially means that the stress field for smaller contacts decline more rapidly than for larger ones though they are equal in magnitude on the surface. A direct consequence of this is that a crack growing under smaller contacts will be less severely stressed as it moves away from the interface and therefore it is likely that it will propagate slower than a crack of the same size growing under a larger contact. On the other hand, the Dang Van mesoscopic criterion suggests that the fatigue initiation process is a punctual phenomenon, which will start and be controlled by the level of microscopic stresses at the most severely stressed point of the component. The aforementioned results provide us with strong evidences that this is not the case when steep stress gradients are present. Therefore, a modification of the Dang Van criterion or the development of a new multiaxial fatigue model, which can incorporate such characteristic, is of fundamental importance. The authors intend to address this issue in future work.

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Appendix

Algorithm to compute the residual stress field, which is given by the center of the minimum hypershepere circumscribing the stress path in the deviatoric space.

1. Discretize the stress path $\mathbf{S}(t)$ (in terms of deviatoric stresses) into n points $\mathbf{S}_k := \mathbf{S}(t_k), k = 1, \ldots, n$;
2. Build an initial estimate ρ_{k-1} for the center of the ellipsoid. A natural choice is the centroid of the macroscopic load path

$$\rho_{k-1} = \frac{1}{n} \sum_{k=1}^{n} \mathbf{S}_k \; ; \qquad (11);$$

3. Set an initial (small) estimate for radius of the hypersphere, R_{k-1}.
4. For each stress state $\mathbf{S}_k$ and while convergence of the algorithm is not achieved:
   4.1. Compute the distance of the point from the current center of the hypersphere ρ_{k-1}:

$$D_k = J_2(\mathbf{S}_k - \rho_{k-1}); \qquad (12)$$

   4.2. Compute the amount by which the point $\mathbf{S}_k$ lies outside the hypersphere:

$$P_k = D_k - R_{k-1}. \qquad (13)$$

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