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Faculdade de Economia, Administração, Contabilidade e Gestão de Políticas Públicas

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Treasury Auction Issuance Strategies and Results

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Estratégias de Emissão e Desempenho dos Leilões de Títulos do Tesouro

Tese apresentada ao Programa de Doutorado em Economia da Universidade de Brasília como requisito à obtenção do título de Doutor em Ciências Econômicas.

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Ao meu pai.

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"If you would know the value of money, go and try to borrow some."

Benjamin Franklin

Abstract

Esta tese avalia estratégias de emissão de dívida pública, com foco na frequência (quinzenal ou semanal) dos leilões, critérios de seleção de propostas (preço uniforme ou discriminatório), competitividade dos leilões e formatos (leilão híbrido ou puro) utilizados pelo Tesouro Nacional entre 2014 e 2023. A análise do aumento da frequência dos leilões de NTN-F para semanal, revela menores oscilações nas taxas de captação nos dias antes e após leilões e manutenção do volume negociado no secundário. Os dados sugerem que a oscilação de taxas ao redor de leilões (efeito "V" nos preços) está associada à capacidade limitada de absorção de risco pelos demandantes. A investigação via modelos reduzidos sobre os critérios de seleção de propostas mostra evidências de menores prêmios nos leilões de preço único. Ademais, contrariamente ao esperado pela teoria, encontramos prêmios superiores em leilões híbridos comparativamente aos puros. Investigamos também redes de participantes e competição em leilões. Nossas simulações e testes apontam para ausência de coordenação estratégica de propostas. Na análise de grupos de proponentes, identificou-se que dealers lançam propostas mais competitivas, provavelmente em razão de obrigações e privilégios da função.

Palavras-chaves: Leilões de Títulos Públicos; Leilões de Preços Múltiplos, Leilões de Preço Uniforme, Equivalência de Receita; Choques de Oferta.

Abstract

This thesis examines public debt issuance strategies, focusing on the frequency (biweekly or weekly) of auctions, bid selection criteria (uniform or discriminatory pricing), competitiveness and formats (hybrid or pure auctions) used by the Brazilian National Treasury between 2014 and 2023. First, we assess the impact of increasing the frequency of NTN-F auctions to weekly, observing reduced fluctuations in yields before and after auctions, as well as stable secondary market traded volumes. Data suggest rate volatility around auctions ("V" effect in prices) is associated with limited risk absorption capacity of bidders. Reduced model analysis of bid selection criteria presents evidence of lower premiums in uniform-price auctions. Also, contrary to theoretical expectations, there were significant higher premiums in hybrid auctions relative to traditional ones. In addition we examined networks and competition in auctions. Our simulations and tests point to absence in strategic bid coordination. Analyzing groups of bidders, we found that dealers bid more aggressively, probably due to obligations and privileges of dealership.

Key-words: Treasury Auctions; Discriminatory Auctions; Uniform Auctions; Revenue Equivalence; Supply shocks.

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1 Introduction

The rapid growth of government debts highlights the pressing necessity for continuous improvement of debt management strategies. In that regard, recent policy experiences in Brazil provide interesting conclusions on government security markets.

As of 2025, Brazil's federal government debt surpasses R$ 7 trillion and requires approximately R$ 1.5 trillion annually in refinancing. Since 2018, several changes in treasury auctions were introduced to refine its debt issuance strategies.

The initial focus of this investigation centers on the increased frequency of NTN-F (fixed income bond) auctions, which have been conducted weekly since early 2018, instead of every other week. This policy adjustment provides a natural experiment, facilitating comparative analysis of price dynamics for NTN-F bonds relative to other fixed-rate instruments issued weekly, such as LTNs (zero-coupon bond), in days preceding and afterwards auctions.

We believe the main hypothesis for fluctuations in bond yields around auction days is the limited capacity of major buyers to absorb auction-related risks. Our conclusion is closer to (LOU; YAN; ZHANG, 2013) who attributes the cause to this price movements to dealers' limited risk-bearing capacity and imperfect capital mobility, than to (AMIN; TéDONGAP, 2023) who cites slow moving capital as the principal source.

Our findings support this hypothesis, as price trajectories presented smoother behavior following the adoption of weekly auctions. Specifically, the 'V-shape' effect (a decrease in bond prices in the days leading up to auctions, followed by an increase in the days after) was less pronounced under weekly auctions.

This diagnosis is reinforced by our empirical analysis, including panel regression estimates that account for possible omitted variables and unobserved heterogeneity across entities; difference-in-differences specifications to capture the effects of policy shifts; and IV-2SLS estimates to address potential endogeneity.

The results indicate that auction frequency, bond duration, and higher-risk supply significantly influence the magnitude of rate fluctuations. In addition, external risk factors played a more significant role in explaining price changes for NTN-Fs, which are predominantly held by foreign investors, in contrast to LTNs. Simply put: milder, more frequent auctions and less risk neutralized 'V' movements.

Also, this new price behavior was not accompanied by a decrease in market trading volume nor abnormal returns by risk-takers, undermining other hypothesis that strategic behavior or slow moving capital are the main drivers of the 'V' effect.

In essence, there was no evidence of an increased issuance frequency causing a crowding-out effect in the bond secondary market (participants would not strategically wait for auctions to trade securities instead of buying in the secondary market). On top of that, time-series analysis of hedge fund returns demonstrated no significant relationship between hedge fund daily returns and auction cycles.

In the second part of the thesis, we turn to institutional changes introduced after the outbreak of COVID-19, specially for LFTs (floating rate bonds) and NTN-Bs (inflation-linked bonds).

These include adjustments in bid selection criteria for LFTs (uniform-price vs. pay-as-bid) and modifications in auction formats for LFTs and NTN-Bs from hybrid (i.e. a certain amount offered can be freely allocated to each security in a group of two or three bonds) to traditional multiple-unit auctions of one security.

Both changes in format and criteria serve as experiments to assess the effects of competition and test revenue equivalence - the idea that, under certain conditions, auction types should yield the same revenue for the issuer.

To evaluate revenue equivalence in bid selection criteria, we apply reduced-form models (regression analysis) to LFT and NTN-B data. Our results revealed significantly reduced profits for bidders under uniform-price auctions for some OLS estimates.

Key independent variables in these regressions include supply adjustments or auction cutoff percentages (which align premiums with secondary market levels), bid-to-cover (demand/supply ratio), selection criteria dummies, auction format, auction size, risk supply (market value potential losses of an amount offered from a 1 bp yield increase in the respective security), and market risk factors.

We also found breaks in average auction profits or bid-to-cover ratios time series for NTN-B, unlike LFT - although evidence suggests increased competition following the shift to a uniform-price criterion, according to those variables.

In spite of these results, it is important to acknowledge the lack of empirical consensus on which selection criteria to adopt. There are relevant papers pointing to both directions for greater seller revenue, either uniform-price or multiple-price, not only through reduced-form (regression analysis), but also by structural models(simulating auctions through recovered valuations), as reviewed in the literature in tables 16 and 17.

One interesting result arose from hybrid auction evaluations. Contrary to theoretical models which propose hybrid auctions may help address demand information issues, potentially improving returns for the issuer, our results point in the opposite direction.

This empirical study yields the unexpected finding that there were higher participant profits in hybrid formats, specially in NTN-B, probably related to risk arising from uncertainty in treasury allocation. In hybrid formats, the issuer can allocate the amount offered freely, potentially causing winner's curse in uniform-price auctions.

Lastly, we test relationships of bidders according to their bids in LFT and NTN-B auctions through complex networks and measures of group coherence and exclusivity. These networks were compatible with our simulation of competitive bidders, without evidence of strategic behavior. In addition, our analysis of bids showed dealers tend to be more competitive in general, as expected by dealer's market share obligations and greenshoe benefits.

Sections 2, 3 and 4 are almost independent short papers, with a common bond auction thread. Section 2 presents literature review, empirical strategy, data description, results and key findings on issuance's price dynamics, 'V-shape' patterns. The next two sections repeat the same structure of empirical analysis on revenue equivalence and on competition/demand features. The final section presents our thesis' conclusions concisely.

2 Issuance's Price Dynamics

In order to finance its deficits and stock of debt, most governments space out transactions in time, scheduling auctions. Those primary market issuances could be considered supply shocks of government bonds in different periods (large amounts of securities sold regularly), conversely affecting prices. In that regard, treasury auctions have been extensively studied throughout several markets in order to understand price behaviors and auction mechanics.

One of the prominent phenomena identified is the 'V-shape' price pattern, commonly observed in secondary markets around auction dates. This price behavior is characterized by a drop in security prices prior to auctions, hitting a bottom at the auction clearing time, followed by a price recovery immediately thereafter. Such patterns are documented for various issuers, such as the United States, Japan, Spain, Germany and Italy, respectively by Lou, Yan e Zhang (2013), Hamao e Jegadeesh (1998), Alvarez e Mazon (2019) and Beetsma et al. (2016).

This 'V-shaped' price pattern is attributed to several hypothesized mechanisms. One explanation is imperfectly anticipated supply shocks (uncertainty regarding the amount of government bond sales and price effects arising from natural buyers). According to this explanation, investors reduce their positions as noise decreases closer to auctions with more available information, creating temporary downward price pressure (SIGAUX, 2020). Another potential mechanism is limited risk-bearing capacity, which amplifies pre-auction price declines as primary dealers adjust their inventories in anticipation of significant supply (LOU; YAN; ZHANG, 2013). Additionally, speculative and strategic behavior by market participants may play a role, as they withhold bids in secondary markets and/or shade their auction bids to influence clearing prices. Finally, imperfect capital mobility, characterized by the slow adjustment of capital flows toward auctioned securities, can extend temporary price distortions (AMIN; TéDONGAP, 2023).

Our empirical strategy will delve deeper into these hypothesis for price anomalies in Brazilian Treasury auctions, focusing on fixed-income securities, LTN and NTN-F, encompassing data from nearly 450 auction dates over a decade.

Alongside auction specific variables, such as bid-to-cover (demand/supply ratio), total supply (amount offered), supply adjustments (issuance/supply ratio), profit(auction clearing rate minus secondary market rate), yield changes, risk-supply and auction frequency, this research incorporates market data controls for exchange rate, volatility, credit risk and interest rates. Moreover, we use hedge fund return data, traded volume and holder data in our robustness analysis.

In the next sections we aim to empirically: establish evidence of 'V-shape' price dynamics using daily price data; test prevailing hypothesis, such as dealers' risk-bearing capacity, slow-moving capital or strategic behavior to identify underlying causes; and to examine the impact of auction frequency on price movements, especially comparing pre-2018 fortnightly NTN-F auctions with post-2018 weekly auctions, to understand how auction policies mitigate 'V-shaped' patterns.

2.1 Literature Review

Papers in this section look into price movements (e.g. shape, volatility, direction) prior, throughout and after auctions.

One the first researches on this field (NYBORG; SUNDARESAN, 1996) used treasuries trading data from the when-issued market. Those are forward contracts on treasuries, negotiated between auction announcement and the actual issue date of a given security, in which settlement occurs on the issue date. Securities' transactions are based on its expected yield and dealers can be long on short in this market, covering its positions through auction purchases. As for investors, when-issued market is helpful for acquiring certain positions without the need for primary or secondary market transactions. Essentially, this is a very powerful private source of information for dealers regarding demand depth, participation and price discovery.

His findings suggested uniform-price auctions had higher trading volume in the when-issued market compared to auctions under the discriminatory pricing, particularly prior to auctions - suggesting greater information dissemination, which would reduce uncertainty before auctions and the winner's curse. Also, dealers mark-ups in the when-issued market were not significant under uniform-price regime, whereas positive in discriminatory auctions.

Recent literature begun to address another interesting phenomena in security price dynamics, the "V-shaped" effect on bond prices surrounding auctions.

The main hypothesis for this auction cycles of prices are limited risk-bearing capacity by dealers accompanied by imperfect capital mobility(LOU; YAN; ZHANG, 2013): dealers would short securities in order to hedge future auction purchases, whereas end investors positions were established at a slower rate. He showed in his paper significant returns were possible under this strategy, however it was not done possibly by imperfect capital mobility.

According to Lou, Yan e Zhang (2013), issuers would benefit from an increased auction frequency with lower auction sizes. This would in turn dilute dealer behavior. Although, in this thesis, we did find a smoother 'V' with an increased NTN-F auction frequency, external risk factors played an important role in this security, mostly owned by foreign investors.

Other research related risk-bearing capacity to ‘V-shape’ patterns, such as Beetsma et al. (2016), Alvarez e Mazon (2019) and Smales (2020).

Beetsma et al. (2016) investigated auction price cycles around new public debt issues in Germany and Italy. They pointed out volatility as a contributing factor for this phenomenon, reinforcing the risk-bearing capacity of dealers hypothesis as the likely source of these cycles.

Contrary to auction theory, Alvarez e Mazon (2019) observed consistent bidders’ losses in Spanish treasury issuances - secondary markets prices were lower than auction clearing prices (overpricing). He investigated determinants to this overpricing, arguing dealer institutional regulation was the main reason for it due to increased participation requirements on primary markets. According to him, results signal auction losses were related to "V-shaped" patterns and market manipulation.

Whereas Smales (2020) examined futures market data to assess the impact on rates of U.S. treasury auctions. Corroborating Lou, Yan e Zhang (2013) hypothesis, he claimed increases in prices, volatility, and traded volumes of securities after auctions were due to dealer’s activities covering shorts (buying back positions protecting assets from price declines).

From a different perspective, Amin e Tédongap (2023) inspected price cycles before and after auctions. Based on primary and secondary market data, they concluded slow-moving capital hypothesis would be more plausible for ‘V-shaped’ patterns in TIPS (Treasury Inflation-Protected Securities) auctions due to investors awaiting auctions to purchase securities rather than dealers shorting positions.

They also found evidence of strategic behavior prior to auctions, where direct and indirect bidders reduce demand in days prior to auctions. Unlike (AMIN; TéDONGAP, 2023), analysis of hedge fund returns in Brazil and security trading volumes did not support slow-moving capital and strategic behavior as ‘V’ source.

In a theoretical approach, emphasizing movements prior to auctions, Sigaux (2020) developed a model explaining price decreases before Italy’s treasury auctions, where investors imperfectly anticipate demand, deciding whether to establish long positions or to go short. The result of long positions increases prices, bringing it above expect prices. According to the model, this is followed by noise decline over time, accompanied by reduced purchases, driving prices lower as there is more available information and auctions take place.

In essence, these research encompass mostly risk as main source of ‘V’ patterns, with notable different hypothesis arising from imperfectly anticipated demand for the bonds issued by governments, slow-moving capital and strategic behavior. The shift in Brazilian treasury auction frequency enabled us to test these hypotheses, alongside market data from hedge fund daily returns, market data on traded volume and security holders.

2.2 Empirical Strategy

This research incorporates a public policy experiment in Brazil, specifically the change in auction frequency for NTN-F securities, which enables checking risk-bearing hypothesis proposed by (LOU; YAN; ZHANG, 2013).

The impact of this policy change was assessed using distribution comparisons, statistical tests, and linear regressions measuring price changes around auctions before and after the shift to weekly NTN-F issuances. Additionally, a difference-in-differences (DiD) approach was employed to compare the magnitude of 'V-shaped' patterns between 48-month LTN's and 7-year/10-year NTN-Fs, pre and post policy change.

We began by documenting the 'V-shaped' price patterns observed around Treasury auctions, analyzing their distribution across different benchmarks for Brazilian government bonds LTN and NTN-F. LTNs were categorized into three maturity (benchmark) groups: 6–12 months, 24 months, and 48 months. Whereas NTN-Fs included benchmarks with maturities of 7 and 10 years.

Subsequently, we tested the existence of significant price changes prior and following auctions for each benchmark with two distinct statistics: T-tests and Wilcoxon Statistic. The latter provides robustness to our results due to its non-parametric nature. Those tests were also computed before and after NTN-F auctions became weekly for both sets of maturities.

We ran tests on d_{ij}, which is the sample mean of bond's benchmark j rate change between auction clearing price and closing price of date i, ranging from -3 to +2, where 0 is the auction date. In other words, rate differences from three days prior to auctions and to the next three closing prices after the particular benchmark auction observation.

Exclusively for NTN-Fs, this test is performed additionally for each period and for differences in d_{ij} for each sample: 2014–2017 and 2018–2023.

We also carry out the non parametric Wilcoxon test as a robustness check on the previous exercise. As before, the test was performed separately for each period (2014–2017 and 2018–2023), as well as for the entire sample across all benchmarks.

Next, we turn to the main suspects for auction price cycles, limited risk-bearing capacity by dealers, imperfect capital mobility or strategic behavior: dealers would short securities to hedge future auction purchases, whereas end investors establish positions at a slower rate or would wait for auctions to establish positions.

The following regression analysis considered distinct models evaluating those hypothesis, including pooled regressions incorporating dummy variables for each benchmark (maturity group), comparisons between fixed and random effects, and DiD.

Now, let the dependent variable $d_{i,t}$ represent bond yield changes from clearing to one specific closing date (we used the third closing date, d+2, although robustness analysis for d+0 and d+1 presented similar results), for bond i at auction t.

The panel regression model can be specified as:

$$d_{i,t} = \beta_0 + \beta_1 \text{Market\_Risk}_t + \beta_2 \text{Benchmark\_Risk}_{i,t} + \beta_3 \text{Dummies}_{i,t} + \epsilon_{i,t} \tag{2.1}$$

where Market_Risk_t measures market risk factors, such as exchange-rate, VIX, CDS, DXY and Selic Rate; $\text{Benchmark\_Risk}_{i,t}$ captures benchmark-specific risk factors, such as benchmark rate variations, amount issued, dv01 supplied, bid-to-cover, supply adjustments, number of bids etc.; and $\text{Dummies}_{i,t}$ are dummy variables for structural and/or benchmark-specific effects, such as auction frequency or auction type.

To control for unobserved heterogeneity across bonds, we introduce bond-specific fixed effects. This could captures, for instance, regulatory demand or bond specific demand arising from a particular specialist dealer of that benchmark. Alternatively, we estimate a random effects model, assuming this heterogeneity is uncorrelated with the regressors.

To account for structural breaks, such as policy shifts in 2018, we employ a Difference-in-Differences (DiD) approach. Since both LTN and NTN-F are fixed-rate bonds and LTN has been for decades issued weekly and NTN-F frequency became weekly in 2018 we model DiD as:

$$\begin{aligned} d_{i,t} = \gamma_0 &+ \gamma_1 \text{Market\_Risk}_t + \gamma_2 \text{Benchmark\_Risk}_{i,t} \\ &+ \gamma_3 \text{d\_2018}_t + \gamma_4 \text{d\_bench}_i + \gamma_5 (\text{d\_2018}_t \times \text{d\_bench}_i) \\ &+ \epsilon_{i,t} \end{aligned} \tag{2.2}$$

where d_2018_t is a dummy variable for the post-2018 period and d_bench_i indicates whether bond i is subject to new benchmark auction frequency rules (i.e. NTN-F 7y or 10y), we mark as 0 the 48-month LTN, which in our sample is the closest in duration to NTN-Fs.

As another robustness check accounting for potential endogenous variables we estimate equations through IV-2SLS with lagged variables.

Our first results point out to risk-bearing capacity as a fundamental source of 'V-shape' size, not for only dealers, but for end investors too. Also, we show shorter cycles of auctions decrease the magnitude of this price dynamic.

In order to test speculation hypothesis, we performed a brazilian hedge fund index data analysis (Índice de Hedge Funds ANBIMA - IHFA). This assessment looked into whetter there was any return seasonality on days around auctions through autocorrelation analysis and Fourier Transform, not finding enough evidence for speculation as 'V-shape' patterns primary source, where $R(t)$ represents the returns of the hedge fund index IHFA at business day t. We tried to uncover if there was any relation or peak in returns of hedge funds in specific days of the week which would coincide with auction price dynamics.

Lastly, we inspected traded volume and turnover of outstanding government bonds. Our results suggest dealers' smaller short cycle is probably a smoothing factor for prices, as there is no clear indicator of significant impact on traded volume for those securities nor signs of a crowding out effect by increasing the number of auctions. In short, there was not a declining trend in secondary market due to increased primary market activity leading investors to predominantly await auctions to make strategic purchases.

2.3 Data

Our data comprises LTN and NTN-F auctions between 2014 and 2023, those are fixed income government securities. We group LTN maturties into benchmarks of 6–12 months, 24 months, and 48 months; NTN-Fs are divided into 7 and 10 year benchmarks. For both types of securities we fit those categories of benchmarks according to maturity windows. We end up with about 450 auctions for LTNs and 350 for NTN-Fs.

For clarity, we organize our variables as market risk factors, benchmark-specific risk factors, dummies and deltas (yield changes from a certain auction clearing rate to a specific date).

Market risk variables reflect the broader economic environment, influencing investor behavior and auction outcomes. Benchmark-specific risk factors, such as benchmark rate variations, amount issued, dv01 supplied, bid-to-cover and supply adjustments take into account idiosyncrasies of each auction/maturity. Lastly, deltas are our main dependent variables, from where we capture 'V-Shape' price patterns.

In our sample, we have **USD/BRL**, the exchange rate between Brazilian Real and the US Dollar, higher values might indicate riskier environments for locals and cheaper prices for foreign investors. Our sample captures the move from 2.50 up to the 5 range.

Also **VIX (Volatility Index)**, a gauge for global market uncertainty, arising from implied volatility on near term index options on the S&P 500. Most of the sample is below 20, averaging 18, which is historically low.

We include **CDS (Credit Default Swap Spreads)**, which measures Brazilian 5-year sovereign credit risk. Three quarters of the sample is below 255 basis points, also historically low.

SELIC, Brazil's policy interest rate, directly influencing yield curves. It ranges from 1.9% to 14.15%. Higher SELIC levels correlate with increased yields across all benchmarks.

UST10Y is the 10y treasury yield.

And **DXY (US Dollar Index)**, an index of exchange rates between major currencies and the US Dollar, it correlates with USD/BRL, and international risk-aversion periods.

As benchmark-specific we include **dv01_of**, computed as the price change from a 1 basis-point increase in the bond yield times the amount issued that date, divided by 1 million here. We end up not using it too much due to LFTs extreme issuances driving this indicator upward, and not really reflecting risk.

Dv01_of_bench, computed as the price change from a 1 basis-point increase in the bond yield times the amount issued that date for the specific benchmark, divided by 1 million here. As expected, 48-month benchmarks have significant larger means and medians. Median 48-month benchmark supply is almost ten times larger than 6&12-month LTNs. NTN-F benchmarks dv01 supply is closer to LTN 24-month benchmark since the amount offered is quite lower than LTNs.

Number_bids, the bid count for each auction. Very similar for both NTN-Fs, around 10, with higher values for LTN benchmarks. 48-month and 24-month present close to 23 and 17 bids per auction, whereas 6&12-month LTNs are closer to 13.

Number_bids_ac, the accepted bid count for each auction, approximately 50-60% for all benchmarks.

Ac_ratio corresponding to supply adjustments, reducing the amount issued and profits. It is computed as the ratio between Issuance and Supply. We adjust this indicator for hybrid auctions dividing the supply by the number benchmarks, making it appear above 1 when one benchmark is allocated more than the supply divided by the number of benchmarks.

Bid_to_cover captures the ratio of amount bid relative to initial supply. It tends to be a little bit higher for 6&12-month benchmark securities, probably due to supply restrictions, otherwise demand is around 60% higher than supply (bid_to_cover close to 1.6).

Profit, also regarded as auction premium, is the difference between the auction's clearing rate and the secondary market rate. It is almost zero on average, or slightly negative by 1 basis-point on average across LTNs and NTN-Fs.

Supply, the amount offered for each benchmark. 48-month LTNs has been the most offered fixed-income securities (not including SELIC-linked bonds, such as LFTs), with 4 million bonds auctioned on average, closely followed by 24-month and 6&12-month

benchmarks. The latter, has recently been practically pegged to 1 million bonds a week. Its median reflect the issuer strategy. NTN-Fs have similar average issuance, around 700,000 bonds per issuance.

Issuance, the amount issued for each benchmark, are very close to supply since supply adjustments are very low (ac_ratio close to one).

Clearing Rate, the last accepted bid auction yield for the specific security. This definition works well for 'V-shape' analysis, however we change it for revenue equivalence investigations, becoming average yield.

Closing Rate, the secondary market end of the day yield for the specific security.

Maturity_months is the number of months until maturity, by construction, it is very close to the benchmark classification.

Anb-5, a proxy of the yield changes between last auction date closing price and current auction opening yield - also very close to zero across all benchmarks.

Deltas, as expected by the 'V-shape' price patterns, exhibit higher means for days further from auctions, either before or after, where: delta-3 is the difference in yield for the specific benchmark 3 closing dates prior to the auction to the clearing rate; delta+0 is the difference from the auction date closing yield to the clearing rate; and delta+2 is the yield difference between the third closing rate after the auction and the clearing rate.

Figures 1 and 2 document clearly 'V-shape' price patterns for LTNs and NTN-Fs. The darker line is the mean delta and the shaded areas comprise 2 standard-deviations from it. At a first glance, we find longer durations increasing deltas for both types of bonds. All benchmarks present clear 'V-shape' patterns, except the less risky of them (6&12-month benchmark), which on top of that has restricted supply. Also, deviations are larger as we move away from auctions as other variables influence yield changes over time.

Tables 1, 2, 3, 4, 5, encompass descriptive statistics for each of the three LTN benchmarks and the two NTN-F benchmarks.

Table 1 covers data on the 24-month benchmarks. This intermediary LTN supplied risk (dv01_of_bench) is about half of its 48-month counterpart and double the shorter duration LTNs. Median offering is 2 million bonds, with the first and third quarter ranging from 1.5 million to 3.5 million. This indicators show more risk and therefore more probable 'V-shape' dynamics, than 6&12-month LTNs. Supply adjustments are very close to zero, since ac_ratio is nearing 1, as in all LTNs. Demand is about two-thirds greater than its supply as we can see in the bid_to_cover. In tandem, these show good demand in our sample and low need for supply adjustment diminishing profits. Median and mean profits are almost zero reflecting decent calibration of supply by the issuer. Deltas reflect

'V-shape' patterns with means above zero across the table. Market variables reflecting the macroeconomic environment are very close between tables as our samples have the same date range. They are discussed briefly on its description.

The 48-month LTN benchmark in Table 2 exhibits the highest duration risk among the zero-coupon fixed-rate bonds in our sample, as reflected in its average dv01_of_bench, which is more than twice that of the 24-month benchmark. Issuance size is also notably larger, with a median offering of 3.5 million bonds and an interquartile range from 2 to 5 million, indicating greater market absorption capacity, specially due to regulatory demand throughout great part of the sample. Despite the increased maturity and duration risk, supply adjustments remain minimal, with acceptance ratios tightly clustered around one, suggesting stable allocation practices. Investor demand, measured by the bid_to_cover ratio, is strong and consistent, averaging around 1.63. Mean and median profits near zero suggests that auction pricing is well calibrated, minimizing excess returns to bidders. Price adjustments following auctions display the typical 'V-shape', with positive average deltas on and after the auction date, marginally higher than in the 24-month case - probably due to higher risk supply.

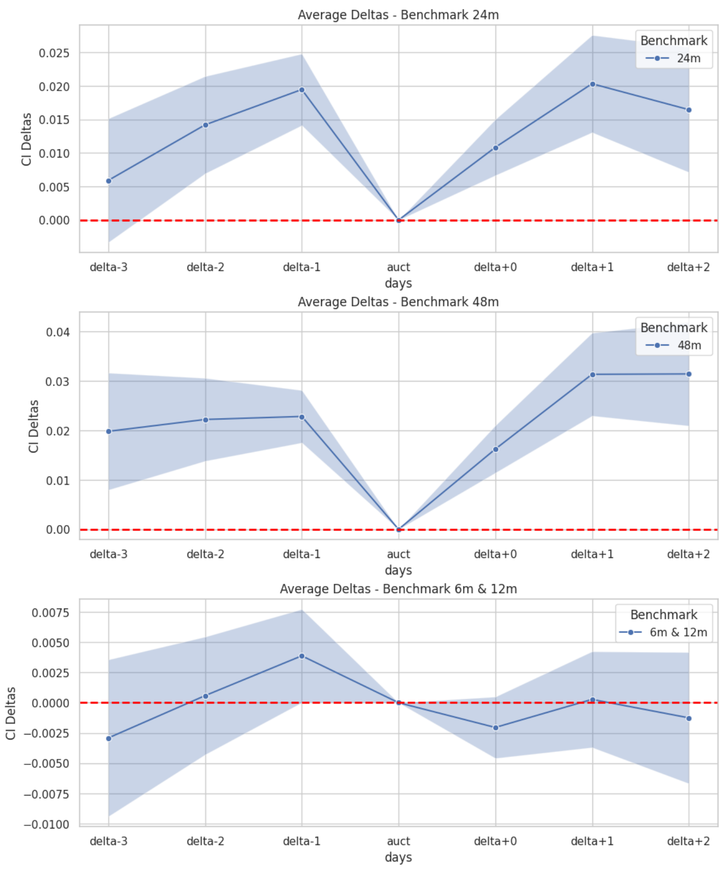
The shortest zero-coupon in the sample — those maturing within 6 to 12 months— exhibit the lowest duration risk, as evidenced by their average dv01_of_bench, which stands at just 0.19 (Table 3). This figure is less than half that of the 24-month benchmark and approximately one-fifth that of the 48-month bonds, confirming the relatively limited exposure to interest rate risk. Limited offerings set interquartile range to zero, with median of 1 million supplied bonds and a highly skewed distribution, as indicated by a long upper tail reaching up to 30 million. This cap on supply brought bid_to_cover ratio averages above 2 — the highest across all LTN benchmarks - and to profits below zero. Acceptance ratios remain tightly centered around one, again indicating minimal supply adjustment in the allocation process. As expected by the limited risk-bearing hypothesis, deltas are close to zero with a less pronounced 'V-shape' pattern as there is less risk involved in these benchmarks.

Table 4 shows descriptive statistics for the 7y NTN-F benchmark. As its supply is reduced in relation to LTN benchmarks due to its duration, demand profile and number of auctions (close to 350 for NTN-Fs and 450 for LTNs), average dv01_of_bench is 0.30, close to the 24-month benchmark. Issuance is relatively measured, with a median of 500,000 bonds and an interquartile range from 150,000 to 1 million, indicating a more cautious supply approach. Auction participation is modest, averaging around 10 bids per auction, with roughly half accepted, and an acceptance ratio below one suggests more caution in bidding. The bid_to_cover ratio, averaging 1.66, indicates healthy but demand, while profits remain slightly negative on average, reflecting tight calibration by the issuer. Price dynamics around the auction shows deltas consistently above zero, suggesting presence of

'V' price patterns.

The 10y NTN-F benchmark in Table 5 presents the longest maturity in this sample, with average dv01_of_bench at 0.39 —higher than the 7y counterpart, reflecting increased interest rate exposure, although not as big as the 48-month LTN due to its smaller supply. Issuance remains measured, with a median of 500,000 bonds and an interquartile range from 150,000 to 1 million, mirroring the 7y benchmark. Auctions are relatively thin, with a median of just 8 bids submitted and 4 accepted, while the acceptance ratio of 0.90 suggests selective supply allocation. Relative demand is shorter than LTN benchmarks, as indicated by a lower average bid_to_cover ratio of 1.51. Profits are close to zero and slightly negative on average, consistent with the 7y benchmark. Price dynamics surrounding the auction reveal persistent positive deltas, with post-auction days showing upward adjustments consistent with 'V-shape' dynamics.

Figure 1 – Shaded LTN



2.4 Results - Issuance's Price Dynamics

Our results point out to risk-bearing capacity as a fundamental source of 'V-shape' size, not for only dealers, but for end investors too. Also, we show shorter cycles of auctions decrease the magnitude of this price dynamic.

Figure 2 – Shaded NTN-F

Table 1 – Descriptive Statistics LTN 24-month Benchmark

Variable	count	mean	std	min	25%	50%	75%	max
dv01_of	436.00	651.50	955.73	0.11	3.87	7.48	1166.20	9515.64
dv01_of_bench	436.00	0.41	0.27	0.01	0.23	0.33	0.55	1.64
number_bids	436.00	17.16	5.36	2.00	14.00	17.00	21.00	34.00
number_bids_ac	436.00	10.07	4.69	1.00	7.00	10.00	13.00	26.00
ac_ratio	436.00	0.98	0.08	0.12	1.00	1.00	1.00	1.00
bid_to_cover	436.00	1.66	0.63	0.61	1.24	1.52	1.92	6.00
profit	436.00	-0.00	0.02	-0.06	-0.01	-0.00	0.00	0.21
USD/BRL	435.00	4.26	0.90	2.57	3.34	4.06	5.17	5.89
DXY	424.00	114.85	4.84	103.08	111.64	114.65	117.66	128.17
VIX	428.00	18.26	6.33	9.19	13.52	16.70	21.67	50.91
UST10Y	425.00	2.29	0.93	0.55	1.66	2.21	2.85	4.98
CDS	436.00	224.30	73.49	92.52	173.62	217.65	253.08	495.45
SELIC	436.00	9.20	4.28	1.90	6.15	9.65	13.65	14.15
Supply	436.00	2711353.21	1814061.49	50000.00	1500000.00	2000000.00	3500000.00	12000000.00
Issuance	436.00	2662692.32	1825016.65	50000.00	1500000.00	2000000.00	3500000.00	12000000.00
Clearing Rate	436.00	9.64	3.26	3.44	7.28	9.64	12.40	16.31
Closing rate	420.00	9.62	3.26	3.37	7.24	9.66	12.37	16.31
maturity_months	436.00	23.82	2.13	18.00	22.00	24.00	25.25	30.00
delta-3	420.00	0.00	0.18	-0.61	-0.11	0.01	0.09	1.04
delta-2	420.00	0.01	0.14	-0.42	-0.07	0.01	0.08	0.82
delta-1	419.00	0.02	0.11	-0.41	-0.04	0.02	0.07	0.63
delta+0	420.00	0.01	0.08	-0.60	-0.03	0.01	0.05	0.37
delta+1	435.00	0.02	0.14	-0.84	-0.05	0.03	0.10	0.62
delta+2	436.00	0.02	0.18	-0.94	-0.07	0.03	0.12	0.68
anb-5	403.00	-0.02	0.22	-0.89	-0.16	-0.04	0.08	1.34

Table 2 – Descriptive Statistics LTN 48-month Benchmark

Variable	count	mean	std	min	25%	50%	75%	max
dv01_of	441.00	644.41	949.50	0.11	3.83	7.41	1136.16	9515.64
dv01_of_bench	441.00	0.90	0.67	0.01	0.48	0.78	1.13	5.27
number_bids	441.00	23.08	8.77	1.00	18.00	23.00	28.00	55.00
number_bids_ac	441.00	13.88	7.02	1.00	9.00	14.00	19.00	38.00
ac_ratio	441.00	0.97	0.12	0.03	1.00	1.00	1.00	1.33
bid_to_cover	441.00	1.63	0.65	0.33	1.22	1.47	1.87	5.33
profit	441.00	-0.00	0.03	-0.12	-0.01	-0.00	0.00	0.31
USD/BRL	440.00	4.25	0.90	2.57	3.33	4.03	5.16	5.89
DXY	429.00	114.80	4.80	103.08	111.76	114.63	117.56	128.17
VIX	433.00	18.13	6.23	9.19	13.49	16.59	21.58	50.91
UST10Y	430.00	2.30	0.92	0.55	1.68	2.23	2.86	4.98
CDS	441.00	224.60	72.99	92.52	173.74	218.31	254.21	495.45
SELIC	441.00	9.21	4.25	1.90	6.40	10.15	13.65	14.15
Supply	441.00	3901247.17	2942950.67	50000.00	2000000.00	3500000.00	5000000.00	20000000.00
Issuance	441.00	3800021.09	2928794.62	40000.00	2000000.00	3137500.00	5000000.00	20000000.00
Clearing Rate	441.00	10.18	2.70	4.82	8.28	10.32	12.14	16.82
Closing rate	426.00	10.15	2.71	4.77	8.21	10.33	12.12	16.65
maturity_months	441.00	42.79	3.67	36.00	40.00	43.00	46.00	51.00
delta-3	426.00	0.01	0.23	-0.89	-0.10	0.01	0.11	1.72
delta-2	426.00	0.02	0.17	-0.49	-0.07	0.01	0.10	1.52
delta-1	425.00	0.02	0.10	-0.40	-0.04	0.02	0.07	0.66
delta+0	426.00	0.02	0.10	-0.56	-0.03	0.02	0.07	0.35
delta+1	440.00	0.03	0.16	-0.65	-0.06	0.03	0.12	0.60
delta+2	441.00	0.03	0.20	-0.60	-0.09	0.04	0.14	0.74
anb-5	410.00	-0.01	0.28	-0.95	-0.17	-0.03	0.11	1.90

Table 3 – Descriptive Statistics LTN 6&12-month Benchmark

Variable	count	mean	std	min	25%	50%	75%	max
dv01_of	430.00	652.52	957.04	0.11	3.89	7.60	1152.64	9515.64
dv01_of_bench	430.00	0.19	0.29	0.00	0.05	0.09	0.17	1.99
number_bids	430.00	12.86	7.02	1.00	8.00	11.00	15.75	43.00
number_bids_ac	430.00	6.79	5.32	1.00	3.00	5.00	8.00	36.00
ac_ratio	430.00	0.97	0.12	0.06	1.00	1.00	1.00	1.00
bid_to_cover	430.00	2.10	0.93	0.06	1.44	2.00	2.65	6.96
profit	430.00	-0.01	0.01	-0.09	-0.02	-0.01	-0.00	0.05
USD/BRL	429.00	4.25	0.90	2.57	3.33	4.04	5.16	5.89
DXY	418.00	114.79	4.82	103.08	111.62	114.65	117.54	128.17
VIX	422.00	18.22	6.36	9.19	13.49	16.59	21.62	50.91
UST10Y	419.00	2.28	0.93	0.55	1.67	2.20	2.84	4.98
CDS	430.00	224.30	73.90	92.52	173.42	217.19	252.92	495.45
SELIC	430.00	9.18	4.28	1.90	6.21	9.15	13.65	14.15
Supply	430.00	2838372.09	4188633.99	50000.00	1000000.00	1000000.00	3000000.00	30000000.00
Issuance	430.00	2785647.21	4153857.53	25000.00	1000000.00	1000000.00	3000000.00	30000000.00
Clearing Rate	430.00	9.31	3.99	2.01	6.33	9.65	13.30	15.71
Closing rate	427.00	9.29	3.99	2.02	6.34	9.59	13.28	15.57
maturity_months	430.00	8.81	3.52	3.00	6.00	9.00	12.00	15.00
delta-3	427.00	-0.01	0.13	-0.75	-0.07	-0.01	0.05	0.73
delta-2	427.00	-0.00	0.09	-0.58	-0.05	-0.01	0.04	0.40
delta-1	426.00	0.00	0.07	-0.36	-0.03	0.00	0.03	0.38
delta+0	427.00	-0.00	0.05	-0.66	-0.02	-0.00	0.02	0.15
delta+1	430.00	-0.00	0.08	-0.46	-0.03	0.01	0.04	0.19
delta+2	430.00	-0.00	0.11	-0.61	-0.04	0.01	0.06	0.29
anb-5	426.00	-0.02	0.14	-0.53	-0.09	-0.03	0.04	1.08

Table 4 – Descriptive Statistics NTN-F 7y Benchmark

Variable	count	mean	std	min	25%	50%	75%	max
dv01_of	353.00	569.13	859.23	0.11	3.79	6.25	1137.54	6007.93
dv01_of_bench	353.00	0.30	0.33	0.01	0.07	0.19	0.40	2.51
number_bids	353.00	9.91	6.31	1.00	5.00	9.00	13.00	29.00
number_bids_ac	353.00	5.59	4.10	1.00	2.00	5.00	8.00	22.00
ac_ratio	353.00	0.90	0.23	0.07	1.00	1.00	1.00	1.33
bid_to_cover	353.00	1.66	0.77	0.09	1.22	1.60	2.00	5.30
profit	353.00	-0.01	0.02	-0.19	-0.02	-0.01	0.00	0.03
USD/BRL	353.00	4.41	0.87	2.60	3.72	4.36	5.21	5.89
DXY	342.00	115.28	4.72	103.28	112.02	115.09	118.21	128.17
VIX	346.00	18.75	5.82	9.22	14.02	17.58	22.41	39.16
UST10Y	343.00	2.31	1.00	0.55	1.60	2.19	2.91	4.98
CDS	353.00	218.16	79.57	92.52	168.68	206.59	243.19	521.36
SELIC	353.00	8.66	4.37	1.90	5.40	7.40	13.65	14.15
Supply	353.00	715722.38	749564.72	50000.00	150000.00	500000.00	1000000.00	4000000.00
Issuance	353.00	684122.10	753889.35	4500.00	150000.00	500000.00	1000000.00	4000000.00
Clearing Rate	353.00	10.20	2.53	5.96	8.46	10.26	11.99	16.40
Closing rate	353.00	10.19	2.53	5.93	8.41	10.30	11.99	16.27
maturity_months	353.00	71.88	10.90	15.00	66.00	71.00	79.00	97.00
delta-3	353.00	0.02	0.25	-0.92	-0.09	0.01	0.13	1.72
delta-2	353.00	0.02	0.18	-0.62	-0.07	0.02	0.10	1.61
delta-1	352.00	0.02	0.10	-0.36	-0.04	0.02	0.07	0.66
delta+0	353.00	0.01	0.10	-0.42	-0.04	0.02	0.06	0.33
delta+1	353.00	0.03	0.16	-0.49	-0.07	0.02	0.12	0.57
delta+2	353.00	0.03	0.21	-0.58	-0.09	0.03	0.14	0.85
anb-5	352.00	-0.00	0.28	-0.79	-0.17	-0.02	0.13	1.71

Table 5 – Descriptive Statistics NTN-F 10y Benchmark

Variable	count	mean	std	min	25%	50%	75%	max
dv01_of	347.00	540.22	832.66	0.19	3.73	5.80	1133.84	6007.93
dv01_of_bench	347.00	0.39	0.47	0.02	0.09	0.26	0.52	5.10
number_bids	347.00	9.29	6.51	1.00	4.00	8.00	13.00	49.00
number_bids_ac	347.00	5.48	4.16	1.00	2.00	4.00	8.00	22.00
ac_ratio	347.00	0.90	0.22	0.05	1.00	1.00	1.00	1.00
bid_to_cover	347.00	1.51	0.64	0.07	1.13	1.42	1.80	4.28
profit	347.00	-0.01	0.02	-0.20	-0.02	-0.01	0.00	0.08
USD/BRL	347.00	4.39	0.86	2.60	3.70	4.23	5.18	5.89
DXY	338.00	115.38	4.62	103.28	112.28	115.24	118.49	128.17
VIX	341.00	18.48	5.80	9.55	13.71	17.03	22.08	39.16
UST10Y	339.00	2.36	0.98	0.55	1.63	2.27	2.95	4.98
CDS	347.00	216.51	71.48	92.52	170.42	209.47	243.40	521.36
SELIC	347.00	8.73	4.24	1.90	5.90	7.40	13.15	14.15
Supply	347.00	665129.68	692993.57	50000.00	150000.00	500000.00	1000000.00	4500000.00
Issuance	347.00	630642.07	693804.28	5000.00	150000.00	325000.00	1000000.00	4500000.00
Clearing Rate	347.00	10.54	2.27	6.51	8.99	10.80	12.14	16.71
Closing rate	345.00	10.50	2.25	6.47	8.95	10.75	12.13	16.49
maturity_months	347.00	120.43	7.06	104.00	115.00	120.00	126.00	133.00
delta-3	345.00	0.03	0.24	-0.88	-0.09	0.02	0.14	1.70
delta-2	345.00	0.02	0.18	-0.88	-0.07	0.02	0.11	1.62
delta-1	344.00	0.03	0.10	-0.40	-0.03	0.02	0.08	0.69
delta+0	345.00	0.02	0.10	-0.30	-0.04	0.02	0.07	0.35
delta+1	347.00	0.03	0.16	-0.51	-0.05	0.03	0.11	0.50
delta+2	347.00	0.03	0.20	-0.54	-0.08	0.03	0.15	0.75
anb-5	341.00	-0.00	0.27	-0.79	-0.16	-0.02	0.12	1.68

After spotting 'V-shape' patterns in Brazilian National Treasury auctions, we delve deeper in the data. Here we present: statistical tests rejecting the null hypothesis that deltas are zero for both LTN and NTN-F; statistical tests showing the difference in the size of deltas are different from zero when we divide the data into two samples, the first slice for auctions every fortnight and the second for weekly auctions; we estimate deltas on a set of control variables through POLS with benchmark dummies, panel regressions with fixed and random effects and differences-in-differences (DiD) regressions corroborating the risk-bearing hypothesis; then we proceed to robustness analysis contrarian to strategic behavior, as Brazilian hedge-fund returns do not present enough evidence for speculation as 'V-shape' patterns primary source; finally, we inspect NTN-F turnover and secondary market traded volume spotting no clear signal of significant impact on traded volume for those securities arising from auction frequency.

Figure 3 tells the story of the policy change. While NTN-F auctions were every fortnight, the sample from 2014 up to the end of 2017, rates increased more sharply prior to auctions and decreased further after them. This pattern in rates (prices) has become smoother after issuances became weekly. The darker line is the mean delta and the shaded areas comprise 2 standard-deviations from it. We also see a more pronounce effect in the longer/riskier NTN-F benchmark, 10y.

Figure 3 – Shaded NTN-F Experiment

Tables 6 and 7 present mean delta by type of bond and benchmark. In a nutshell, longer LTN benchmarks and all NTN-F benchmarks show delta significantly above 0 on

T-Tests. Results are similar in the non-parametric Wilcoxon rank-sum test statistic.

Table 6 – Results by Benchmark and Delta - LTN

Benchmark	Delta	Mean	T-Statistic	P-Value (T-Test)	Wilcoxon Statistic	P-Value (Wilcoxon)	95% CI Lower	95% CI Upper	Observations
24m	delta-3	0.005917	0.710893	0.4775	52440.0	0.4511	-0.010440	0.022274	467
24m	delta-2	0.014208	2.172464	0.0303*	48208.5	0.0397*	0.001356	0.027060	467
24m	delta-1	0.019483	4.044838	0.0001*	40854.5	0.0000*	0.010018	0.028948	466
24m	delta+0	0.010852	2.884869	0.0041*	42878.0	0.0001*	0.003460	0.018244	467
24m	delta+1	0.020349	3.166633	0.0016*	45153.5	0.0000*	0.007723	0.032976	484
24m	delta+2	0.016479	1.996834	0.0464*	48851.5	0.0011*	0.000264	0.032695	485
48m	delta-3	0.019809	1.872418	0.0618	51280.5	0.1088	-0.000979	0.040598	474
48m	delta-2	0.022203	2.961552	0.0032*	48561.0	0.0096*	0.007471	0.036935	474
48m	delta-1	0.022818	4.807252	0.0000*	39784.5	0.0000*	0.013491	0.032145	473
48m	delta+0	0.016211	3.793871	0.0002*	41913.0	0.0000*	0.007815	0.024608	474
48m	delta+1	0.031335	4.248915	0.0000*	44751.5	0.0000*	0.016845	0.045825	490
48m	delta+2	0.031411	3.412180	0.0007*	49074.0	0.0003*	0.013324	0.049498	491
6m & 12m	delta-3	-0.002913	-0.504538	0.6141	53082.5	0.1932	-0.014257	0.008431	477
6m & 12m	delta-2	0.000583	0.134459	0.8931	54986.5	0.5541	-0.007940	0.009106	477
6m & 12m	delta-1	0.003874	1.126123	0.2607	51960.0	0.1272	-0.002886	0.010634	476
6m & 12m	delta+0	-0.002045	-0.903953	0.3665	55559.0	0.7469	-0.006490	0.002400	477
6m & 12m	delta+1	0.000274	0.077774	0.9380	51137.0	0.0253*	-0.006644	0.007191	481
6m & 12m	delta+2	-0.001249	-0.259390	0.7954	52733.5	0.0866	-0.010709	0.008211	481

Table 7 – Results by Benchmark and Delta - NTN-F

Benchmark	Delta	Mean	T-Stat.	P-Value (T-Test)	Wilcoxon Stat.	P-Value (Wilcoxon)	95% CI Lower	95% CI Upper	Obs.
10y	delta-3	0.044031	3.198832	0.0015*	27715.5	0.0017*	0.016964	0.071098	370
10y	delta-2	0.033289	3.112177	0.0020*	27435.5	0.0008*	0.012255	0.054322	370
10y	delta-1	0.030618	5.058844	0.0000*	22532.5	0.0000*	0.018716	0.042519	369
10y	delta+0	0.016169	3.284448	0.0011*	26029.5	0.0001*	0.006489	0.025850	370
10y	delta+1	0.031432	3.538466	0.0005*	26143.5	0.0000*	0.013965	0.048899	375
10y	delta+2	0.037349	3.436289	0.0007*	27976.0	0.0005*	0.015977	0.058721	375
7y	delta-3	0.032543	2.192401	0.0290*	31333.0	0.0350*	0.003357	0.061730	379
7y	delta-2	0.034754	2.972238	0.0031*	29868.0	0.0040*	0.011763	0.057745	379
7y	delta-1	0.028579	4.186018	0.0000*	25812.0	0.0000*	0.015155	0.042003	378
7y	delta+0	0.010408	2.122139	0.0345*	28646.5	0.0013*	0.000764	0.020051	379
7y	delta+1	0.031095	3.373672	0.0008*	27997.5	0.0002*	0.012972	0.049218	379
7y	delta+2	0.036875	3.230966	0.0013*	29441.5	0.0021*	0.014434	0.059316	379

Then, we proceed to sliced sample in Tables 8 and 9. Although deltas are significantly above zero in both slices in Table 8, these results are less common in the second sample (2018-2023). T-Tests on delta differences between samples in Table 9 are significant only in delta+2 for the 7y benchmark and in delta-2 for the 10y benchmark at the 5% level. Wilcoxon Statistics are significant in the 5% level for three 7y and 10y benchmark deltas. This last results signal less pronounced 'V-shape' for both benchmarks, as we tested differnce in magnitude of these dynamics after the policy shift.

Next, we start regression analysis. POLS for LTN, Table 10, display results compatible wih the risk-bearing hypothesis: the amount of risk offered (dv01_of_bench) is positively related with price rebounds after auctions. In specifications 2, 3 and 5 containing dummies for benchmarks, d_bench24 and d_bench48, respectively LTN 24m and LTN 48m, coefficients for those are significantly different from zero on the 10% and in the 1% threshold. This reflects our graphical analysis of 'V-shape' patterns increase in the longer duration bonds in Figures 1 and 2, as well statistical tests in Tables 6 and 7.

Our panel estimates, Table 11, showed similar results for POLS, fixed effects (FE) and random effects (RE) with p-value above 0.9 from Hausman Test - the same for NTN-F panel regressions. It is noteworthy that higher USD/BRL and anb-5 (yield changes between last auction date closing price and current auction opening yield) led to lower

Table 8 – Results by Benchmark and Delta (2 samples) - NTN-F

Period	**Benchmark**	**Delta**	**Mean**	**T-Stat.**	**P-Value (T-Test)**	**Wilcoxon Stat.**	**P-Value (Wilcoxon)**	**95% CI Lower**	**95% CI Upper**	**Obs.**
2014-2017	10y	delta-3	0.088381	2.718998	0.0078*	1418.0	0.0009*	0.023850	0.152912	96
2014-2017	10y	delta-2	0.070156	2.705586	0.0081*	1378.0	0.0005*	0.018678	0.121634	96
2014-2017	10y	delta-1	0.047113	4.612268	0.0000*	1155.0	0.0000*	0.026834	0.067391	96
2014-2017	10y	delta+0	0.024546	2.555405	0.0122*	1580.0	0.0063*	0.005477	0.043615	96
2014-2017	10y	delta+1	0.046851	2.674342	0.0088*	1492.0	0.0023*	0.012072	0.081630	96
2014-2017	10y	delta+2	0.066641	3.016561	0.0033*	1426.5	0.0010*	0.022783	0.110498	96
2014-2017	7y	delta-3	0.053767	1.622202	0.1079	1872.0	0.0118*	-0.011983	0.119516	102
2014-2017	7y	delta-2	0.049885	2.012522	0.0468*	1779.0	0.0047*	0.000714	0.099057	102
2014-2017	7y	delta-1	0.040356	3.946604	0.0001*	1380.0	0.0001*	0.020071	0.060640	102
2014-2017	7y	delta+0	0.012175	1.265738	0.2085	2067.5	0.0620	-0.006906	0.031255	102
2014-2017	7y	delta+1	0.048679	2.893002	0.0047*	1633.5	0.0009*	0.015300	0.082059	102
2014-2017	7y	delta+2	0.076501	3.414314	0.0009*	1563.5	0.0004*	0.032054	0.120948	102
2018-2023	10y	delta-3	0.028523	1.944524	0.0529	16510.5	0.1138	-0.000355	0.057400	273
2018-2023	10y	delta-2	0.020007	1.787509	0.0750	16649.5	0.1162	-0.002028	0.042042	273
2018-2023	10y	delta-1	0.024818	3.386763	0.0008*	13508.0	0.0001*	0.010391	0.039244	273
2018-2023	10y	delta+0	0.013135	2.283217	0.0232*	14753.5	0.0033*	0.001809	0.024461	273
2018-2023	10y	delta+1	0.026028	2.477044	0.0139*	14666.0	0.0020*	0.005341	0.046714	273
2018-2023	10y	delta+2	0.026924	2.125608	0.0344*	16085.5	0.0452*	0.001987	0.051861	273
2018-2023	7y	delta-3	0.024791	1.520210	0.1296	17844.0	0.3916	-0.007313	0.056895	276
2018-2023	7y	delta-2	0.028890	2.189120	0.0294*	17228.5	0.1556	0.002910	0.054870	276
2018-2023	7y	delta-1	0.024227	2.834207	0.0049*	15407.0	0.0090*	0.007399	0.041055	276
2018-2023	7y	delta+0	0.009597	1.675524	0.0950	15330.0	0.0098*	-0.001679	0.020874	276
2018-2023	7y	delta+1	0.024567	2.230207	0.0265*	15965.0	0.0226*	0.002881	0.046253	276
2018-2023	7y	delta+2	0.022058	1.668484	0.0964	17269.5	0.1648	-0.003968	0.048085	276

Table 9 – Benchmarks and Deltas Tests for Differences in Samples - NTN-F

Benchmark	Delta	Mean Diff	T-Stat.	P-Value (T-Test)	Wilcoxon Stat.	P-Value (Wilcoxon)	Obs.2014-2017	Obs.2018-2023
7y	delta-3	-0.0290	0.8639	0.3882	15815	0.0652	102	276
7y	delta-2	-0.0210	0.7946	0.4273	16098	0.0320*	102	276
7y	delta-1	-0.0161	1.0488	0.2950	16527	0.0094*	102	276
7y	delta+0	-0.0026	0.2324	0.8164	14388	0.7407	102	276
7y	delta+1	-0.0241	1.1587	0.2473	15634	0.0985	102	276
7y	delta+2	-0.0544	2.1217	0.0345*	16359	0.0155*	102	276
10y	delta-3	-0.0599	1.9096	0.0570	15218	0.0187*	96	273
10y	delta-2	-0.0501	2.0609	0.0400*	15447	0.0091*	96	273
10y	delta-1	-0.0223	1.6197	0.1062	15155	0.0225*	96	273
10y	delta+0	-0.0114	1.0143	0.3111	13873	0.3923	96	273
10y	delta+1	-0.0208	1.0138	0.3114	14303	0.1823	96	273
10y	delta+2	-0.0397	1.5851	0.1138	14751	0.0670	96	273

delta+2. This could be either from stops (closing losing positions by dealers/investors) after auctions or even upward trend in local rates.

Probably due to higher benchmark duration, NTN-F estimates in Tables 12 and 13 exhibit baseline delta+2 higher than LTN. Differences in risk from 7y to 10y benchmarks did not enable capturing significant relationships of dv_of_bench or d_bench10y (dummy for 10y NTN-F benchmark). Consistent with expectations, ac_ratio, higher issuance relative to auction initial supply, is negatively related to delta+2. This might be for various reasons, including supply restrictions in riskier periods.

Another interesting results: domestic risk factors move deltas for LTNs, whereas external risk factors impact more significantly NTN-Fs. The latter results steer us to risk-bearing hypothesis for end investors too, as locals/dealers are the main investors of LTNs (hence affected by local risk offered and local volatility), while foreigners predominantly hold NTN-Fs, being mostly affected by international factors, such as VIX, and exchange rate.

DiD estimates in Table 14 compared delta+2 from LTN 48-month benchmark and NTN-F 7y or NTN-F 10y through the full sample. We added dummies for the years prior to 2018, when auctions were every fortnight and deltas higher according to our data, dummies for either NTN-F benchmark and another for both year and NTN-F.

In those, it was spotted weak evidence of reduced deltas after NTN-F auctions became weekly. However, we did perform one last OLS estimate on NTN-F 10y benchmark with a dummy for years prior to 2018, with a significant coefficient on the 5% level, Table 15.

Table 10 – OLS Results for Multiple Specifications - LTN

Dependent Variable: $\delta + 2$	**Spec. 1**	**Spec. 2**	**Spec. 3**	**Spec. 4**	**Spec. 5**
const	0.0752 (0.094)	0.0521 (0.092)	0.0819 (0.095)	0.0756 (0.087)	0.0794 (0.087)
anb-5	-0.0657 (0.041)	-0.0721* (0.041)	-0.0657 (0.041)	-0.0728* (0.041)	-0.0770* (0.040)
USD/BRL	-0.0200** (0.008)	-0.0165** (0.008)	-0.0215*** (0.008)	-0.0167** (0.008)	-0.0102* (0.006)
VIX	0.0019 (0.001)	0.0019 (0.001)	0.0020 (0.001)	0.0016 (0.001)	
UST10Y	0.0035 (0.006)	0.0038 (0.007)	0.0065 (0.005)		
SELIC	0.0011 (0.002)	0.0008 (0.001)			
dv01_of_bench	0.0267* (0.015)		0.0247* (0.015)		
number_bids	0.0007 (0.001)	0.0013 (0.001)	0.0008 (0.001)	0.0012* (0.001)	
ac_ratio	-0.0659 (0.084)	-0.0638 (0.086)	-0.0665 (0.085)	-0.0669 (0.083)	-0.0486 (0.081)
bid_to_cover	0.0059 (0.006)	0.0052 (0.006)	0.0058 (0.006)	0.0061 (0.006)	0.0045 (0.006)
d_bench24		0.0149 (0.012)		0.0153 (0.012)	0.0194* (0.011)
d_bench48		0.0247 (0.016)		0.0247 (0.015)	0.0355*** (0.012)
Observations	1204	1204	1204	1213	1236
R-squared	0.031	0.030	0.030	0.029	0.022
BIC	-814.751	-806.863	-821.310	-831.209	-861.329

Table 11 – Fixed Effects and Random Effects Regression Results - LTN

Variable	**Fixed Effects**	**Random Effects**
const	0.1104** (0.050)	0.1120** (0.049)
bid_to_cover	0.0092 (0.007)	0.0073 (0.007)
ac_ratio	-0.0648 (0.047)	-0.0631 (0.047)
dv01_of_bench	0.0264** (0.011)	0.0335*** (0.009)
anb-5	-0.0702*** (0.022)	-0.0673*** (0.022)
VIX	0.0018* (0.001)	0.0017* (0.001)
USD/BRL	-0.0222*** (0.007)	-0.0228*** (0.007)
R-squared	0.024	0.029
Observations	1213	1213

Table 12 – OLS Results for Multiple Specifications - NTN-F

Dependent Variable: $\delta + 2$	**Spec. 1**	**Spec. 2**	**Spec. 3**	**Spec. 4**	**Spec. 5**
const	0.1512** (0.070)	0.1495** (0.070)	0.1690** (0.070)	0.1845*** (0.065)	0.1817*** (0.060)
anb-5	-0.0374 (0.039)	-0.0371 (0.039)	-0.0398 (0.040)	-0.0351 (0.040)	-0.0348 (0.039)
USD/BRL	-0.0176 (0.012)	-0.0178 (0.012)	-0.0244** (0.012)	-0.0261** (0.012)	-0.0255** (0.011)
VIX	0.0023 (0.002)	0.0024 (0.002)	0.0035* (0.002)	0.0034* (0.002)	0.0034* (0.002)
UST10Y	-0.0266** (0.012)	-0.0265** (0.011)	0.0030 (0.008)		
SELIC	0.0096*** (0.003)	0.0096*** (0.003)			
dv01_of_bench	-0.0045 (0.027)		0.0043 (0.029)		
number_bids	-0.0016 (0.002)	-0.0018 (0.002)	-0.0000 (0.002)	-0.0002 (0.002)	
ac_ratio	-0.0871* (0.050)	-0.0884* (0.051)	-0.1094** (0.051)	-0.1073** (0.050)	-0.1087** (0.047)
bid_to_cover	-0.0057 (0.015)	-0.0047 (0.014)	-0.0013 (0.015)	-0.0015 (0.014)	-0.0016 (0.014)
d_bench10y		0.0028 (0.016)		0.0042 (0.016)	0.0043 (0.016)
Observations	675	675	675	680	680
R-squared	0.046	0.046	0.025	0.025	0.025
BIC	-176.706	-176.719	-168.971	-176.947	-183.458

Then, we based our tests to unearth strategic behavior/timing auctions or slow moving capital through returns of hedge funds. IHFA (hedge fund index) return time series by Anbima from 2014 until the end of 2023 inspection of seasonality through autocorrelation, Fourier Transform and heatmaps in Figure 4 detected no evidence of auction cycles

Table 13 – Fixed Effects and Random Effects Regression Results - NTN-F

Variable	Fixed Effects	Random Effects
const	0.1833*** (0.055)	0.1835*** (0.055)
bid_to_cover	-0.0015 (0.013)	-0.0019 (0.013)
ac_ratio	-0.1091*** (0.042)	-0.1087*** (0.042)
dv01_of_bench	0.0009 (0.025)	0.0014 (0.025)
anb-5	-0.0347 (0.029)	-0.0346 (0.029)
VIX	0.0034** (0.002)	0.0034** (0.002)
USD/BRL	-0.0255** (0.012)	-0.0254** (0.012)
R-squared	0.025	0.025
Observations	680	680

Table 14 – DiD Results LTN 48m vs NTN-F 7y and 10y

Variable	48m vs 7y	48m vs 10y
const	0.1618** (0.078)	0.1201 (0.076)
bid_to_cover	0.0037 (0.013)	-0.0113 (0.015)
ac_ratio	-0.1453** (0.059)	-0.1198* (0.064)
anb-5	-0.0446 (0.037)	-0.0314 (0.037)
VIX	0.0030* (0.002)	0.0026 (0.002)
USD/BRL	-0.0131 (0.013)	-0.0025 (0.013)
d_2018	0.0207 (0.027)	0.0338 (0.028)
d_bench7y	-0.0180 (0.018)	
d_2018_7y	0.0442 (0.036)	
d_bench10y		-0.0080 (0.017)
d_2018_10y		0.0247 (0.036)
R-squared	0.033	0.024
Observations	746	736

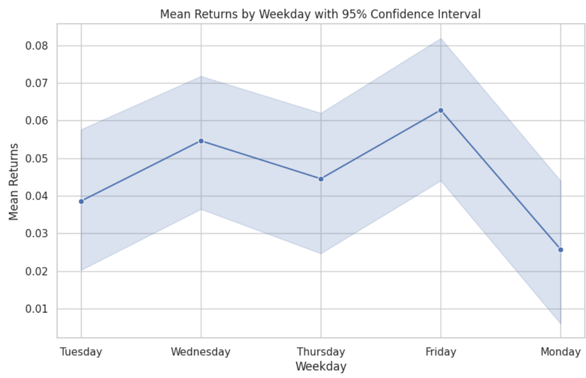
Table 15 – OLS NTN-F 10y Dummy 2018

Variable	Coefficient (Std. Error)	P>|z|
const	0.0412 (0.096)	0.667
bid_to_cover	-0.0271 (0.026)	0.295
ac_ratio	-0.0725 (0.071)	0.309
anb-5	-0.0174 (0.053)	0.742
VIX	0.0023 (0.002)	0.349
USD/BRL	0.0096 (0.019)	0.608
d_2018	0.0758** (0.036)	0.037
R-squared	0.034	
Observations	335	

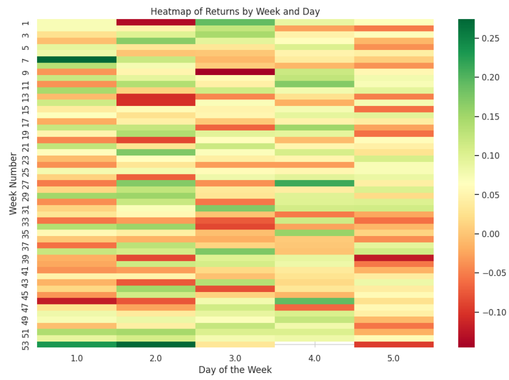
impacting local funds returns. This effectively ruled out speculation as main hypothesis for ‘V-Shape’ price patterns, as those funds are major risk-takers on the Brazilian market. There was no seasonality related to auction cycles, except maybe for a weekend effect on the top row of Figures in 4, (a) and (b), where the day of the week 5 is monday in (b). Nor did the Fourier analysis reveal any significant peaks indicating clear frequencies in returns, nor did autocorrelation suggest the presence of patterns in (d) and (c).

Ultimately, we checked traded volume and turnover of outstanding government bonds in Figure 5, without noticing lesser liquidity after NTN-F auctions became weekly. There were just cycles of turnover related to issuance of new benchmarks(on-the-run/off-the-run shifts). Therefore, dealer short cycle did not affect turnover or NTN-F outstanding traded volume. Nor we found strategic reduction of trading volumes in order to capture higher premiums on auctions by end investors, which by (b) and (d) remained mostly concentrated as foreign investors throughout the sample. Moreover, Figure 11 trend does not seem to be correlated to auction frequency.

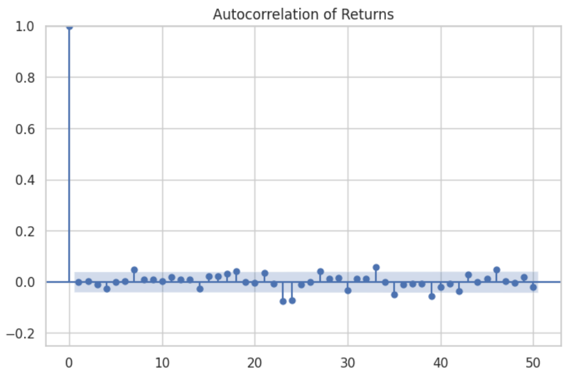
Figure 4 – IHFA: Returns, Heatmap, Autocorrelation, and Fourier Analysis.



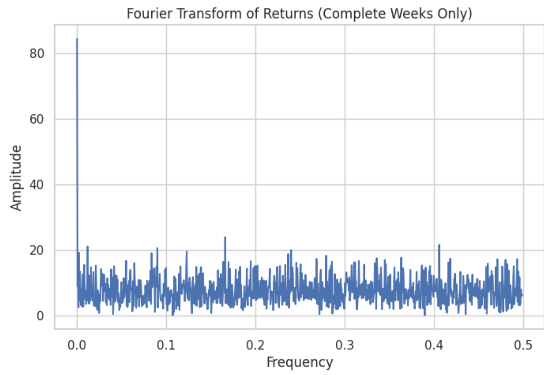
(a) Returns IHFA



(b) Heatmap IHFA



(c) Autocorrelation IHFA



(d) Fourier Analysis IHFA

As robustness checks, we estimated panel regressions with different deltas as dependent variables for both NTN-F and LTN with results pointing to the same path as delta+2. We also present on annex residuals for the original panel estimates. Lastly, we estimate the main regression using a two-stage least squares IV approach, instrumenting all potentially endogenous variables with their respective one-period lags.

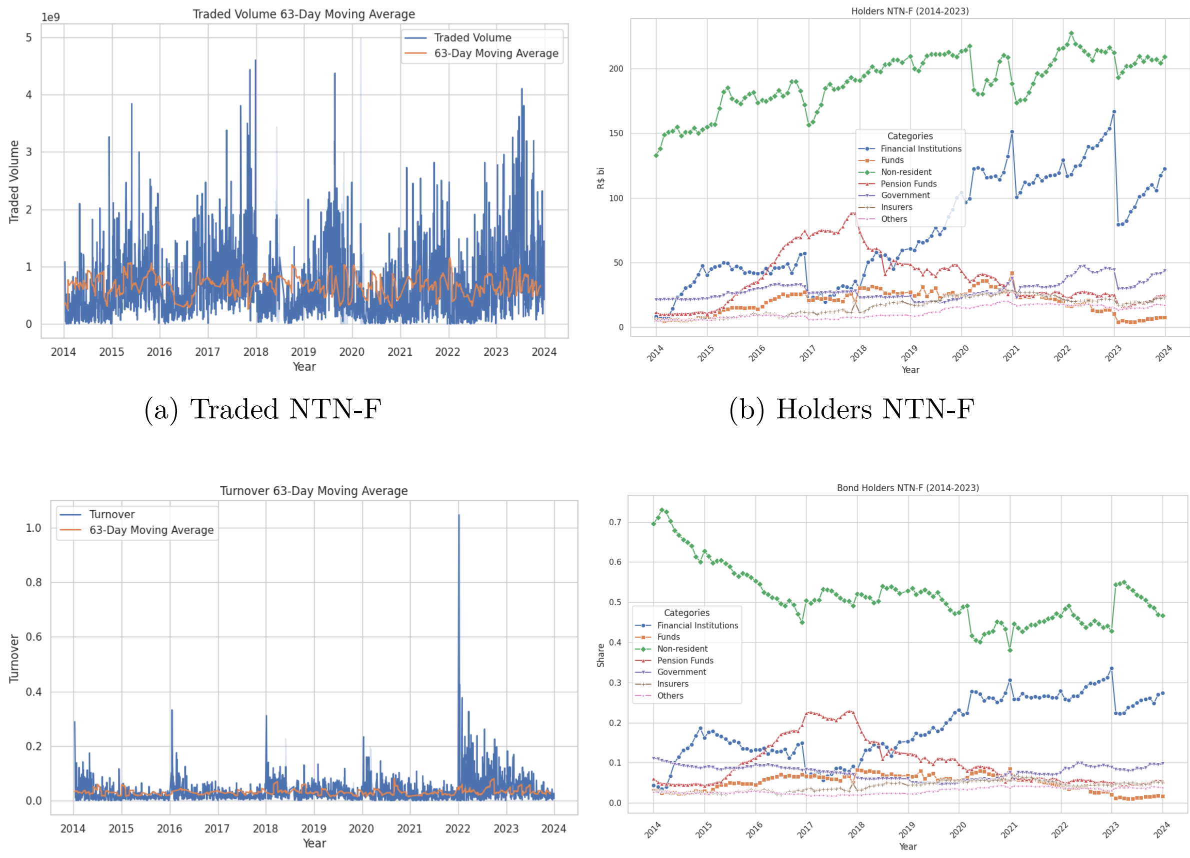
While the IV estimates are generally consistent in sign with the baseline fixed effects and random effects models, they are substantially less precise. All coefficients are statistically insignificant at conventional levels, and the model fit is poor. This imprecision is likely due to the weak predictive power of the lagged instruments in the first stage.

These results suggest that, although endogeneity cannot be ruled out, the main findings are robust in direction (tables 39 and 42).

2.5 Conclusion

This section examines the phenomenon of ‘V-shape’ price patterns observed in Treasury auctions, characterized by a decline in security prices prior to auctions, and a subsequent recovery. Drawing on data from Brazilian National Treasury auctions, the

Figure 5 – Traded Volume, Turnover, Holders, and Holders Share NTN-F.



(a) Traded NTN-F (b) Holders NTN-F

(c) Turnover NTN-F (d) Holders Share NTN-F

analysis aims to establish empirical evidence for the existence of these dynamics, explore their underlying causes, and evaluate the influence of auction frequency on their magnitude. The transition from fortnightly to weekly auctions, implemented in 2018 for NTN-F securities, provides a natural experiment to investigate these effects.

The findings highlight that risk-bearing capacity, encompassing both dealers and end investors, serves as a fundamental driver of the observed V-shape patterns. The analysis reveals that shorter auction cycles mitigate the magnitude of these price dynamics, therefore dampening pre-auction yield increases and post-auction decreases.

Statistical tests reject the null hypothesis of zero price deviations around auctions, confirming the presence of significant deltas in both LTN and NTN-F securities, specially in longer duration bonds, which innately bear increased interest rate risk.

Additionally, the division of the data into pre and post 2018 periods demonstrates that the size of these deltas differs significantly between fortnightly and weekly auction cycles. Panel regressions, including fixed and random effects models as well as differences-in-differences (DiD) estimations, further corroborate the risk-bearing hypothesis as established by Lou, Yan e Zhang (2013). Although, not only by dealers, but by other major players such as NTN-F foreign investors.

In addressing alternative explanations, the study finds no evidence to support strategic behavior or slow-moving capital as primary contributors to V-shape patterns as in Amin e Tédongap (2023). Time-series analyses of hedge fund returns, using autocorrelation, Fourier Transform, and heatmap visualizations, reveal no seasonality related to auction cycles, aside from a potential weekend effect. These results effectively rule out speculation as a major driver, despite the significant role of hedge funds as risk-takers in Brazilian markets.

Moreover, investigations of liquidity and turnover dynamics in the secondary market indicate the shift to weekly auctions has not adversely impacted the traded volume or turnover of NTN-F securities. There was no crowding out in secondary markets by issuances, which would in turn lead to strategic purchases preferably in auctions. The observed turnover cycles are instead attributed to shifts between on-the-run and off-the-run securities driven by benchmark issuance.

This section aimed to enhance understanding of market responses to Treasury auctions and insights into the optimization of auction mechanisms. It underscores the central role of risk-bearing capacity in shaping price dynamics while demonstrating the efficacy of shorter auction cycles in reducing supply shocks adverse price movements.

3 Revenue Equivalence - Reduced Models

Through several experiments by the Brazilian Treasury we aim to measure which mechanism design yields greater revenue to the issuer of government securities. We use a standard method in this literature, regression analysis with dummies for experiments, referred broadly as reduced models. Results point to lower participant profits in traditional auctions and in uniform-price auctions.

Mechanism selection for government bond auctions is a common theme for papers investigating which design generates greater seller revenue. A wave of empirical papers surged after theoretical studies concluded it is impossible to detect which type of auction is best without data evidence.

Earlier research primarily relied on policy experiments – revenue evaluations on different types of auctions. They were mostly based on a measure of auction profit, the gap between secondary market and auction clearing prices. Typically, this line of research examined whether auction profits differed between samples of discriminatory-price (pay-as-bid) and uniform-price auctions (every bid above clearing price pays the clearing price).

Papers point left and right on this matter - the evidence today is still far from conclusive, as we can see in table 17, many papers conclude uniform price auctions are superior and others find the opposite. We hope to bring new evidence in this debate in the next sections.

The main hypothesis behind those studies is that auction mechanisms generating lower profits are more effective at reflecting market prices, better aligned with extracting bidders' true valuations.

We bring new evidence to this line of research from hybrid auctions in Brazil, where the Treasury is able to offer a fixed amount and, after the end of the bidding process, allocate any amount to each benchmark in the hybrid offer, as long as the total amount issued is less or equal the total amount offered.

For example: a hybrid issuance of 300,000 in 5-year and 10-year NTN-Bs grants flexibility for debt management to issue any combination of the two maturities, provided the aggregate amount remains within the 300,000 limit. In contrast, a traditional auction would offer 150,000 5y NTN-B and 150,000 10y NTN-B separately.

In order to investigate revenue equivalence across auction's bid selection criteria and formats, as aforementioned, we employ the straightforward approach: we regress a measure of profit on a comprehensive set of control variables. The independent variables include broader market and risk controls, alongside benchmark/issuance-specific factors, to capture variations in market conditions.

Additionally, we incorporate dummy variables to test the impact of policy experiments, such as shifts in bid selection criteria and auction formats, focusing on NTN-B and LFT securities. The recent changes in uniform/discriminatory price selection criteria and between traditional/hybrid auctions fueled this section with novel evidence.

The analysis begins with multiple OLS specifications to establish baseline relationships and proceeds to panel regressions to account for unobserved heterogeneity across benchmarks and time.

Our regression results reveal weak evidence of lower profits under uniform-price auctions; and, contrary to theoretical predictions, we find higher profits in hybrid auctions. In hybrid auctions, the Brazilian treasury allocates freely between benchmarks the amount offered. This ability to issue benchmarks with higher bids after the bidding process is over may cause winner's curse risk, preventing participants from bidding more aggressively.

Finally, we test for structural breaks from those shifts in policies on profit and bid-to-cover, detecting evidence for both in NTN-B bonds.

Several policy experiments helped our investigation. Until early 2012, LFT auctions were under a hybrid format, transitioning to a traditional non-hybrid format in January of that year. Bid selection for LFT auctions continued on a uniform-price basis until March 2012, after which a multiple-price selection criterion was adopted. Amid the increased risk and funding needs caused by the Covid-19 pandemic in March 2020, auctions returned to a uniform-price criterion and hybrid format.

In April 2022, with a single benchmark in use, the hybrid format was no longer required. However, with the reintroduction of dual benchmarks in early 2023, hybrid auctions resumed until the end of June that year. By the end of 2023, the use of non-hybrid auctions provided an additional basis for analysis.

During the pandemic, the issuance of a 3-year benchmark marked a notable deviation from the typical format for NTN-B, which traditionally employed hybrid auctions with uniform-price selection. Beginning in 2021, a series of changes were introduced, including the addition of new benchmarks, an increase in auction frequency for key benchmarks, and a reversion to the traditional format.

The remainder of this section comprises literature review, empirical strategy, data, results and a brief conclusion.

3.1 Literature Review

This literature review commences with a brief analysis of studies related to auction theory, moving on shortly after to multi-unit sealed-bid deals and treasury auctions.

Wilson (1979) proposed the first model of divisible goods auction where participants have private information about asset values. Back e Zender (1993), under the common value assumption, extended Wilson (1979)'s model to auctions of government securities, where sellers auction a fixed quantity of goods, agents are symmetric, and risk-neutral. According to him, agents would be concerned with marginal cost rather than price, submitting a steep demand schedule which would enable bidders in a uniform-price auction to obtain a collusive outcome even in a nooncoperative equilibrium.

There is also a literature on refinements in the previously theoretical models, such as relaxing the assumption of risk neutrality and offering quantities with a degree of randomness by the issuer. The bridge from the theoretical model to empirical studies was the insight by Guerre, Perrigne e Vuong (2000) that auction participants are playing under a Bayesian Nash equilibrium. In this case, it would be possible to use the distribution of residual supply as an estimate of participants' ex-ante beliefs. Having participants' beliefs then inverting the optimality condition would be enough to recover unobserved marginal valuation.

In most researches, the revenue equivalence theorem(MYERSON, 1981) between auction formats are central themes. Notably, treasury auctions studies confront uniform-price auctions (all participants pay the price of the last accepted bid) and discriminatory-price auctions (pay-as-bid or multiple-price) results.

For that matter, a theoretical model by AUSUBEL et al. (2014) showed the optimal use of multiple bids with decreasing prices (bid shading) distorts allocative efficiency in uniform-price auctions. Such shading occurs since winning bidders affect market prices, hence providing incentives for demand reduction. This happens because by shading bids, a participant is able to reduce prices paid for inframarginal units. However, in certain environments, according to participants' risk aversion and symmetry hypothesis, uniform-price auctions could outperform discriminatory-prices. Consequently, comparing expected revenues between uniform-price and multiple price should be done empirically.

Typically, there are two strands of empirical research in the analysis of government debt auctions: a structural approach that aims to estimate value assigned to assets from submitted bids, and a reduced form approach which uses econometric models to compare outcomes across different auction formats or policy experiments. Structural models recover participants marginal valuations, evaluating then efficiency and revenues from auctions, whereas reduced models, employed subsequently, usually estimate through regression analysis the effects on auction revenues and profits from other variables (e.g. auction type, volatility, amount of greenshoe benefits etc.).

3.1.1 Revenue Equivalence - Structural Models

The first breakthrough in these models came with Hortaçsu (2002). He tries to answer what is the most effective way to sell government securities estimating bidders' true marginal valuation. He modeled bidders' strategic behavior in a discriminatory-price auction as an incomplete information game with independent private values, in which bidders were risk-neutral and symmetric.

Deriving optimality conditions provided a path to bidders' marginal valuation, which enabled him to reconstruct a competitive outcome and to contrast it to the actual data on discriminatory-price auctions. Given that the issuers' revenue in competitive outcomes serves as an upper bound to revenues generated from a uniform-price or Vickrey auction, Hortaçsu (2002) approach is able to compare actual discriminatory-price results to the best case in a uniform-price auction format.

Through Turkish Treasury auctions data, the author employed a non-parametric method to estimate bidders' valuation and simulated competitive outcomes. To accomplish this task, he proposed a resampling technique based on random draws of participants bids.

Although Hortaçsu (2002) suggested superior results for discriminatory-price auctions, upon updating his research in Hortaçsu e McAdams (2010), he concluded a switch in auction regimes in Turkey from discriminatory-price to uniform-price auctions would not significantly increase issuer's revenue.

Sbai e Armantier (2006) examined French Treasury auctions, incorporating into a different structural model information asymmetries and risk aversion, concluding uniform-price auctions would outperform multiple-price auctions. They proposed an alternative version of the k-th price averaging auction format, which would be the preferable mechanism choice.

A second breakthrough occurred when Kastl (2011) refined the method used by Hortaçsu (2002) in a model considering private information and treating demands as step functions. Participants then were constrained by discrete proposals with a limited number of bids - as in most practical applications.

In this Bayesian Nash Equilibrium, bidders, in some cases, submit proposals in which prices are above their valuations. Thus, expected revenue in uniform-price auctions may be higher than estimates based on marginal valuations for competitive outcomes - previously considered upper bounds for assessing uniform-price auctions.

Moreover, Kastl (2011) empirical strategy allowed computing valuations in a uniform-price setting. His findings for Czech treasury auctions suggest uniform-price auctions extract most of the participants' surplus, being efficient in terms of allocation and revenue maximization.

Drawing on Brazilian treasury data, Melo (2014) conducted a comparison of multiple-price and uniform-price auctions using two approaches: the structural model proposed by Hortaçsu (2002) and a reduced-form analysis of changes in auction formats over time. Initially, he examined the transition of the Treasury Financial Letters (LFT) auction from uniform-price to multiple-price, suggesting higher revenues for the issuer in uniform-price auctions. Subsequently, Melo (2014) applied the aforementioned structural model with modifications based on Silva (2002)'s method (dividing participants into two symmetric groups), concluding that multiple-price auctions are slightly superior in terms of revenue compared to uniform-price auctions, even though results were not statistically significant.

The results of this subsection are summarized in Table 16, they are far from consensual on the best mechanism design.

Table 16 – Structural Model Results

Author	**Mechanism Choice Results**
Hortaçsu (2002)	Superiority of multiple-price auctions.
Sbai e Armantier (2006)	Superiority of uniform-price auctions.
Hortaçsu e McAdams (2010)	No significant differences found between formats.
Kastl (2011)	Superiority of uniform-price auctions.
Melo (2014)	Superiority of multiple-price auctions.

3.1.2 Revenue Equivalence - Reduced Models

First empirical researches on mechanism selection for government bond auctions primarily relied on policy experiments – revenue evaluations on different types of auctions. These comparisons were typically based on a measure of auction profits, which is the difference between secondary market and auction clearing prices. Usually, this line of research tested whether these auction profits across samples of either discriminatory-price or uniform-price auctions were equal or not.

The common hypothesis in those studies is that mechanisms with smaller auction profits reflect better market prices, capable of extracting bidders' true valuations. In most reduced-form models, the dependent variable is auction profits, and control variables include measures of risk, bid variance, levels of interest rates, number of proposals etc.

One of the earliest works in this research line stemmed from suspicions of excessive profits in U.S. Treasury auctions, following news involving Salomon Brothers bank holding large quantities of government bonds, leading to artificial price increases (JEGADEESH, 1993).

An early reduced model by Umlauf (1993) tackled mechanism design issue through Mexico's regime change period between multiple-price and uniform-price auctions for T-Bills. He argued in a common-values setting, an uniform-price format would inhibit collusion and improve sellers' revenue. His regression results of auction profits on variables related to participants' proposals and auction type dummies suggest higher revenue in uniform-price auctions and some form of collusion among banks during the multiple-price period.

The U.S. Treasury Department conducted a similar study on its uniform-price auctions experience. In this paper, Malvey e Archibald (1998) evaluated whether the Treasury revenue from this technique reduces financing costs by encouraging broader participation and more aggressive bids. Findings on U.S. Treasury auction results and secondary market outcomes, suggest smaller differences in average between the secondary market and auction clearing prices in uniform-price settings, implying higher revenues for the issuer. Additionally, they observed greater dispersion of gains among agents, reducing the profits of major dealers.

Goldreich (2007) also tested underpricing on U.S. Treasuries uniform-price and discriminatory-price deals (higher yields on winning bids than when-issued market yields, a measure of auction profits). Until 1998, some securities were still sold by the U.S. government through discriminatory-prices. Starting August 1998, all treasuries were issued under uniform-prices.

He found higher average underpricing in the discriminatory-price sample as well as Malvey e Archibald (1998). Additionally, in a sample encompassing both systems, he regressed undrepricing on auctions' features. The results reinforced his conclusions, in line with common values framework, indicating inferior underpricing in the uniform-price system.

Another revenue equivalence test on mechanism choice investigated Japanese government bond issuances. Japan's ministry of finance conducted a reverse switch, in relation to the U.S., in auction types. It went from an uniform-price to a discriminatory-price regime. Hattori e Takahashi (2021) expected higher revenues for pay-as-bid auctions in symmetric bidders' setting as AUSUBEL et al. (2014) predicted.

He tested for higher revenues regressing auction profits on control variables and auction type dummies. Hattori e Takahashi (2021) results indicated higher revenues in multiple-price settings. He also found symmetry evidence in auction profits, corroborating his hypothesis of symmetric bidders. His robustness exercises included results on levels of auctions profits on other maturities issued in discriminatory-prices prior and after the switch in auction types through a difference-in-differences approach.

Whereas Barbosa et al. (2022) examined chinese experience through a comparison of two issuers with same sovereign risk. They estimated reduced-form models on transactions from government-affiliated agencies in China (Ex-Im Bank and Chinese Development Bank) using both strategies to sell government securities (pay-as-bid and uniform-price).

After conducting various robustness analyses, they found no significant difference in expected revenues due to changes in auction formats.

Mariño e Marszalec (2023) examined 500 auctions in the Philippines, where, as in Brazil, both multiple-price and discriminatory-price auctions are used, with supply management based on proposals. However, Philippines actively switch between bid selection criteria. As explanatory variables of auction performance, they included not only auction characteristics but also variables reflecting fiscal and market conditions. The results suggest that uniform-price auctions widen spreads between auction results and the secondary market, indicating superiority of multiple-price auctions in terms of revenues. Regarding the concentration of winning proposals, uniform-price auctions show higher dispersion. The authors also measured participant heterogeneity, ranking them according to their participation in purchased lots. According to the authors, major winners make more bids, demand larger volumes, and exhibit greater dispersion among proposals.

In a research project for the Brazilian Treasury Department, Bugarin (2017) analyzed auction proposals for different types of securities, comparing bond rates with market variables and auction variables, finding no significant differences in revenues between uniform-price and multiple-price auctions. Bugarin (2017) also found evidence of correlation on participant's proposals, indicating some form of collusion. Bugarin (2017) employed GLS and GMM estimates of yield to account for possible endogeneity. Instead, we opted for multiple ols, fixed and random effects profit estimates with robust errors, since nor the present specification nor robustness analysis point to endogeneity.

More recently, in his master's dissertation, Barbosa (2024) conducted a comprehensive analysis of bid-shading behavior among participants and revenue equivalence in LFT auctions. His findings revealed no statistically significant differences between the various bid selection criteria employed in these auctions. Although in a similar approach, our results pointed to a different direction, with higher revenue in uniform-price auctions and in traditional format auctions, robust to multiple specifications and estimation methods, for NTN-B and to to some specifications for LFT.

Table 17 summarizes this section's results. Just as in the previous section, there were mixed results regarding the best mechanism choice.

Our first contribution to the literature involves testing revenue equivalence hypothesis comparing hybrid to traditional auctions. We also apply the same analytical structure to uniform-price and discriminatory-price auctions, bringing new evidence to the debate of which mechanism generates greater issuer revenue through different estimates and robustness checks on results.

Table 17 – Reduced Model Results

Author	**Mechanism Choice Results**
Umlauf (1993)	Superiority of uniform-price auctions..
Malvey e Archibald (1998)	Superiority of uniform-price auctions.
Goldreich (2007)	Superiority of uniform-price auctions.
Bugarin (2017)	No significant differences found between formats.
Hattori e Takahashi (2021)	Superiority of multiple-price auctions.
Barbosa et al. (2022)	No significant differences found between formats.
Mariño e Marszalec (2023)	Superiority of multiple-price auctions.
Barbosa (2024)	No significant differences found between formats.

3.2 Empirical Strategy

As most reduced models applied to revenue equivalence problems we define a measure of auction profit or premium and check whether a regime shift on auction types were positive or not to the issuer.

Since 2020, various shifts in auction types and formats by the Brazilian National Treasury provided relevant data to our research. Up to March 2020, multiple-price selection was adopted for LFT. In April 2020, risks arising from the Covid-19 pandemic led to the reintroduction of the hybrid format and uniform-price mechanism. By April 2022, with a single benchmark, the hybrid format was discontinued but resumed in early 2023 with dual benchmarks, continuing until June. By late 2023, non-hybrid auctions offered new opportunities for analysis.

Also during the pandemic, the issuance of a shorter inflation-linked benchmark (3y) marked a departure from the NTN-B's hybrid uniform-price format. This one benchmark was issued in a non-hybrid multiple-price selection criterion. From early 2021, changes included new benchmarks, increased auction frequency, and a return to the non-hybrid uniform-price format for all benchmarks.

After inspecting main variables, such as profit (gap from clearing rate to market), ac_ratio (supply adjustments) and bid-to-cover (relative demand) for both benchmarks, we estimate OLS regressions of profit on several sets of control variables in order to establish a baseline model. Among this controls, there are market risk and benchmark/issuance specific variables, alongside dummies for benchmarks and auction types/formats. These first OLS regressions serve a second purpose as a robustness check on the model specification.

We then estimate panel regressions (FE and RE) with the baseline model arranged previously. As before, the primary objective was to determine whether transitions between uniform-price and multiple-price auctions, as well as between hybrid and non-hybrid auctions enhanced seller revenue. Our results pointed some evidence to higher profits in hybrid formats and lower on uniform-price auctions. Hausman tests on the models did not discard the random effects estimates. The estimates were actually very close in FE and RE.

Lastly, as another robustness analysis, we checked structural breaks in profit and bid_to_cover, detecting evidence for both in NTN-B bonds.

Our specification is quite similar to 'V-shape' analysis. We do however make adjustments on the profit variable, using the gap from market to average clearing price and not to the last clearing price as before;

Hence, the dependent variable $profit_{i,t}$ represents bond yield changes from average clearing rate to the secondary market, for benchmark i at auction t.

The panel regression model can be specified as:

$$profit_{i,t} = \beta_0 + \beta_1 \text{Market\_Risk}_t + \beta_2 \text{Benchmark\_Risk}_{i,t} + \beta_3 \text{Dummies}_{i,t} + \epsilon_{i,t} \quad (3.1)$$

where Market_Risk_t measures market risk factors, such as exchange-rate, VIX, CDS, Selic Rate; Benchmark_Risk_i,t captures benchmark-specific risk factors, such as benchmark rate variations, amount issued, dv01 supplied, bid-to-cover, supply adjustments, number of bids etc; and Dummies_i,t are dummy variables for structural and/or benchmark-specific effects, such as auction type and formats.

To control for unobserved heterogeneity across bonds, we introduce bond-specific fixed effects. This could captures, for instance, regulatory demand or bond specific demand arising from a particular specialist dealer of that benchmark. Alternatively, we estimate a random effects model, assuming this heterogeneity is uncorrelated with the regressors.

We also checked for potential endogenous variables estimating equations through IV-2SLS with lagged variables (Tables 43 and 44).

Finally, we performed Chow tests on OLS estimations of profit and bid-to-cover to capture breaks in the time-series after policy shifts.

3.3 Data

Since policy changes regarding traditional/hybrid and uniform/discriminatory price auctions are restricted to NTN-B and LFT securities, our data comprises their issuances between 2018 and 2023. Those are respectively inflation-linked and floating rate bonds. We group LFT maturities into benchmarks of 3 years and 6 years; NTN-B are divided into our own classification of various benchmarks. We set NTN-B benchmarks as: 3 years; 5 years; 7, 10 and 15 years; 20, 25 and 30 years; and 40 years. For both types of securities we fit those categories of benchmarks according to maturity windows and issuance schedule. We end up with 240 auctions for the longer LFT benchmark and 136 for the shorter. Since the NTN-B 3y benchmark is newer, it has only 77 auctions in our sample, while the others range from 140 to 215 auctions.

As in our previous data section, we organize our variables as market risk factors, benchmark-specific risk factors and dummies. Deltas are used only for 'V-shape' analysis, therefore are not present here.

Market risk variables reflect the broader economic environment, influencing investor behavior and auction outcomes. Benchmark-specific risk factors, such as benchmark rate variations, amount issued, dv01 supplied, bid-to-cover and supply adjustments take into account idiosyncrasies of each auction/maturity. In revenue equivalence tests, our main dependent variable is profit, which is slightly modified from the previous definition to grasp the average auction profits, and not the gap from secondary market to the last bid as before.

As market risk factors, we have **USD/BRL**, the exchange rate between Brazilian Real and the US Dollar, higher values might indicate riskier environments for locals and cheaper prices for foreign investors. Our sample captures the move from 3.30 up to the 5 range.

VIX (Volatility Index), a gauge for global market uncertainty, arising from implied volatility on near term index options on the S&P 500. Most of the sample is below 20, which is historically low.

CDS (Credit Default Swap Spreads), which measures Brazilian 5-year sovereign credit risk. Three quarters of the sample is below 230 basis points, also historically low.

SELIC, Brazil's policy interest rate, directly influencing yield curves. It ranges from 1.9% to 13.65%. Higher SELIC levels correlate with increased yields across NTN-Bs and higher premiums - spreads above SELIC - for LFT.

UST10Y is the 10y treasury yield.

And **DXY (US Dollar Index)**, an index of exchange rates between major currencies and the US Dollar, it correlates with USD/BRL, and international risk-aversion periods.

As benchmark-specific risk factors we include **dv01_of**, computed as the price change from a 1 basis-point increase in the bond yield times the amount issued that date, divided by 1 million here. We end up not using it too much due to LFTs extreme issuances driving this indicator upward, and not really reflecting risk.

Dv01_of_bench is computed as the price change from a 1 basis-point increase in the bond yield times the amount issued that date for the specific benchmark, divided by 1 million here. NTN-Bs higher dv01 supply are the 40y benchmarks (143 avg.) and the 5y (125), the latter due to its amount issued. 20y, 25y & 30y benchmarks and 3y benchmark present lower dv01 according to supply and duration, respectively. This variable is not really applicable for LFTs because of the ways it is priced, in addition to its issuances.

Number_bids, the bid count for each auction. While the two shorter durations 3y and 5y benchmarks have above 20 bids per auction, 7y, 10y & 15y benchmarks have about 15 bids per auction, while the rest are in the 10 bid per auction ballpark. LFT 3y benchmark is bid close to 10-12 times per auction, in contrast with its longer peer, bid almost thrice as much.

Number_bids_ac, the accepted bid count for each auction, approximately 55-70% for all benchmarks.

Ac_ratio corresponding to supply adjustments, reducing the amount issued and profits. It is computed as the ratio between Issuance and Supply. We adjust this indicator for hybrid auctions dividing the supply by the number benchmarks, making it appear above 1 when one benchmark is allocated more than the supply divided by the number of benchmarks.

Bid_to_cover captures the ratio of amount bid relative to initial supply. As in the number of bids, the two shorter durations 3y and 5y benchmarks have above higher relative demand (above 2), while the rest are between 1.2 and 1.4. LFT 3y benchmark is relatively less bid, with bid-to-cover of 1, on the other hand LFT 6y benchmark indicator is 40% above that.

Profit, also regarded as auction premium, is the difference between the auction's clearing rate and the secondary market rate. It is almsot zero on average to LFTs and negative in 2 basis-point on average NTN-Bs.

Supply, the amount offered for each benchmark. Brazilian National Treasury NTN-B 5y benchmark median supply is 750,000 per auction, while 500,000 for 3y and 7y, 10y and 15y benchmarks. Longer NTN-Bs are less issued, close to 150,000 median. LFTs median supply is 750,000 bonds per auction.

Issuance, the amount issued for each benchmark, are very close to supply in securities more restricted to traditional auctions since supply adjustments are very low (ac_ratio close to one). Since in hybrid auctions we adjust supply definition, and by its nature, ac-ratio range from 60% to above 1 in some cases.

Clearing Rate is the average bid auction yield for the specific security.

Closing Rate, the secondary market end of the day yield for the specific security.

Maturity_months is the number of months until maturity, as expected it is very close to the benchmark classification.

Anb-5, a proxy of the yield changes between last auction date closing price and current auction opening yield - also very close to zero across all benchmarks.

Tables 18, 19, 20, 21, 22, 23 and 24 encompass descriptive statistics for each of the five NTN-B groups.

The 5-year NTN-B benchmark (Table 18) shows relatively high interest rate sensitivity, with an average dv01_of_bench of 124.66 and substantial issuance (median of 750,000) within a moderately wide interquartile range (300,000 to 1 million). Auction participation is active, with a median of 25 bids submitted and 14 accepted, while the acceptance ratio above 1 reflects market preference over 10y benchmarks in hybrid auctions. Bid_to_cover ratio averages 2.08, suggesting healthy demand, yet profits remain negative, due to competitive bidding. Overall, compared to longer NTN-Bs, this benchmark is more liquid and more frequently issued. The macroeconomic environment during this sample period featured a mean exchange rate of 4.70 BRL/USD, an average VIX of 20.18 indicating low global risk aversion, and a mean SELIC rate of 7.59%. The DXY, UST10Y, and CDS averaged 116.62, 2.42%, and 201 basis points respectively, reflecting relatively favorable external conditions.

Longer duration NTN-Bs in the 20–30y range (Table 19) exhibit lower issuance volumes and less intense bidding activity. The median dv01_of_bench is 60.21, substantially lower than the 5y benchmark, as a result of modest issued amount. Auction demand is relatively weaker, with a lower bid_to_cover ratio (1.24) and an acceptance ratio of 0.71. Profits remain fairly negative and issuance volumes are highly concentrated in the 50,000 to 150,000 range.

As for the 40y NTN-B benchmarks (Table 20), its long duration exposure (mean dv01_of_bench of 143.41) drives median issuance to just 150,000, with an interquartile range capped at 300,000, indicating a cautious approach to supply. Demand is variable, with a wide bid_to_cover dispersion (mean of 1.45, max of 8.57), and allocation remains selective (acceptance ratio of 0.85) with profits negative on 2 basis-points. Although auction participation is moderate, results suggest constrained supply paired with investor demand for longer inflation-linked instruments, often tied to asset liability management strategies.

NTN-Bs spanning 7 to 15y in Table 21 strike a balance between issuance and demand consistency. The mean dv01_of_bench is 147.69, among the highest, reflecting a combined higher sensitivity to interest rates (longer duration) and relative high supply. Median issuance is around 500,000 bonds, with strong demand (median of 16 bids) and moderate allocation (acceptance ratio of 0.77) comparative to 5y benchmarks. The average bid_to_cover ratio of 1.34 suggests light demand, though profits remain negative, in line with other NTN-B benchmarks. This segment offers a well-supported maturity profile for duration seekers while allowing selective supply control by the issuer.

The recently introduced 3y NTN-B benchmark (22 is among the most liquid in the inflation-linked curve, with the highest bid_to_cover ratio (2.24) and strong auction participation (median of 21 bids). The average dv01_of_bench of 89.19 indicates relatively low duration exposure compared to longer bonds. Supply tends to be large and variable (median 500,000; max 8 million), yet almost fully allocated (acceptance ratio near 1). Profits remain negative, suggesting competitive pricing conditions.

Table 18 – Descriptive Statistics NTN-B 5y Benchmark

Variable	count	mean	std	min	25%	50%	75%	max
dv01_of	148.00	1352.62	832.18	99.10	722.42	1260.07	1825.95	4749.88
dv01_of_bench	148.00	124.66	92.21	7.59	53.42	106.06	165.85	530.83
number_bids	148.00	23.30	8.43	1.00	17.75	25.00	29.00	40.00
number_bids_ac	148.00	14.51	6.71	1.00	10.00	14.00	19.00	36.00
ac_ratio	148.00	1.17	0.39	0.03	1.00	1.00	1.49	2.00
bid_to_cover	148.00	2.08	0.98	0.03	1.43	1.85	2.57	6.80
profit	148.00	-0.02	0.01	-0.07	-0.03	-0.02	-0.01	0.01
USD/BRL	148.00	4.70	0.73	3.20	3.97	4.96	5.28	5.87
DXY	146.00	116.62	4.35	107.03	113.79	116.04	119.79	128.44
VIX	147.00	20.18	6.66	10.08	14.91	18.97	24.02	53.54
UST10Y	146.00	2.42	1.10	0.52	1.56	2.62	3.10	4.83
CDS	148.00	201.07	50.10	92.51	165.45	201.94	238.21	346.99
SELIC	148.00	7.59	4.12	1.90	4.40	6.40	12.65	13.65
Supply	148.00	764527.03	531499.40	50000.00	300000.00	750000.00	1000000.00	3000000.00
Issuance	148.00	657532.09	533410.43	4500.00	268425.00	500000.00	1000000.00	3000000.00
Clearing Rate	148.00	4.36	1.35	1.62	3.10	4.68	5.54	6.38
Closing rate	148.00	4.36	1.36	1.62	3.13	4.66	5.54	6.46
maturity_months	148.00	60.18	4.03	44.00	57.00	60.00	63.00	68.00
anb-5	148.00	-0.01	0.19	-0.87	-0.10	0.01	0.09	1.09

Table 19 – Descriptive Statistics NTN-B 20y, 25y & 30y Benchmarks

Variable	count	mean	std	min	25%	50%	75%	max
dv01_of	140.00	1208.44	740.65	74.01	701.70	1061.48	1565.55	3829.24
dv01_of_bench	140.00	64.50	34.58	21.98	46.48	60.21	74.00	258.19
number_bids	140.00	9.45	4.05	1.00	6.00	9.00	12.00	21.00
number_bids_ac	140.00	6.73	3.28	1.00	4.00	6.00	9.00	17.00
ac_ratio	140.00	0.71	0.42	0.01	0.28	0.85	1.00	1.93
bid_to_cover	140.00	1.24	0.87	0.07	0.57	1.15	1.64	4.54
profit	140.00	-0.02	0.01	-0.08	-0.02	-0.02	-0.01	0.02
USD/BRL	140.00	4.74	0.73	3.20	4.05	5.04	5.29	5.82
DXY	139.00	116.68	4.35	107.03	113.88	116.03	119.73	127.92
VIX	140.00	20.09	6.38	10.08	15.24	19.34	23.27	53.54
UST10Y	139.00	2.40	1.13	0.52	1.54	2.51	3.15	4.88
CDS	140.00	197.78	48.99	92.51	164.53	197.13	227.19	346.99
SELIC	140.00	7.56	4.15	1.90	4.34	6.40	12.65	13.65
Supply	140.00	137142.86	70805.09	50000.00	100000.00	150000.00	150000.00	500000.00
Issuance	140.00	77970.71	79886.08	500.00	26500.00	50000.00	97225.00	500000.00
Clearing Rate	140.00	4.90	0.95	2.85	4.14	5.04	5.76	6.48
Closing rate	140.00	4.90	0.95	2.84	4.14	5.06	5.77	6.50
maturity_months	140.00	248.05	46.64	187.00	205.50	239.50	279.00	336.00
anb-5	140.00	-0.01	0.12	-0.41	-0.07	-0.01	0.06	0.57

Tables 23 and 24 encompass descriptive statistics for the two LFT benchmarks.

Although, the 6y LFT exhibits the highest benchmark-level interest rate sensitivity in the floating-rate segment, with an average dv01_of_bench of 607.48, it does not reflect overall risk, but only risk for the bond basis. Issuance is substantial and frequent, with

Table 20 – Descriptive Statistics NTN-B 40y Benchmark

Variable	count	mean	std	min	25%	50%	75%	max
dv01_of	143.00	1375.26	817.96	99.10	745.74	1263.53	1831.16	4749.88
dv01_of_bench	143.00	143.41	157.34	23.69	78.65	90.58	166.46	1098.82
number_bids	143.00	12.54	7.05	1.00	8.00	11.00	16.00	33.00
number_bids_ac	143.00	7.99	4.40	1.00	5.00	7.00	10.00	29.00
ac_ratio	143.00	0.85	0.45	0.06	0.47	1.00	1.00	1.98
bid_to_cover	143.00	1.45	1.03	0.06	0.79	1.30	1.87	8.57
profit	143.00	-0.02	0.02	-0.09	-0.03	-0.02	-0.01	0.02
USD/BRL	143.00	4.72	0.72	3.20	4.04	4.97	5.29	5.87
DXY	141.00	116.68	4.37	107.03	113.89	116.11	119.80	128.44
VIX	142.00	20.30	6.71	10.08	14.90	19.06	24.20	53.54
UST10Y	141.00	2.39	1.11	0.52	1.55	2.51	3.11	4.83
CDS	143.00	197.83	50.06	92.51	163.35	196.27	228.42	346.99
SELIC	143.00	7.57	4.16	1.90	4.40	6.40	12.65	13.65
Supply	143.00	211888.11	215831.02	50000.00	100000.00	150000.00	300000.00	1500000.00
Issuance	143.00	164313.29	234730.24	3000.00	36400.00	72550.00	150000.00	1500000.00
Clearing rate	143.00	4.99	0.88	3.23	4.34	5.10	5.81	6.48
Closing rate	143.00	4.99	0.88	3.23	4.33	5.13	5.82	6.45
maturity_months	143.00	438.97	17.79	406.00	423.50	442.00	453.00	469.00
anb-5	143.00	-0.01	0.11	-0.35	-0.08	-0.01	0.05	0.57

Table 21 – Descriptive Statistics NTN-B 7y, 10y & 15y Benchmarks

Variable	count	mean	std	min	25%	50%	75%	max
dv01_of	215.00	1395.68	865.02	74.01	784.31	1278.87	1936.87	5478.20
dv01_of_bench	215.00	147.69	119.42	10.34	58.29	121.95	213.65	643.90
number_bids	215.00	16.72	6.85	2.00	12.00	16.00	20.50	38.00
number_bids_ac	215.00	11.20	4.91	1.00	8.00	11.00	14.00	30.00
ac_ratio	215.00	0.77	0.32	0.00	0.52	1.00	1.00	1.59
bid_to_cover	215.00	1.34	0.78	0.04	0.83	1.38	1.72	4.46
profit	215.00	-0.02	0.01	-0.06	-0.03	-0.02	-0.01	0.01
USD/BRL	215.00	4.89	0.63	3.20	4.73	5.10	5.31	5.87
DXY	212.00	117.22	4.44	107.03	113.89	116.59	120.60	128.44
VIX	214.00	20.45	6.10	10.93	16.00	19.39	24.00	53.54
UST10Y	212.00	2.54	1.14	0.52	1.58	2.71	3.52	4.88
CDS	215.00	202.82	46.89	92.51	168.49	203.35	231.59	346.99
SELIC	215.00	8.39	4.37	1.90	4.65	6.65	13.15	13.65
Supply	215.00	501395.35	400493.75	50000.00	150000.00	500000.00	750000.00	2000000.00
Issuance	215.00	298658.14	286903.58	1000.00	89900.00	167250.00	471450.00	1500000.00
Clearing Rate	215.00	4.87	1.08	2.57	3.95	5.22	5.76	6.46
Closing rate	215.00	4.86	1.08	2.56	3.95	5.21	5.77	6.44
maturity_months	215.00	129.14	33.61	81.00	112.00	120.00	128.00	214.00
anb-5	215.00	0.00	0.14	-0.56	-0.09	0.01	0.08	0.69

Table 22 – Descriptive Statistics NTN-B 3y Benchmark

Variable	count	mean	std	min	25%	50%	75%	max
dv01_of	77.00	1345.02	855.18	74.01	809.04	1261.43	1929.43	3829.24
dv01_of_bench	77.00	89.19	117.79	4.69	30.55	52.80	91.40	690.08
number_bids	77.00	21.05	9.24	2.00	15.00	21.00	25.00	48.00
number_bids_ac	77.00	11.66	6.65	1.00	7.00	11.00	15.00	33.00
ac_ratio	77.00	0.95	0.16	0.10	1.00	1.00	1.00	1.00
bid_to_cover	77.00	2.24	1.02	0.20	1.61	1.90	2.64	5.46
profit	77.00	-0.02	0.01	-0.06	-0.03	-0.02	-0.02	0.06
USD/BRL	77.00	5.18	0.31	3.77	5.02	5.16	5.36	5.78
DXY	76.00	117.86	4.51	110.66	113.67	118.59	121.52	127.92
VIX	77.00	20.95	5.06	12.69	17.46	19.78	23.33	35.55
UST10Y	76.00	2.70	1.20	0.81	1.56	2.92	3.71	4.88
CDS	77.00	211.59	38.60	146.11	177.47	213.38	232.24	301.14
SELIC	77.00	9.48	4.59	1.90	5.15	12.15	13.65	13.65
Supply	77.00	853246.75	1225156.75	50000.00	300000.00	500000.00	750000.00	8000000.00
Issuance	77.00	842168.18	1229886.01	8800.00	300000.00	500000.00	750000.00	8000000.00
Closing Rate	77.00	4.78	1.49	0.75	4.08	5.46	5.77	6.61
Clearing rate	77.00	4.78	1.50	0.75	4.11	5.44	5.76	6.67
maturity_months	77.00	36.10	4.01	26.00	33.00	36.00	39.00	43.00
anb-5	77.00	0.05	0.20	-0.64	-0.07	0.04	0.16	0.68

a median of 750,000 and a wide upper tail reaching 3.5 million, confirming its role as a primary instrument for liquidity management. Auctions attract significant participation, median of 25 bids, with 18 accepted, and an average acceptance ratio near one. Despite this strong participation, profits are essentially zero, consistent with highly competitive bidding and efficient supply calibration. Bid_to_cover ratio averages 1.45, with wide dispersion. Post-auction price dynamics are virtually flat, reflecting the nature of LFTs, whose returns are directly tied to the SELIC rate. Macroeconomic variables are similar to NTN-B, since the sample's date range is the same.

The more recent 3y LFT benchmark has milder and dispersed issuance, with a median of 182,150 and substantial variation across auctions. Auction participation is notably lower than in the 6y benchmark, with a median of 10 bids and only 6 accepted. The acceptance ratio is also considerably lower at 0.60, reflecting selective allocation by the issuer in hybrid auctions. Bid_to_cover is the lowest among LFTs, averaging just 1.01. Profits remain slightly negative, indicating tight pricing, and deltas are effectively flat before and after auctions, as expected in a floating-rate instrument with minimal price volatility.

3.4 Results - Reduced Models

The analysis begins with an examination of key variables, including profit (the gap between the clearing rate and market rate), ac_ratio (supply adjustments), and bid-to-cover (a measure of relative demand), for both benchmarks in Figure 6.

Moving averages on red lines shed light on trend in lower profits for LFT and NTN-

Table 23 – Descriptive Statistics LFT 6y Benchmark

Variable	count	mean	std	min	25%	50%	75%	max
dv01_of	234.00	1700.96	826.08	157.03	1200.63	1509.63	2200.80	6007.93
dv01_of_bench	234.00	607.48	296.22	62.64	380.84	574.92	775.42	2242.12
number_bids	234.00	33.81	23.32	6.00	20.00	25.00	35.00	98.00
number_bids_ac	234.00	21.49	14.83	2.00	13.00	18.00	26.00	76.00
ac_ratio	234.00	1.00	0.42	0.03	0.77	1.00	1.22	2.00
bid_to_cover	234.00	1.45	0.76	0.17	0.96	1.25	1.73	5.74
profit	234.00	-0.00	0.01	-0.07	-0.01	-0.00	-0.00	0.04
USD/BRL	234.00	4.90	0.65	3.18	4.76	5.12	5.32	5.89
DXY	229.00	117.19	4.44	106.49	113.98	116.48	120.59	128.44
VIX	230.00	21.03	6.72	9.22	16.25	19.38	24.93	50.91
UST10Y	229.00	2.41	1.20	0.55	1.45	2.51	3.46	4.88
CDS	234.00	204.55	48.23	97.02	170.18	207.53	233.90	337.79
SELIC	234.00	7.89	4.46	1.90	3.77	6.40	13.15	13.65
Supply	234.00	920299.15	483997.48	100000.00	500000.00	750000.00	1000000.00	3500000.00
Issuance	234.00	662758.76	445138.99	7450.00	324025.00	526750.00	1000000.00	2097700.00
Clearing Rate	234.00	0.16	0.11	0.01	0.04	0.17	0.28	0.40
Closing rate	228.00	0.17	0.11	0.00	0.04	0.17	0.28	0.39
maturity_months	234.00	72.99	2.39	69.00	71.00	73.00	74.00	79.00
anb-5	221.00	-0.00	0.02	-0.19	-0.00	-0.00	0.00	0.07

Table 24 – Descriptive Statistics LFT 3y Benchmark

Variable	count	mean	std	min	25%	50%	75%	max
dv01_of	136.00	1817.46	911.19	189.38	1205.50	1692.61	2341.29	6007.93
dv01_of_bench	136.00	190.90	126.66	21.46	108.54	175.13	246.65	761.48
number_bids	136.00	12.37	8.88	1.00	7.00	10.00	15.00	43.00
number_bids_ac	136.00	7.29	5.80	1.00	4.00	6.00	8.00	28.00
ac_ratio	136.00	0.60	0.37	0.01	0.33	0.56	0.75	1.93
bid_to_cover	136.00	1.01	0.86	0.01	0.57	0.72	1.26	5.86
profit	136.00	-0.01	0.01	-0.05	-0.01	-0.00	-0.00	0.00
USD/BRL	136.00	5.23	0.27	4.72	5.02	5.23	5.41	5.89
DXY	132.00	117.12	3.90	110.56	113.42	117.19	120.31	124.12
VIX	134.00	21.31	6.78	12.07	16.87	19.28	23.61	50.91
UST10Y	132.00	2.18	1.40	0.55	0.93	1.56	3.67	4.88
CDS	136.00	203.54	39.36	136.40	173.86	199.19	224.54	337.79
SELIC	136.00	7.23	4.92	1.90	2.52	5.15	13.15	13.65
Supply	136.00	802941.18	546868.69	100000.00	500000.00	750000.00	1000000.00	3500000.00
Issuance	136.00	272584.93	330079.33	2000.00	101800.00	182150.00	302487.50	2003900.00
CLearing Rate	136.00	0.09	0.04	0.02	0.07	0.09	0.12	0.28
Closing rate	136.00	0.09	0.04	0.02	0.07	0.09	0.12	0.29
maturity_months	136.00	26.74	8.54	11.00	20.75	28.00	34.25	38.00
anb-5	136.00	-0.00	0.02	-0.17	-0.00	0.00	0.00	0.12

B after policy adjustments, respectively shifts to uniform-price auctions and traditional auctions. While NTN-B variance in profits seem to be kept in similar range after the shift to non-hybrid auctions, profits in LFT appear to have higher variance since the return to uniform-price criterion.

With regards to bid-to-cover, it remained quite stable for NTN-B and without a clear trend in LFT securities, except for auction frequency.

Supply adjustments (ac_ratio) moving average remained quite similar for NTN-B, in spite of variable breaks due to its definition. In this research, supply for hybrid auctions is divided by the number of objects, and if most of the issuance is directed to one benchmark, ac_ratio goes beyond 1, which is impossible in non-hybrid formats. Because we are dealing with moving averages, it goes beyond 1 in LFT during hybrid shifts.

Then, OLS estimates of profit on various control variables are applied to establish a baseline model. These control variables include market risk factors, benchmark/issuance-specific characteristics, and dummy variables for benchmarks and auction types or formats. In addition to building the baseline model, the OLS regressions in tables 25 and 26 serve as a robustness check, ensuring the validity of the model's specification.

There is one relevant distinction between models: LFT specifications do not contain hybrid dummies due to multicollinearity problems. LFT has mostly followed either traditional auctions with multiple-price criterion or hybrid auctions with uniform-price criterion. Few exceptions in the data happened when there was only one benchmark and therefore hybrid auctions did not apply and the latter return to hybrid auctions with two benchmarks by the end of the sample. Therefore, LFT hybrid and type dummies were practically equal for the whole data, making the distinction for hybrid auctions less clear. Coefficients for types of auction seem consistent with NTN-B estimates, though.

Profits from three of the five specifications are significantly below zero for uniform-price auctions in the 1% level.

NTN-B OLS estimates bring us interesting results: hybrid auctions appear to have higher profits for bidders than non-hybrid. It is a striking result given current theory propose this format may resolve issues related to demand information by giving the issuer ability to adjust supply more easily. Profits in NTN-B estimates are negatively related to uniform-price auctions, nonetheless discriminatory criterion were used in the 3y benchmark for a short period and its dummy coefficient was not significant.

We also ran panel regressions including all benchmarks for each security with fixed effects (FE) and random effects (RE) - Tables 27 and 28.

This time LFT dummy for uniform-price did not generate significant relationship with profit, although its coefficient remained below zero. Surprisingly, all other variables

Table 25 – OLS Results for Multiple Specifications - LFT

Dependent Variable: *profit*	**Spec. 1**	**Spec. 2**	**Spec. 3**	**Spec. 4**	**Spec. 5**
const	0.0166** (0.008)	0.0168** (0.007)	0.0167** (0.008)	0.0302*** (0.007)	0.0273*** (0.007)
anb-5	-0.0587** (0.026)	-0.0567** (0.025)	-0.0590** (0.027)	-0.0571** (0.027)	-0.0525** (0.026)
USD/BRL	-0.0038* (0.002)	-0.0042** (0.002)	-0.0039* (0.002)	-0.0068*** (0.002)	-0.0070*** (0.002)
VIX	0.0002*** (0.000)	0.0002*** (0.000)	0.0002*** (0.000)	0.0002** (0.000)	0.0002** (0.000)
UST10Y	0.0009 (0.001)	0.0010 (0.001)	0.0011*** (0.000)		
SELIC	0.0001 (0.000)	0.0000 (0.000)			
dv01_of_bench	-0.0000 (0.000)		-0.0000 (0.000)		
number_bids	-0.0001* (0.000)	-0.0001** (0.000)	-0.0001* (0.000)	-0.0001** (0.000)	
ac_ratio	0.0058*** (0.001)	0.0060*** (0.001)	0.0058*** (0.001)	0.0057*** (0.001)	0.0053*** (0.001)
bid_to_cover	-0.0045*** (0.001)	-0.0043*** (0.001)	-0.0044*** (0.001)	-0.0043*** (0.001)	-0.0045*** (0.001)
d_type	-0.0080*** (0.003)	-0.0079*** (0.003)	-0.0078*** (0.003)	-0.0036 (0.003)	-0.0004 (0.002)
d_bench3y		0.0011 (0.001)		0.0007 (0.001)	0.0011 (0.001)
Observations	348	348	348	351	351
R-squared	0.351	0.351	0.351	0.334	0.328
BIC	-2516.613	-2516.854	-2522.354	-2541.582	-2544.509

Table 26 – OLS Results for Multiple Specifications - NTNB

Dependent Variable: *profit*	**Spec. 1**	**Spec. 2**	**Spec. 3**	**Spec. 4**	**Spec. 5**
const	-0.0112 (0.012)	-0.0152 (0.012)	-0.0117 (0.012)	-0.0186 (0.011)	-0.0219** (0.011)
anb-5	-0.0071** (0.003)	-0.0071** (0.003)	-0.0075** (0.003)	-0.0076** (0.003)	-0.0069** (0.003)
USD/BRL	-0.0006 (0.001)	-0.0007 (0.001)	-0.0007 (0.001)	-0.0003 (0.001)	-0.0004 (0.001)
VIX	0.0000 (0.000)	0.0001 (0.000)	0.0001 (0.000)	0.0001 (0.000)	0.0001 (0.000)
UST10Y	-0.0016* (0.001)	-0.0016* (0.001)	-0.0005 (0.001)		
SELIC	0.0004 (0.000)	0.0004* (0.000)			
dv01_of_bench	-0.0000 (0.000)		-0.0000 (0.000)		
number_bids	-0.0002** (0.000)	-0.0002** (0.000)	-0.0002** (0.000)	-0.0002** (0.000)	
ac_ratio	0.0029* (0.002)	0.0038** (0.002)	0.0029* (0.002)	0.0038** (0.002)	0.0028* (0.002)
bid_to_cover	-0.0016** (0.001)	-0.0015** (0.001)	-0.0017*** (0.001)	-0.0016*** (0.001)	-0.0018*** (0.001)
d_type	-0.0061 (0.010)	-0.0061 (0.009)	-0.0055 (0.010)	-0.0053 (0.009)	-0.0053 (0.009)
d_hybrid	0.0110*** (0.002)	0.0117*** (0.002)	0.0100*** (0.002)	0.0112*** (0.001)	0.0119*** (0.001)
d_bench3y		0.0034* (0.002)		0.0034* (0.002)	0.0040** (0.002)
d_bench10y		0.0018 (0.001)		0.0016 (0.001)	0.0025* (0.001)
d_bench20y		0.0038** (0.002)		0.0036** (0.002)	0.0057*** (0.002)
d_bench40y		0.0001 (0.002)		0.0000 (0.002)	0.0017 (0.002)
Observations	714	714	714	720	720
R-squared	0.234	0.243	0.232	0.240	0.232
BIC	-4210.300	-4198.657	-4214.616	-4249.752	-4248.880

produced coefficients significant at the 1% level.

NTN-B panel regressions still gave rise to significant coefficients for hybrid dummies, once again favoring traditional auctions in terms of seller revenue - possibly from risk arising from uncertainty in allocation of securities in the hybrid format.

Overall results provide evidence of higher profits in NTN-B hybrid auction formats and mild evidence of lower profits under uniform-price auctions for NTN-B and LFT. Hausman tests indicate that random effects estimates are not rejected and are closely aligned with fixed effects estimates, supporting the reliability of the findings (p-values above 0.8 for LFT and close to 1 for NTN-B).

Further robustness analysis focuses on detecting structural breaks in profit and bid-to-cover time series since policy shifts in either security. Models for those tests were similar to Specifications 5. The exceptions were specifications with bid_to_cover as dependent variable, where profits were not included.

No structural breaks were detected in the average auction profits or bid-to-cover ratios for LFT auctions since April 2020. However, evidence suggests increased competition indicators following the transition to a uniform-price selection criterion.

Chow tests on bid-to-cover and profits this time detected breaks on the 10% level for NTN-B in early 2021, despite visual inspection of bid-to-cover indicate the opposite (Table 29).

Table 27 – Fixed Effects and Random Effects Regression Results - LFT

Variable	**Fixed Effects**	**Random Effects**
const	0.0277*** (0.005)	0.0274*** (0.005)
d_type	-0.0004 (0.002)	-0.0001 (0.002)
ac_ratio	0.0053*** (0.001)	0.0047*** (0.001)
bid_to_cover	-0.0045*** (0.001)	-0.0044*** (0.001)
anb-5	-0.0525*** (0.019)	-0.0537*** (0.019)
USD/BRL	-0.0070*** (0.001)	-0.0069*** (0.001)
VIX	0.0002*** (0.000)	0.0001*** (0.000)
R-squared	0.323	0.325
Observations	351	351

Table 28 – Fixed Effects and Random Effects Regression Results - NTNB

Variable	**Fixed Effects**	**Random Effects**
const	-0.0185*** (0.006)	-0.0187*** (0.006)
d_type	-0.0048 (0.004)	-0.0043 (0.004)
d_hybrid	0.0119*** (0.001)	0.0116*** (0.001)
bid_to_cover	-0.0011** (0.001)	-0.0014*** (0.000)
anb-5	-0.0070** (0.003)	-0.0067** (0.003)
USD/BRL	-0.0003 (0.001)	-0.0002 (0.001)
VIX	0.0001 (0.000)	0.0001 (0.000)
R-squared	0.202	0.214
Observations	720	720

Table 29 – Chow Test Results for LFT and NTN-B Metrics

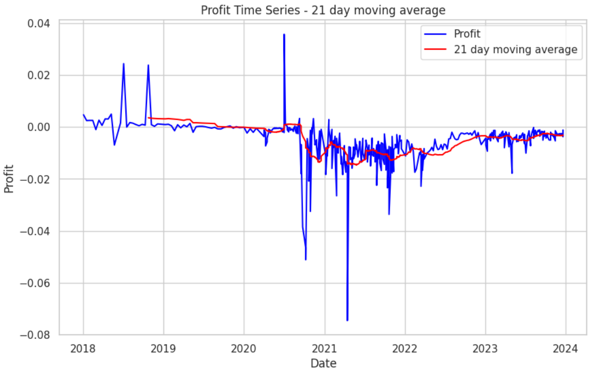
Security	**Variable**	**Chow F-Stat**	**Critical Value**
LFT	bid_to_Cover	0.4769	1.8642
	profit	1.1796	1.7916
NTN-B	bid_to_Cover	2.0189	1.7916
	profit	1.9840	1.7253

As robustness checks, we estimate the main regressions using a two-stage least squares IV approach, instrumenting all potentially endogenous variables with their respective one-period lags. IV estimates are generally consistent in sign with the baseline fixed effects and random effects models, however substantially less precise. Although endogeneity cannot be ruled out, the main findings are close (Tables 43 and 44).

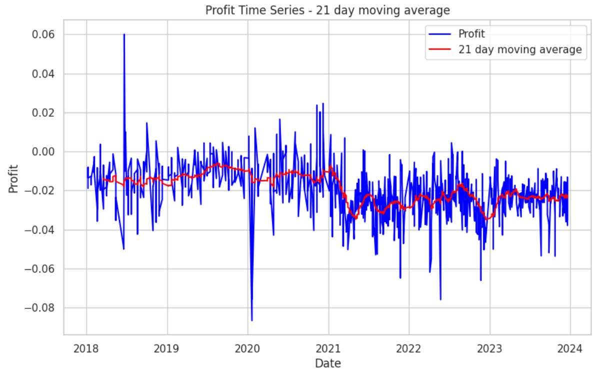
3.5 Conclusion

Auction design and mechanism selection for government bond auctions is a common theme for papers investigating which type and format generates greater seller revenue. A wave of empirical papers surged after theoretical studies concluded it is impossible to detect which type of auction is best without data evidence.

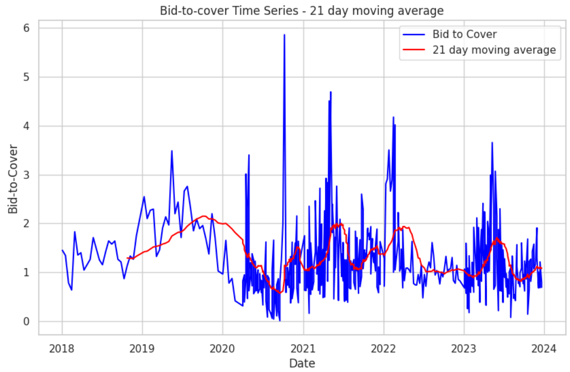
Figure 6 – LFT and NTN-B: Profit, Bid-to-cover, and AC Ratio



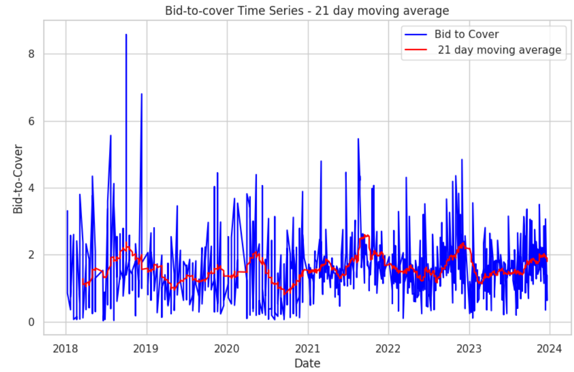
(a) Profit LFT



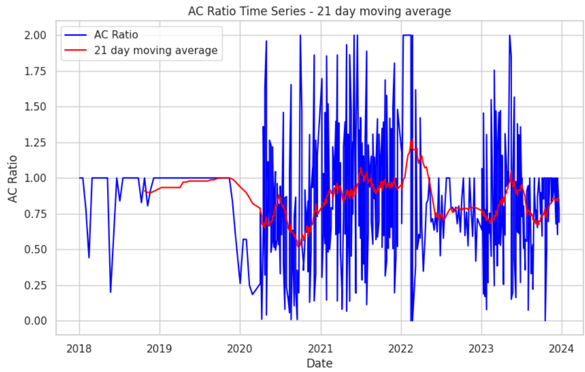
(b) Profit NTN-B



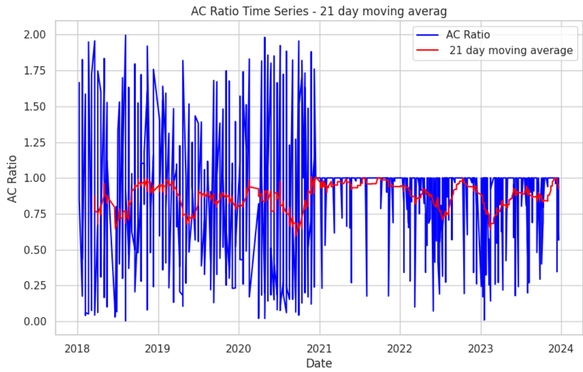
(c) Bid-to-cover LFT



(d) Bid-to-cover NTN-B



(e) AC Ratio LFT



(f) AC Ratio NTN-B

Those researches are founded on the idea that auction mechanisms generating lower profits are more effective at reflecting market prices, and thus, better aligned with extracting bidders' true valuations.

We examined changes in bid selection criteria for LFTs and modifications to auction formats for LFT and NTN-B securities since there were several customizations after 2020. Those shifts served as public policy experiments testing aspects of competition and revenue equivalence.

Even though there is no consensus in the literature, our regression results reveal weak evidence of lower bidder profits under LFT uniform-price auctions, which could lead to conclusions closer to common values or interdependent values hypotheses, as LFTs are floating-rate bonds with less uncertainty about its pricing.

We bring new evidence to the revenue equivalence debate with analysis of hybrid auctions conducted by the Brazilian Treasury. Contrary to theoretical predictions which propose hybrid auctions address informational issues regarding demand, we find higher profits in these type of auctions in comparison to traditional auctions.

This result may be related to risk arising from treasury allocation in hybrid auctions, causing winner's curse in a tighter pricing of securities alongside lower profits.

Finally, as a robustness check on our results, we tested for structural breaks from those modifications on profit and bid-to-cover, detecting evidence for both in NTN-B bonds after auctions were held under a traditional non-hybrid format.

4 Demand Features and Competition

A sovereign issuer strategy encompass its mechanism choice, benchmarks, debt profile, liquidity requirements, auction schedule, dealers' privileges and obligations, including routing orders, primary market share dealership goals, market risk and many other variables. All of those affect demand for government bonds. In that sense, researches on demand features often borrow structural or reduced-form models.

Nonetheless, empirical research on government bond auctions' focused mainly on issuance's price dynamics and bid profits for certain groups (e.g. whether large banks and dealers had higher auction surplus, secondary market manipulation prior to auctions etc.).

This section presents our contribution with one conventional and another modern tool to investigate how bids and bidders influence each other in treasury auctions.

Transparency efforts by government organizations have made new empirical evidence available, which in tandem with advanced computational methods, propelled significant leaps in this field. We jump on that quantitative trend simulating a random network of bidders calibrated by actual bid data and compare it with current bidders networks.

We follow Wachs e Kertész (2019) approach to identify collusion groups in a network of firms based on their co-bidding behavior. The distribution of two network measures were the core of this network analysis: coherence and exclusivity. Together they tell if a group of bidders have similar bids among each other and if they are different from the rest of the bidders.

Our simulations of competitive bidders seem quite consistent with NTN-B and LFT networks of bidders. Albeit, shifts in policy in 2020 for LFT, unlike previous section's analysis, brought LFT indicators to a less competitive state - though minimally.

We then turn to features of certain groups of bidders, taking into account individual data of auction profits. Since our bid-level data has classifications of dealers and non-dealers, we group those and investigate if their obligations and benefits drive their bids more competitively. Estimates suggest dealers bid more aggressively in general (lower profits).

The remainder of this section comprises literature review, empirical strategy, data, results and a brief conclusion.

4.1 Literature Review

There is an extensive body of literature on multi-unit auctions, particularly concerning government bonds. We review briefly several auction demand features, including foreign participation, consequences of dealers' privileges and obligations, information and so on.

Then, we offer a glimpse on quantitative analysis studies of a range of procurement auctions investigating competition.

4.1.1 Demand Features

From a supply standpoint, a sovereign issuer may decide on its mechanism choice and an auction schedule, improving its revenue, competition and efficiency of its transactions. However, other features such as foreign investors' participation, bidders' information, dealers' privileges and obligations, including routing orders, capital requirements, primary market share dealership goals etc., might positively affect demand for government bonds. Not infrequently, related researches borrow structural or reduced-form models.

Influenced by suspicions of market manipulation by some participants in treasury auctions, Jegadeesh (1993) examined whether auction profits were related to the share of winning bids by banks and dealers to check for collusion evidences. He compared profits obtained by buying bonds in auctions with secondary market trade profits for similar bonds, finding no significant differences except for a liquidity premium for new issuances. His results indicate that auction profits would be lower than secondary market bid-ask spreads.

In order to test the collusion hypothesis, Jegadeesh (1993) regressed auction profits on the spread of bids, the number of bids, and the fraction of winning bids by dealers. His results point to weak evidence of lower profits when there is a higher proportion of winning bids from non-dealers and commercial banks, in line with the collusion hypothesis.

More recently, the emphasis on information as a central theme has become more pronounced in the literature. Hortaçsu e Kastl (2012) estimated the benefit to dealers of being able to update their bids in auctions after observing customers' orders. The authors extended Kastl (2011) structural model to account for two types of participants: dealers and customers (investors who submit bid orders through dealers), enabling dealer order updates following customers' orders information.

Based on Canadian Treasury auction data, they estimated that information obtained in bid routing allowed dealers to update their knowledge about their auction competitors through the distribution of bids. This knowledge accounts for between 13% and 27% of dealers' expected auction profits.

In a study on bid shading and bidder surplus in US Treasury auctions, Hortaçsu, Kastl e Zhang (2018) incorporated modifications to the previous model proposed by Hortaçsu e Kastl (2012), considering information asymmetries and three types of participants: dealers, direct bidders and indirect bidders (participants who need to submit their bids through dealers).

The authors point out dealers shade their bids more than other participants (submit bids below their true valuation). To understand whether this bid shading comes from different valuations or market power, they estimate a structural model for three types of participants comparing bid shading between groups. Then, they estimate bidder surplus, finding dealers are less willing to buy bonds at lower rates, having the ability to shade their bids more efficiently, obtaining better auction prices. In their auction efficiency computations, participants' surplus in US Treasury auctions was close to 3 basis points, with efficiency losses around 2 basis points. Our results differ as dealers in Brazil tend to bid more competitively due to greenshoe benefits and market share obligations.

Also addressing information in auctions, Beetsma et al. (2018) developed a theoretical model estimating the effect on asset value due to dealers' reception of private information from clients. Their findings indicate lower post-auction rates in auctions with high bid-to-cover ratios, with a magnified effect in periods of high volatility. They also investigated determinants of bid-to-cover, finding evidence of influence on this variable from past auctions of the same issuing country and others (in Europe), as well as the number of dealers, volatility, and supply (BEETSMA et al., 2020). The authors recommend adjusting supply due to market volatility.

From a different perspective, Boyarchenko, Lucca e Veldkamp (2021) sought to answer whether a dealer who receives client's bids should pass on information to other dealers and/or other clients. To do this, Boyarchenko, Lucca e Veldkamp (2021) calibrated a theoretical model of a Nash Bayesian equilibrium in which there is diffusion of information among dealers and other participants, verifying the effects exchanged knowledge.

The central hypothesis is shared information reduces auction uncertainty and encourages participants to bid more. Welfare effects generated by information sharing depend on how much it reduces or increases information asymmetry. In their simulations, shared information among dealers generates more auction revenue - a possible explanation arises from improved risk-sharing and reduced information asymmetry.

Meanwhile, Cole, Neuhann e Ordoñez (2021) constructed a Walrasian model with agent heterogeneity regarding wealth, market power, risk aversion, and information, concluding the main difference in bid patterns is due to information. In another perspective, Tchuindjo (2022) modeled auctions as a Stackelberg game, with leaders (indirect bidders) and a followers (dealers), in which dealers have information about the leaders' bids and reacts by increasing the competitiveness of their bids. The theoretical result is an increase in revenue for the issuer, corroborating the result of Boyarchenko, Lucca e Veldkamp (2021).

A myriad of papers investigate demand effects emerging from dealer regulation, bank capital requirements, and foreign participation in domestic auctions. Alvarez e Mazon (2019) analyzed characteristics of Spanish Treasury auctions, concluding overbidding causes primary market price to be above secondary market in Spain, mainly due to primary dealers' regulation. It is worth noting their mechanism choice is an hybrid auction type: the wap, in which bids with a price above the wap (weighted average price) pay the wap, while bids with a price below pay the value of their bids. Whereas Klingler e Sundaresan (2023) tested the Treasury bill premium over OIS (overnight indexed swap), pointing out a bond liquidity premium over a swap. According to him, this is related to buyer balance sheet constraints, even when controlling for supply and interest rate levels.

A different application using both reduced and structural models by Elsinger, Schmidt-Dengler e Zulehner (2019) assessed foreign bank competition impact on bidder surplus in Austrian Treasury auctions. As expected, they found evidence of increased competition and a surplus reduction resulting from foreign participants entry into domestic auctions. His empirical strategy incorporated reduced models cleverly through the use of differences in differences, comparing the spread reduction with the issuance of German government bonds.

Regarding those, Shida (2023) conducted an empirical analyses aiming to identify the primary drivers for demand in German government bond auctions. Their dataset included more than 600 auctions between 2005 and 2002. The main findings indicate a higher demand with higher rates, larger premiums in previous auctions, supply, and new syndicates by issuers. However, volatility and bank leverage constraints negatively affect demand. Shida (2023) did not find a robust crowding-out effect from comparable issuers.

In Brazil, Silva (2002) proposed a structural model variation to accommodate two participant groups: dominant participants (those acquiring a significant portion of the lot) and the rest. In this study, Silva (2002) calculated a measure of participants' elasticity, concluding that commercial and national banks exhibit higher demand elasticities, which vary according to the auction. This suggests caution in using a common values setting for treasury auction analysis.

Furthermore, Silva (2003) investigated the bidding strategies of different participant types in auctions (commercial banks, national banks, foreigners, etc.). According to the author, in more competitive auctions, articipants tend to bid higher for bonds, submit a greater number of bids, and increase bid dispersion. The results of higher profits for foreign participants also undermine the common values premise. (BARBOSA, 2024) and the present thesis employ similar approach, although unlike the previous study, here we encounter more aggressive bidding by dealers.

4.1.2 Competition and Networks

Previous research on government bond auctions' strategic behavior focused mainly on issuances' price dynamics and bid profits for certain groups (e.g. whether large banks and dealers had higher auction surplus, secondary market manipulation prior to auctions etc.). This section presents other tools to investigate how bids and bidders influence each other, which are commonly used to check collusive behavior not only in procurement auctions, but in different markets. Unlike most government bonds' researches on competition, usually papers in this area and in cartel detection employ screening tests (descriptive statistics on bid/bidders data), correlation and bidding patterns among competitors.

Quantitative methods are crucial to spot bidder strategic behavior. Transparency efforts by government organizations have made new empirical evidence available, which in tandem with advanced computational methods, propelled significant leaps in this field. Thus, traditional use of screening tests through descriptive statistics evolved to more sophisticated econometric tools.

Despite progresses in quantitative investigation, it is paramount other evidence should also be used by authorities in cases related to cartel prevention and punishment.

Explicitly, statistical tests on filters are designed to identify anomalous bidding patterns or those with a low probability of occurring comparing groups of bids/bidders. Filters are extensively used throughout literature as inputs in econometric and machine learning models.

According to Abrantes-Metz e Bajari (2012), a good filter requires: minimizing false positives and false negatives; easy implementation; it should be costly for firms to conceal collusion; and empirical basis. They review several types of filters for detecting collusion applied in different contexts.

One of the primary filters used on procurement data related bids to cost. In this case, authors conjecture competitors should submit proposals according to their cost of production - bids independent of it should then be considered suspicious. Next, they refer to an interesting kind of filter (not limited to auctions) which investigates collusion through market share consistency and rotation among leaders.

They also mention filters for improbable events, citing a case in which 7 distinct bidders submitted identical 8 digit bids down to the penny. This unlikely event could indicate coordinated bidding. In Brazil, an interesting application of it uses Benford's Law to identify overpricing in procurement bids (CUNHA; PORTUGAL; BUGARIN, 2016). This law states digit distribution should be decreasing from digit 1 to 9, therefore she argues deviations in bid distributions from it might signal collusion. Another famous implementation of Benford's Law was the case of Libor manipulation - where dealers submitted expected rates.

Imhof (2017) finds cartel members submit bids with lower coefficient of variation and higher kurtosis statistics. He notes bid combinations show more negative asymmetry and different price behaviors in cartel periods and afterwards. Whereas Blanckenburg, Kholodilin e Geist (2012) tests descriptive statistics and the use of moments for collusion detection in 11 different markets. The authors did not find robustness in the indicators, but they succeeded in comparing price distributions through the Kolmogorov-Smirnov test.

More recently, Chassang et al. (2022) proposed density tests in bid distribution to detect collusion in procurement auctions. His methodology stemmed from the observation that there was a lack of density in the distribution of proposals near winners. According to them, a missing mass around winning bids was inconsistent with competition.

A separate perspective on cartel detection uses network analysis and clusters to infer relationships among market participants.

An intriguing use of network analysis was conducted in the Indian stock market (PALSHIKAR; APTE, 2008), which used a combination of clustering methods on graphs (networks) of market agents to detect artificial price formation of assets traded on the stock exchange(circular trading). While Dass, Reddy e Iacobucci (2014) examined interactions between bidders in open envelope online auctions to detect central participants - those who influence others - and to predict the price of winning bids in these art auctions.

Morselli e Ouellet (2018) investigated collusion by construction industries in Canada through bid patterns to increase market shares. His research indicate bidding similarity (measured by Jaccard coefficient as a rate firms bid together) has a positive relationship with firms' market shares, especially in cities where there are suspicions of cartel activity - successful companies' bids were more similar in collusive markets. As if companies strategically cooperated to keep higher market shares on repeated games creating a false sense of competition at the same time.

Similarly, Wachs e Kertész (2019) applied network analysis to identify groups that interact intensely in a market as a filter for potential collusive groups, however based on the topology of their networks and how this can facilitate strategic behavior. They adapt an algorithm for community detection that merges nodes into groups by locally optimizing a function proposed by Lancichinetti, Fortunato e Kertész (2009). Using graph theory, Wachs e Kertész (2019) established algorithms for group formation and proposed the use of two indicators together: exclusivity and coherence (to find segregated and similar groups) for coordinated behavior detection. In their dataset, cartel markers were more likely associated to exclusive and cohesive groups. We employ a similar strategy for treasury auctions comparing distributions of simulated networks to actual data.

The last strand of research in competition presented in this paper arises from the recent use of machine learning tools to detect collusive behavior.

Silveira et al. (2021) detect cartels using supervised learning techniques based on moments of the gasoline price distribution. They combine filters on descriptive statistics and features in distributions of prices with machine learning models (lasso and ridge regressions, random forests and neural networks) in order to predict cartels in fuel retail market. Later, Silveira et al. (2023) proposed a method to identify cartels with unsupervised learning. Initially, the authors divided participants into clusters, then used it as targets of an algorithm that classifies groups based on a variety of filters to capture non-competitive behaviors.

In supervised learning, Wallimann, Imhof e Huber (2022) put together various filters in all possible subgroups of 3 or 4 bids within tenders to detect anomalies and apply machine learning model for classification of competitive or collusive proposals, obtaining better results in random forests models. His goal was to propose a method to flag incomplete cartels.

A very modern approach by Huber e Imhof (2023) perfected classification of bid-rigging cartels. Based on procurement data from Switzerland and Japan auctions, the authors use a *deep learning* tool, convolutional neural networks, to identify patterns in graphics of plotted bid data - similar to image recognition. Their results improved accuracy in flagging cartels in a supervised learning environment.

Despite the availability of these diverse groups of tools to detect collusive behavior in different settings, we opted for the use network analysis proposed by Wachs e Kertész (2019), extracting information through relationships of bidders revealed through bid patterns. We employ this analysis due to some reasons: it is a scalable, unsupervised method, applicable to auctions and it provides reasonable intuition and visual interpretations.

4.2 Empirical Strategy

The first step in our analysis is to estimate networks of bidders connected by the similarity of their bids. In this research, an edge (connection) is added to nodes (bidders) if the two bid within 0.5 basis point of each other. The weight of an edge (connection from bidder i to j) is given by the number of similar bids. A common way to mathematically represent a network is through an adjacency matrix, defined as:

$$A_{ij} = \begin{cases} w, & \text{if node } i \text{ connects to node } j \text{ with weight } w, \\ 0, & \text{otherwise.} \end{cases} \tag{4.1}$$

We also set $A_{ij} = A_{ji}$ and $A_{ii} = 0$. Our approach is similar to Wachs e Kertész (2019), a bottom-up method for group detection in which nodes transform into groups by locally optimizing a fitness function proposed by Lancichinetti, Fortunato e Kertész (2009). The fitness function f_G for a group of nodes G is represented as follows:

$$f_G = \frac{s_{\text{in}}^G}{(s_{\text{in}}^G + s_{\text{out}}^G)^\alpha \times |G|^\beta}, \tag{4.2}$$

where s_{in}^G is the sum of the weights of the edges within the group; s_{out}^G it the sum of the weights of the edges adjacent to the group; $|G|$ is the group size; and α, β are parameters controlling the cohesion and size of the groups, respectively.

The fitness of adding a node n to a group G can be defined as the difference in the fitness function before and after adding n:

$$f_G^n = f_{G+\{n\}} - f_G. \tag{4.3}$$

The group detection algorithm can be performed as follows: select a node n and initialize a group containing only n; then select an adjacent node of n that maximizes the fitness function (if adding this node increases the group's fitness, include it in the group); repeat it until no adjacent node increases the group's fitness.

Next, we compute coherence and exclusivity of those groups. Coherence measures how similar connections are within a group by dividing the geometric mean by the arithmetic mean of the edge weights.

Coherence should be near 1 if edge weights are similar within the group. If they differ, the geometric mean will be much smaller than the arithmetic, resulting in a lower coherence. This indicator is useful for understanding how evenly distributed the connection weights are within a set of nodes. It is defined as follows:

$$C = \frac{M}{A} \tag{4.4}$$

where A represents the arithmetic mean and M the geometric mean of edge weights within the group. Specifically, arithmetic mean is given by:

$$A = \frac{\sum_{(i,j)\in E_{\text{in}}} w_{ij}}{|E_{\text{in}}|} \tag{4.5}$$

and the geometric mean is defined as:

$$M = \left(\prod_{(i,j)\in E_{\text{in}}} w_{ij} \right)^{\frac{1}{|E_{\text{in}}|}} \tag{4.6}$$

In these expressions, E_{in} denotes the set of edges within the group. w_{ij} is the weight associated with the edge (i, j), and $|E_{\text{in}}|$ it the total number of edges within the group.

The second metric, exclusivity, signals how much a group binds itself among its nodes in relation to outside nodes. It is close to 1 when most of the group's edge weights are connected to the own group. When most of the edge weights are connected to outside nodes, it gets closer to zero.

A highly exclusive nodes has much more relationships within the group than with outside nodes. It is very useful for detecting clusters or communities inside networks.

This metric is particularly useful for identifying clusters or communities within networks that have a distinct separation from other groups. It is defined as follows:

$$E = \frac{\sum_{(i,j)\in E_{\text{in}}} w_{ij}}{\sum_{(i,j)\in E_{\text{all}}} w_{ij}} \tag{4.7}$$

where E_{in} is the set of edges within the group, E_{all} is the set of all edges involving nodes in the group (both internal and external),w_{ij}, the weight of the edge (i, j).

Together, these two metrics play an important role in detecting clusters of participants in networks with similar behavior.

The next step is to evaluate how those networks stand against random simulations of competitive and suspicious networks of bids, comparing exclusivity and coherence metrics. We set two networks with 23 bidders, however one of them had 3 bidders with lower variance among its bids (standard deviations of 1 and 2 for competitive and suspicious bidders, respectively), higher correlation (0.5) and bid on average 2 basis points above the others tagged as competitive.

These numbers are calibrated to resemble data from our sample in magnitude, where accepted bids are close to 20 in LFT and 13 in NTN-B, individual profits interquartile range are 1 and 3, respectively, with standard deviation above 2 basis points for NTN-B (Tables 31 and 33).

In our 10,000 simulations (200 years of 50 auctions), the tolerance of bid deviation to form an edge between bidders was below 0.5 basis point, which is generally the minimum bid/ask spread in the secondary market. The more bids fell within this tolerance, the more connected the bidders were, resulting in a higher weight on the edge between them.

After comparing densities of group metrics of simulated networks with actual networks groups, we tested differences in group series of exclusivity and coherence from two samples of the same bond - before and after policy changes.

Secondly, we took a little further our bid data and investigated demand features based on individual bid profits. We grouped bidders into dealers and non-dealers. Then, our approach was plain vanilla: dummies for groups, dummies for benchmarks, types and formats of auctions, control variables for market risk controls and benchmark/specific covariates. We test whether in fact dealers, due to obligation and privileges, bid more competitively (if there is indeed private values).

Here, instead the dependent variable $profit_i$ represents bond yield changes from clearing rate to the secondary market, for bid i. This differs from previous model which used a profit measure on the auction level.

The model is as follows:

$$profit_i = \beta_0 + \beta_1 \text{Market\_Risk}_i + \beta_2 \text{Benchmark\_Risk}_i + \beta_3 \text{Dummies}_i + \epsilon_i \tag{4.8}$$

where Market_Risk_i measures market risk factors, such as exchange-rate, VIX, CDS, Selic Rate, Benchmark_Risk_i captures benchmark-specific risk factors, such as benchmark rate variations, amount issued, dv01 supplied, bid-to-cover, supply adjustments, number of bids etc., Dummies_i are dummy variables for structural and/or benchmark-specific effects, such as auction type, formats and groups of bidders.

4.3 Data

Although many variables are similar than previous data sections, here we focus on bid level data, instead of auction level. Our data comprises NTN-B (inflation-linked) and LFT (floating-rate) bids from auctions between 2019 and 2023, with over 10,000 NTN-B bids and close to 8,000 LFT bids.

Since policy changes regarding traditional/hybrid and uniform/discriminatory price auctions are restricted to NTN-B and LFT securities, we still focus our analysis in those bonds. We also group benchmarks as before: LFT maturities into benchmarks of 3 years and 6 year; NTN-B are divided into our own classification of various benchmarks. Again, we set NTN-B benchmarks as: 3 years; 5 years; 7, 10 and 15 years; 20, 25 and 30 years; and 40 years. For both types of securities we fit those categories of benchmarks according to maturity windows and issuance schedule. For networks of bids, we only use bid yield data, unlike demand features which still need other controls.

Once more, we organize our variables as market risk factors, benchmark-specific risk factors and dummies. We only add tables slicing samples between accepted bids and non-accepted bids in order to calibrate our network simulation. Deltas are used only for 'V-shape' analysis, therefore are not present here.

Market risk variables reflect the broader economic environment, influencing investor behavior and auction outcomes. Benchmark-specific risk factors, such as benchmark rate variations, amount issued, dv01 supplied, bid-to-cover and supply adjustments take into account idiosyncrasies of each auction/maturity. In contrast with revenue equivalence analysis, profit is our dependent variable on the bid level, enabling dummies for separate groups of bidders.

As market risk variables, we have **USD/BRL**, exchange rate between Brazilian Real and the US Dollar, higher values might indicate riskier environments for locals and cheaper prices for foreign investors. Our sample captures the move from 4.9 up to 5.9.

VIX (Volatility Index), which is a gauge for global market uncertainty, arising from implied volatility on near term index options on the S&P 500. Most of the sample is below 20, which is historically low.

CDS (Credit Default Swap Spreads), Brazilian 5-year sovereign credit risk. Three quarters of the sample is below 230 basis points, also historically low.

Also, **SELIC**, Brazil's policy interest rate, directly influencing yield curves. It ranges from 1.9% to 13.65%. Higher SELIC levels correlate with increased yields across NTN-Bs and higher premiums - spreads above SELIC - for LFT.

UST10Y is the 10y treasury yield.

And **DXY (US Dollar Index)**, an index of exchange rates between major currencies and the US Dollar, it correlates with USD/BRL, and international risk-aversion periods.

As benchmark specific we include **dv01_of_bench**, computed as the price change from a 1 basis-point increase in the bond yield times the amount issued that date for the specific benchmark bid, divided by 1 million here. This is variable is not really applicable for LFTs because of the ways it is priced, in addition to its issuances.

Number_bids, the bid count for each auction. We group the benchmarks here in the description by type of bond - close to 20 for NTN-Bs and over 35 for LFTs.

number_bids_ac, the accepted bid count for each auction, around 2/3 for both types of bonds.

Ac_ratio corresponding to supply adjustments, reducing the amount issued and profits. It is computed as the ratio between Issuance and Supply - always very close to 1 when you combine all benchmarks.

Bid_to_cover captures the ratio of amount bid relative to initial supply. In this sample, NTN-Bs relative demand is close to 1.8 and LFTs, 1.6.

Profit, also regarded as auction premium, is the difference between the auction's clearing rate and the secondary market rate. It is almsot zero on average to LFTs and negative in 1 basis-point on average NTN-Bs.

Profit_i, individual auction premium, is the difference between the bid 'i' rate and the secondary market rate. It is almsot zero on average to LFTs and negative in 1 basis-point on average NTN-Bs. Unlike profit, it takes into account non-accepted bids.

Anb-5 is a proxy of the yield changes between last auction date closing price and current auction opening yield - also very close to zero across all benchmarks.

Tables 30 and 31 encompass descriptive statistics for grouped NTN-B benchmarks, including a sample segmentation for accepted and non-accepted bids.

The first table presents descriptive statistics for accepted ad non-accepted bids. Accepted bids account for about two thirds of all NTN-B, with average profit negative on 2.5 basis points and interquartile range of 2 basis points. More than three quarters of all bids are within 2 basis points from the secondary market yield.

While 31 comprises data for all 10,609 bids in our sample. Macroeconomic indicators averages such as SELIC (8.17%), USD/BRL (5.01), UST10Y (2.4) and VIX (20.8), albeit several periods of volatility, characterize a relatively favorable market conditions in which these auctions took place. Supply adjustments overall is close to 1, with bid-to-cover next to 1.8. Overall, profits are negative in approximately 1 basis point.

Table 30 – Descriptive Statistics for Accepted and Non-Accepted Proposals - NTN-B

Variable	count	mean	std	min	25%	50%	75%	max
non-accepted profit_i	3830.0000	0.0055	0.0192	-0.0502	-0.0050	0.0040	0.0130	0.3800
accepted profit_i	6829.0000	-0.0250	0.0226	-0.3200	-0.0336	-0.0212	-0.0105	0.0350

Table 31 – Descriptive Bid Level Regressions - NTNB

	count	mean	std	min	25%	50%	75%	max
number_bids	10659.0000	21.1933	8.8094	1.0000	14.0000	21.0000	28.0000	48.0000
number_bids_ac	10659.0000	13.1005	6.3048	1.0000	9.0000	12.0000	17.0000	36.0000
ac_ratio	10659.0000	0.9638	0.3137	0.0100	1.0000	1.0000	1.0000	1.9800
bid_to_cover	10659.0000	1.7701	0.8488	0.0600	1.2802	1.6370	2.1300	5.4640
profit	10659.0000	-0.0075	0.0120	-0.0516	-0.0150	-0.0060	0.0000	0.0350
profit_i	10659.0000	-0.0140	0.0259	-0.3200	-0.0285	-0.0112	0.0000	0.3800
dv01_of_bench	10659.0000	154.7745	142.0011	4.6881	64.1191	113.8530	198.8669	1098.8230
anb-5	10659.0000	0.0031	0.1476	-0.8691	-0.0876	0.0060	0.0790	1.0899
USD/BRL	10659.0000	5.0134	0.5213	3.6670	4.8841	5.1561	5.3543	5.8746
DXY	10510.0000	117.4187	4.2527	110.5179	113.9431	116.5402	120.5927	128.4399
VIX	10603.0000	20.7836	5.7222	12.0700	16.6600	19.6900	24.1100	53.5400
UST10Y	10510.0000	2.4014	1.1495	0.5200	1.5300	2.0600	3.5200	4.8800
CDS	10659.0000	200.6169	46.3540	92.5100	168.0100	199.7500	229.1700	346.9900
SELIC	10659.0000	8.1701	4.6201	1.9000	3.6500	6.4000	13.1500	13.6500

Tables 32 and 33, encompass descriptive statistics for grouped LFT benchmarks. Again, about two thirds of 7,897 bids are accepted, this time with profit interquartile range much tighter in less than 1 basis point. Macroeconomic variables are similar to NTN-B sample, with small differences due to outliers. Profits are slightly below zero, acceptance ratio close to 1, with bid-to-cover close to 1.6.

Table 32 – Descriptive Statistics for Accepted and Non-Accepted Proposals - LFT

Variable	count	mean	std	min	25%	50%	75%	max
non-accepted profit_i	3077.0000	0.0027	0.0237	-0.1000	-0.0010	0.0005	0.0017	0.3400
accepted profit_i	4978.0000	-0.0005	0.0348	-0.3334	-0.0070	-0.0022	-0.0003	0.3200

Table 33 – Descriptive Bid Level Regressions - LFT

	count	mean	std	min	25%	50%	75%	max
number_bids	7897.0000	37.6609	27.2769	1.0000	20.0000	27.0000	44.0000	98.0000
number_bids_ac	7897.0000	20.8496	13.6071	0.0000	11.0000	19.0000	27.0000	70.0000
ac_ratio	7897.0000	0.9759	0.4262	0.0000	0.6941	1.0000	1.1140	2.0000
bid_to_cover	7897.0000	1.5851	0.8382	0.0080	0.9697	1.4339	2.0910	5.8620
profit	7897.0000	-0.0023	0.0079	-0.1000	-0.0037	-0.0008	0.0003	0.0385
profit_i	7897.0000	-0.0027	0.0075	-0.0900	-0.0050	-0.0010	0.0004	0.0899
dv01_of_bench	7897.0000	545.1537	297.8719	21.4637	319.3881	575.9302	655.4973	2242.1230
anb-5	7527.0000	-0.0021	0.0164	-0.1894	-0.0029	-0.0001	0.0005	0.1190
USD/BRL	7897.0000	4.9021	0.6275	3.6800	4.2289	5.1214	5.3466	5.8864
DXY	7731.0000	116.8554	3.8151	110.5570	114.1672	115.8906	119.7987	128.4399
VIX	7792.0000	20.2287	6.1869	12.0700	15.8200	18.8400	23.1300	50.9100
UST10Y	7731.0000	2.2635	1.1214	0.5500	1.4900	1.9000	3.0500	4.8800
CDS	7897.0000	193.6361	45.6227	97.0200	165.0000	185.0100	223.7300	337.7900
SELIC	7897.0000	7.4140	4.2452	1.9000	4.1500	6.4000	12.6500	13.6500

4.4 Results

First we set up networks for NTN-B and LFT according to bid similarity (Figure 7), just for illustration purposes. LFT network appears condensed and more populated, just as dispersion of bid profits and number of bids per auction would lead. NTN-B presents a smaller number of relevant players with higher network centrality.

Subsequently, we divide the samples before and after policy shifts: April 2024 for LFT, when bids started to be selected by uniform-price criterion; and January 2021, just as tradition auctions became the norm for NTN-B, instead of hybrid.

Now, we set up two networks for each type of security, according to the sample, and then apply the group forming algorithm. It is noteworthy randomness of bids might form groups and it does not necessarily mean they are colluded. So, we end up with a network of bidders from 2019 to 2020 and another from 2021 up to the end of 2023 for NTN-B. LFT also has two networks, one from 2019 to March 2020, and another from April 2020 to the end of 2023.

Results for coherence and exclusivity metrics for both samples are a) and c) of Figure 8. Prior to policy shifts groups formed are dotted in red, and afterwards in blue. NTN-B groups seem to have same distribution of coherence and exclusivity before and after auction adjustments (Figure 8 (b)) - this suspicion is endorsed by T-Tests and Mann-Whitney U test on differences of coherence and exclusivity of the two cohorts in Table 34. LFT distributions of coherence and exclusivity present quite the opposite conclusion as witnessed in the same table. T-stats for both exclusivity and coherence are significant the 5% and 1% level, respectively.

Table 34 – Test Results for NTN-B and LFT Group Metrics

Security	**Test**	**Statistic**	**p-value**
NTN-B	Coherence (t-statistic)	0.12	0.90282
	Exclusivity (t-statistic)	0.69	0.49222
	Coherence (U-statistic)	298.50	0.74866
	Exclusivity (U-statistic)	334.00	0.65437
LFT	Coherence (t-statistic)	2.24	0.03016
	Exclusivity (t-statistic)	-4.20	0.00008
	Coherence (U-statistic)	989.00	0.12820
	Exclusivity (U-statistic)	396.00	0.00001

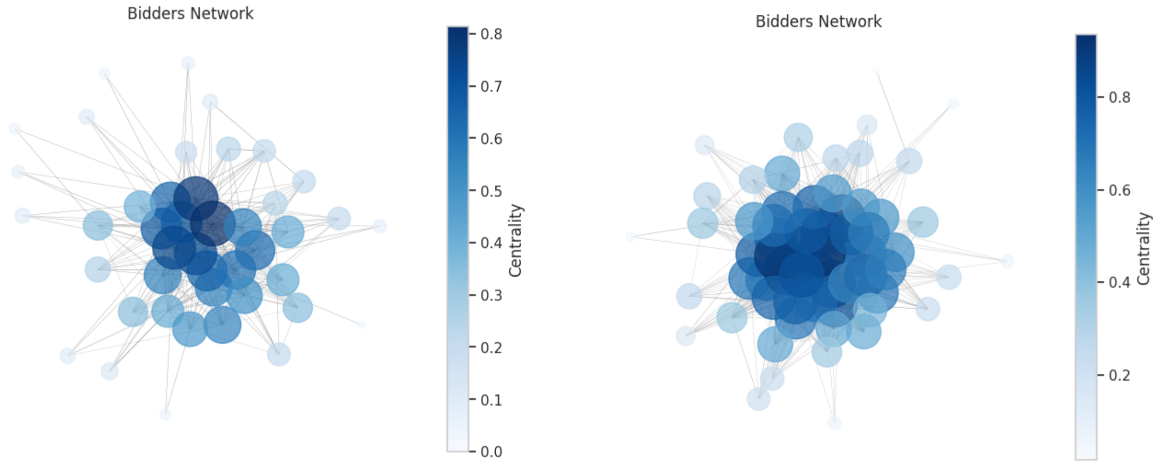
The more recent sample moved LFT groups towards the less competitive side, however it is quite a stretch to interpret it as a non-competitive market. Levels of coherence and exclusivity are still very low, inferior to our conservative simulations of competitive bidders and really close to NTN-B metrics.

When we compare densities of NTN-B and LFT groups to simulated densities of suspicious and competitive groups, respectively in red and blue, we observe a higher concentration of red dots on the top right of e) in Figure 8. This is the zone were suspicious groups might be located according to Wachs e Kertész (2019).

By construction and by the way this markets work, bids should be very similar, therefore highly cohesive. Price discovery, traded futures, electronic trading platforms and other mechanisms make it easier for bids to stand close. However, density of exclusivity might give us some indication of suspicious groups.

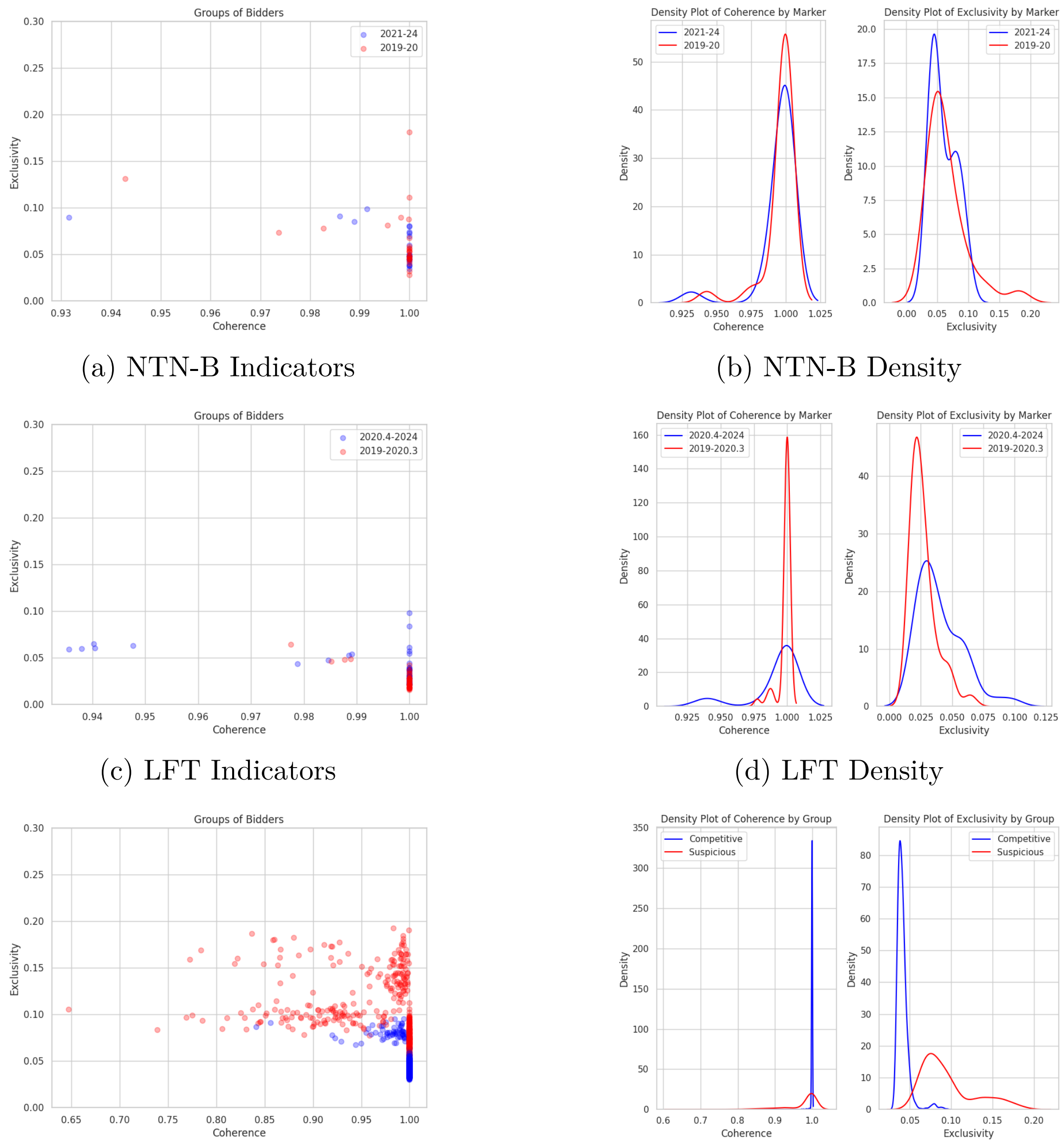
Still, metrics values of suspicious groups from our random simulations - f) in Figure 8 - are much higher than both LFT and NTN-B, making it improbable to infer formation of groups acting as one.

Figure 7 – Coherence and Exclusivity Across Networks



(a) NTN-B Network (b) LFT Network

Figure 8 – NTN-B, LFT and Simulation Network Metrics



(a) NTN-B Indicators (b) NTN-B Density

(c) LFT Indicators (d) LFT Density

(e) Simulation Indicators (f) Simulation Density

The second part of results tend to a more conventional empirical strategy. For multiple specifications in Tables 35 and 36 our estimates of bid level profit on controls provide key insights on how relevant bidders participate in auctions.

With dummies for dealers, our estimates showed dealers tend to have lower profits, which is consistent with dealer's obligations and greenshoe benefits and other researches on overpricing as in Alvarez e Mazon (2019). Although the difference is quite small, it sheds light on a private value component, as is in the interdependent values hypothesis.

Macroeconomic controls such as exchange rate and VIX tend to have an extra effect on NTN-Bs (since it is a riskier asset), nonetheless yield variation was not significant in NTN-B, whereas anb-5 appeared relevant for LFT, in opposite direction to auction profits. Acceptance ratio coefficients are negative as expected, as supply adjustments drive profits downward. However, relative demand represented by bid_to_cover was not negatively related to lower profits in NTN-B, maybe due to lesser restrictive supply adjustments.

Lastly, bid-level coefficients are also coherent with lower bidder profits in traditional auctions and uniform-price criterion.

Table 35 – OLS Results for Multiple Specifications - Bids LFT

Dependent Variable: *profit*	**Spec. 1**	**Spec. 2**	**Spec. 3**	**Spec. 4**	**Spec. 5**
const	0.0171*** (0.004)	0.0117** (0.005)	0.0179*** (0.004)	0.0125*** (0.004)	0.0138*** (0.003)
anb-5	-0.0000 (0.002)	0.0004 (0.002)	-0.0004 (0.002)	-0.0001 (0.002)	-0.0003 (0.002)
USD/BRL	-0.0039*** (0.001)	-0.0038*** (0.001)	-0.0043*** (0.001)	-0.0041*** (0.001)	-0.0041*** (0.001)
DXY					
VIX	0.0003*** (0.000)	0.0003*** (0.000)	0.0004*** (0.000)	0.0004*** (0.000)	0.0004*** (0.000)
UST10Y	-0.0011** (0.001)	-0.0011** (0.001)	-0.0002 (0.000)		
CDS					
SELIC	0.0003* (0.000)	0.0003** (0.000)			
dv01_of_bench	0.0000 (0.000)		0.0000 (0.000)		
number_bids	-0.0000 (0.000)	0.0001 (0.000)	-0.0000 (0.000)	0.0000 (0.000)	
ac_ratio	-0.0081*** (0.001)	-0.0079*** (0.001)	-0.0080*** (0.001)	-0.0079*** (0.001)	-0.0076*** (0.001)
bid_to_cover	0.0043*** (0.000)	0.0042*** (0.000)	0.0043*** (0.000)	0.0042*** (0.000)	0.0043*** (0.000)
d_type	-0.0157*** (0.002)	-0.0141*** (0.002)	-0.0153*** (0.002)	-0.0136*** (0.002)	-0.0144*** (0.002)
d_hybrid	0.0084*** (0.001)	0.0090*** (0.001)	0.0078*** (0.001)	0.0083*** (0.001)	0.0083*** (0.001)
d_bench3y		0.0013 (0.001)		0.0014* (0.001)	0.0013 (0.001)
d_bench20y		0.0039*** (0.001)		0.0036*** (0.001)	0.0032*** (0.001)
d_bench40y		0.0023*** (0.001)		0.0020** (0.001)	0.0018** (0.001)
d_dealer	-0.0044*** (0.001)	-0.0045*** (0.001)	-0.0043*** (0.001)	-0.0044*** (0.001)	-0.0045*** (0.001)
Observations	10491	10491	10491	10584	10584
R-squared	0.056	0.058	0.056	0.058	0.057
BIC	-47583.221	-47587.197	-47588.051	-48019.969	-48027.992

Table 36 – OLS Results for Multiple Specifications - Bids NTN-B

Dependent Variable: *profit*	**Spec. 1**	**Spec. 2**	**Spec. 3**	**Spec. 4**	**Spec. 5**
const	0.0091*** (0.002)	0.0105*** (0.002)	0.0090*** (0.002)	0.0193*** (0.002)	0.0171*** (0.001)
anb-5	-0.0762*** (0.015)	-0.0770*** (0.015)	-0.0762*** (0.015)	-0.0770*** (0.015)	-0.0744*** (0.015)
USD/BRL	-0.0024*** (0.000)	-0.0028*** (0.000)	-0.0024*** (0.000)	-0.0037*** (0.000)	-0.0036*** (0.000)
DXY					
VIX	0.0002*** (0.000)	0.0002*** (0.000)	0.0002*** (0.000)		
UST10Y	0.0010*** (0.000)	0.0011*** (0.000)	0.0009*** (0.000)		
CDS					
SELIC	-0.0000 (0.000)	-0.0001 (0.000)			
dv01_of_bench	-0.0000*** (0.000)		-0.0000*** (0.000)		
number_bids	-0.0000** (0.000)	-0.0000*** (0.000)	-0.0000** (0.000)	-0.0000*** (0.000)	
ac_ratio	-0.0007 (0.000)	-0.0012*** (0.000)	-0.0007 (0.000)	-0.0018*** (0.000)	-0.0019*** (0.000)
bid_to_cover	-0.0004 (0.000)	-0.0001 (0.000)	-0.0004* (0.000)	-0.0000 (0.000)	-0.0002 (0.000)
d_type	-0.0045*** (0.001)	-0.0043*** (0.001)	-0.0045*** (0.001)	-0.0011*** (0.000)	0.0000 (0.000)
d_bench3y		-0.0003 (0.000)		-0.0011*** (0.000)	-0.0010*** (0.000)
d_dealer	-0.0004*** (0.000)	-0.0004*** (0.000)	-0.0004*** (0.000)	-0.0005*** (0.000)	-0.0005*** (0.000)
Observations	7318	7318	7318	7484	7484
R-squared	0.178	0.175	0.178	0.147	0.145
BIC	-53273.086	-53245.208	-53281.896	-54011.437	-54005.572

4.5 Conclusion

In this section, we tested relationships of bidders according to their bids in LFT and NTN-B auctions through complex networks formed by bid similarity. Through an algorithm of group detection, we have have inspected two relevant metrics: coherence and exclusivity.

Those two together tell if a group bids similar to each other and are somewhat apart from others. Combined, they are a powerful tool to identify suspicious bidder behavior.

Although, as a relevant caveat, even in clear identifiable cases, these metrics should not be used alone and without context by authorities.

Nevertheless, complex networks for two samples of either NTN-B and LFT were compatible with our simulation of competitive bidders, without evidence of strategic behavior.

In addition to the previous analysis, we employed a standard empirical strategy through OLS estimates of profit in the bid-level on several controls. Differently than the preceding section, our data was segmented by types of bidders: dealers and non-dealers.

Our analysis of bids showed dealers tend to be more competitive in general, as expected by dealer's obligations and privileges, such as direct access to the Brazilian National Treasury and greenshoe benefits.

Dummy coefficients of types and formats of auctions on bid-level data are consistent with the previous revenue equivalence investigation.

5 Conclusions

This thesis is divided almost as if three separate short papers relating to issuance strategies in treasury auctions. The first addresses auction frequencies and 'V-shape' price patterns; the second examines revenue equivalence of auctions though policy experiments; and the third takes bid-level observations and apply network analysis and conventional approaches exploring demand features and competition. All of them are based on Brazilian National Treasury auctions data.

The findings highlight that risk-bearing capacity, encompassing both dealers and end investors, serves as a fundamental driver of the observed V-shape patterns. The analysis reveals that shorter auction cycles mitigate the magnitude of these price dynamics: the shift to a weekly auction schedule for NTN-F securities has effectively dampened the amplitude of pre-auction yield increases and post-auction decreases.

In addressing alternative explanations, we find no evidence to support the hypothesis that strategic behavior or slow-moving capital are primary contributors to V-shape patterns. Time-series analyses of hedge fund returns does not find seasonality related to auction cycles, aside from a potential weekend effect. These results effectively rule out speculation as a major driver, despite the significant role of hedge funds as risk-takers in the Brazilian market.

Investigations of liquidity and turnover dynamics in the secondary market indicate the shift to weekly auctions has not adversely impacted the traded volume or turnover of NTN-F securities. There was no crowding out in secondary markets by issuances, which would in turn lead to strategic purchases preferably in auctions. The observed turnover cycles are instead attributed to shifts between on-the-run and off-the-run securities driven by benchmark issuance.

In the second short paper, as most reduced models applied to revenue equivalence problems, we define a measure of auction profit or premium and check whether a regime shift on auction types were positive or not to the issuer. Recent changes in policies in treasury auctions by the Brazilian National Treasury ignited this research on which mechanism design yields greater revenue to the issuer.

Although there is no consensus in the literature, our results reveal weak evidence of lower bidder profits under uniform-price auctions; and, contrary to theoretical predictions, we find higher profits in hybrid auctions.

Finally, in the third paper our contribution encompasses two approaches. The first brings a modern tool to investigate how bids and bidders influence each other in treasury auctions and another more conventional one.

We simulated a random network of bidders calibrated by actual bid data and compare it with bidders networks from two samples for each bond: NTN-B and LFT.

The distribution of two measures were the core of this network analysis: coherence and exclusivity. Together they tell if a group of bidders have similar bids among each other and if they are different from the rest of the bidders. Our simulations of competitive bidders seem quite consistent with NTN-B and LFT networks of bidders.

We then take a conventional approach on demand features of certain groups of bidders, taking into account individual data of auction profits. Since our bid-level data has classifications of dealers and non-dealers, we group those and investigate if their obligations and benefits drive their bids more competitively. Our analysis of bids showed dealers tend to be more competitive in general, as expected by dealer's obligations and privileges, such as direct access to the Brazilian National Treasury and greenshoe benefits.

All of these policy shifts provide emerging markets issuers data on how to navigate in ever changing conditions. As Brazilian financial markets grew deeper and stock of debt rose significantly, treasury auctions and regulations became more frequent and complicated with several lines and different types of bonds. Our conclusions point towards a competitive market, with interdependent values, where traditional and uniform auction would be more appropriate. In addition, our data implies an increased amount of auctions constitutes a smoothing factor to risk supply.

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Annex

ANNEX A – Robustness Analysis LTN

Figure 9 – Residuals Panel Regressions LTN

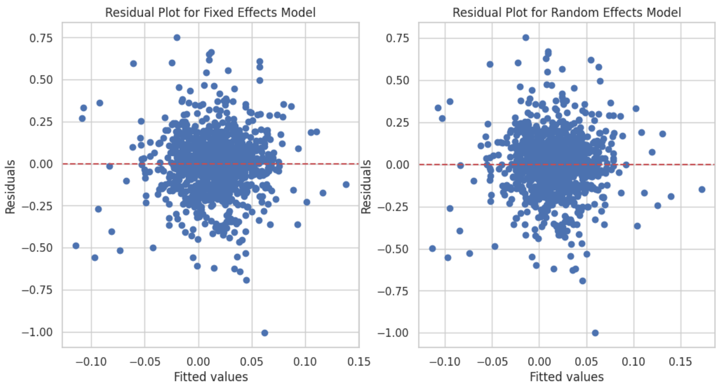


Table 37 – Fixed Effects and Random Effects Regression Results LTN - Delta+1

Variable	**Fixed Effects**	**Random Effects**
const	0.0593 (0.039)	0.0610 (0.039)
bid_to_cover	0.0049 (0.005)	0.0028 (0.005)
ac_ratio	-0.0191 (0.037)	-0.0172 (0.037)
dv01_of_bench	0.0156* (0.009)	0.0234*** (0.007)
anb-5	-0.0378** (0.018)	-0.0346** (0.018)
VIX	0.0013* (0.001)	0.0013* (0.001)
USD/BRL	-0.0159*** (0.006)	-0.0165*** (0.006)
R-squared	0.015	0.020
Observations	1213	1213

Table 38 – Fixed Effects and Random Effects Regression Results LTN - Delta+0

Variable	**Fixed Effects**	**Random Effects**
const	0.0079 (0.023)	0.0085 (0.023)
bid_to_cover	0.0040 (0.003)	0.0024 (0.003)
ac_ratio	-0.0164 (0.022)	-0.0144 (0.022)
dv01_of_bench	0.0127** (0.005)	0.0169*** (0.004)
anb-5	-0.0228** (0.010)	-0.0210** (0.010)
VIX	0.0024*** (0.000)	0.0023*** (0.000)
USD/BRL	-0.0098*** (0.003)	-0.0101*** (0.003)
R-squared	0.032	0.038
Observations	1213	1213

Table 39 – IV-2SLS LTN (Instruments: Lagged Variables)

Variável	**IV-2SLS**
const	0.0455 (0.723)
bid_to_cover	0.0012 (0.024)
ac_ratio	-0.0207 (0.774)
dv01_of_bench	0.0219 (0.020)
anb-5	-0.3238 (0.377)
VIX	0.0006 (0.002)
USD/BRL	-0.0097 (0.018)
R-squared	-0.085
Observations	1151

ANNEX B – Robustness Analysis NTN-F

Figure 10 – Residuals Panel Regressions NTN-F

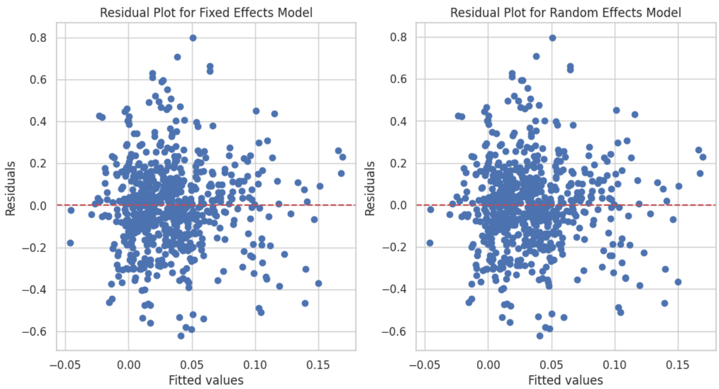


Table 40 – Fixed Effects and Random Effects Regression Results NTN-F - Delta+1

Variable	**Fixed Effects**	**Random Effects**
const	0.1033** (0.044)	0.1034** (0.044)
bid_to_cover	-0.0032 (0.010)	-0.0034 (0.010)
ac_ratio	-0.0769** (0.034)	-0.0767** (0.033)
dv01_of_bench	0.0022 (0.020)	0.0025 (0.020)
anb-5	-0.0502** (0.023)	-0.0501** (0.023)
VIX	0.0028** (0.001)	0.0028** (0.001)
USD/BRL	-0.0123 (0.009)	-0.0123 (0.009)
R-squared	0.025	0.025
Observations	680	680

Table 41 – Fixed Effects and Random Effects Regression Results NTN-F - Delta+0

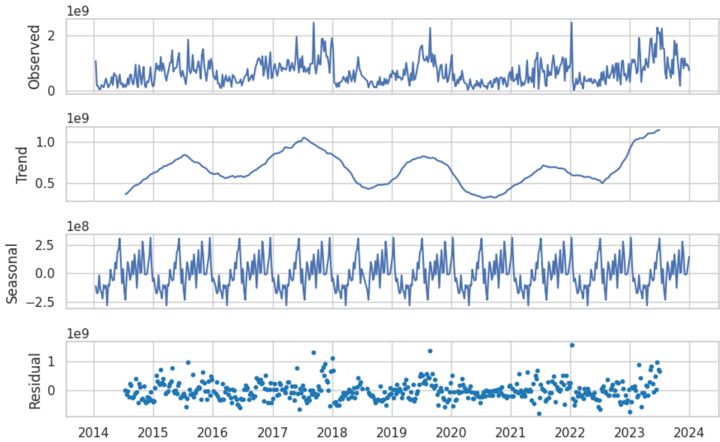
Variable	**Fixed Effects**	**Random Effects**
const	0.0021 (0.026)	0.0023 (0.026)
bid_to_cover	-0.0076 (0.006)	-0.0080 (0.006)
ac_ratio	0.0021 (0.020)	0.0025 (0.020)
dv01_of_bench	0.0084 (0.012)	0.0089 (0.012)
anb-5	-0.0198 (0.014)	-0.0197 (0.014)
VIX	0.0027*** (0.001)	0.0027*** (0.001)
USD/BRL	-0.0070 (0.005)	-0.0070 (0.005)
R-squared	0.025	0.025
Observations	680	680

Table 42 – IV-2SLS NTN-F (Instruments: Lagged Variables)

Variable	**IV-2SLS**
const	0.3904 (0.277)
bid_to_cover	0.0015 (0.046)
ac_ratio	-0.3533 (0.383)
dv01_of_bench	0.0139 (0.099)
anb-5	-0.1368 (0.285)
VIX	0.0027 (0.003)
USD/BRL	-0.0218 (0.021)
R-squared	-0.047
Observations	659

ANNEX C – NTN-F Traded Volume

Figure 11 – Time Series Decomposition Traded Volume NTN-F



ANNEX D – Robustness Analysis NTN-B

Table 43 – IV-2SLS NTN-B (Instruments: Lagged Variables)

Variable	**IV-2SLS**
const	-0.0242*** (0.009)
d_type	0.0051 (0.008)
d_hybrid	0.0120*** (0.003)
bid_to_cover	-0.0039*** (0.001)
anb-5	0.0191 (0.099)
USD/BRL	0.0002 (0.002)
VIX	-0.0000 (0.000)
R-squared	0.104
Observations	712

ANNEX E – Robustness Analysis LFT

Table 44 – IV-2SLS LFT (Instruments: Lagged Variables)

Variable	**IV-2SLS**
const	0.0247*** (0.007)
d_type	0.0001 (0.005)
ac_ratio	0.0047 (0.005)
bid_to_cover	-0.0032 (0.005)
anb-5	0.0118 (0.193)
USD/BRL	-0.0069** (0.003)
VIX	0.0002** (0.000)
R-squared	0.276
Observations	332